Financial Development and Housing Kuznets Curve*

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Abstract

This paper documents a housing Kuznets curve in the data: countries with the lowest and highest levels of financial development have relatively low house price-to-rent ratios, while countries falling in the middle range exhibit the highest housing prices relative to rents. This pattern is rationalized through a model that incorporates financial frictions, where both capital and land can be used as collateral assets. Financial development allows the economy to transit from the constrained to the unconstrained state. The price-to-rent ratio can rise above the levels in either of the steady states, providing an explanation for the observed housing Kuznets curve.

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1 Introduction

Housing market prices show considerable variation across countries and time periods. This has long intrigued economists because of the significant role housing markets play in modern macroeconomies. This paper explores the relationship between a country's financial system and its housing market dynamics, focusing on the impacts of financial development and financial frictions.

We document a novel inverse U-shaped relationship, termed "*the housing Kuznets curve*", between the degrees of financial development in various economies and their respective house price-to-rent ratios. We take three steps to establish this empirical finding. First, we present time-series evidence from four economies: the United States, Germany, Japan, and Greece. In each of these economies, we consistently observe an inverse U-shaped pattern over time between the price-to-rent ratio and private debt-to-GDP ratio. This suggests that the level of financial development may have a nonmonotonic impact on the dynamics of a country's housing prices.

We then document cross-sectional evidence on the housing Kuznets curve. We collect data on the average house price-to-rent ratio for a broad range of economies between 2015 and 2020 and examine how the house price-to-rent ratio varies based on a country's financial development level, proxied by the private debt-to-GDP ratio. Our result confirms the Kuznets curve in the cross section. In the group of economies with relatively low financial development, higher levels of financial development correspond to higher housing price-to-rent ratios. One could imagine comparing Eastern European countries such as Poland and Hungary with China, which displays both a higher level of financial development and a higher price-to-rent ratio. However, in the group of countries with relatively high financial development (such as Korea and the United States), further increases in financial development lead to lower price-to-rent ratios.

Finally, we conduct a panel regression analysis, considering a quadratic function of the private debt-to-GDP ratio while controlling for various factors such as GDP per capita, inequality, and country fixed effects. Again, we identify a statistically significant Kuznets curve relationship, where the price-to-rent ratio initially rises and then falls as the debt-to-GDP ratio changes.

To explain the housing Kuznets curve, we introduce a macroeconomic model that incorporates a housing sector and financial frictions. In this model, there are two types of assets: capital and land, which serve as both production inputs and collateral. We assume that firms can borrow to finance investments and have some capacity to borrow without collateral (such as credit lines granted by banks). When borrowing exceeds this capacity, collaterals such as capital and land become relevant. We show that the model can exhibit two stable steady states:

one corresponding to a standard neoclassical real business cycle model and another involving a binding financial constraint, leading to a land bubble and a relatively higher price-to-rent ratio. In our model, the multiplicity comes from the occasionally binding collateral constraint, which leads to an S-shaped policy function with *a unique recursive equilibrium*. In our model, there are not multiple equilibria as in Schmitt-Grohé and Uribe (2021b) because in our model different steady states feature different capital stocks, while Schmitt-Grohé and Uribe (2021b) consider a small open economy without capital. Schmitt-Grohé and Uribe (2021a) also study a model with an occasionally binding collateral constraint and show that an occasionally binding *flow* collateral constraint leads to endogenous cycles around one unique steady state. In our model, the occasionally binding *stock* collateral constraint leads to deterministic dynamics around multiple steady states.

We first conduct a steady-state analysis. We find that when capital and land are sufficiently complementary in the production function, the model can exhibit two stable steady states in the unique recursive equilibrium. In one of these steady states, the financial constraint is not binding, and since the financial friction is the only friction in the model, the steady state corresponds to the unique steady state in the standard neoclassic real business cycle model. When the credit constraint is binding, land can generate a liquidity premium, which means that the land price is higher than the discounted value of its marginal productivity. We define the discounted value of the liquidity premium as the land bubble, and bubbles lead to a higher price-to-rent ratio. We then compare the steady states under different credit constraint parameters and find that credit loosening has two competing effects on land bubbles. On the one hand, a looser credit constraint means that land can be used to borrow more, leading to a higher liquidity premium; on the other hand, credit loosening means that the constraint is less likely to bind, leading to a smaller liquidity premium. When the credit constraint is loose enough, the land bubble disappears. The two competing effects generate an inverse U-shaped relationship between credit loosening and land bubbles or the price-to-rent ratio, implying the "housing Kuznets curve".

We then study the dynamic properties of our model. After calibrating our model to the US economy, we find that due to the existence of credit constraints, the policy functions are S-shaped, implying two possible regions. If the initial capital stock is lower than a critical level, the economy will converge to the constrained low-level steady state, implying a long-lasting, bubbly recession. However, if the initial capital stock is high enough, or the credit constraint becomes loose enough, the economy will escape the trap and converge to the high-level unconstrained steady state.

As the last quantitative experiment with our model, we calculate the transition path during a general credit loosening, capturing general financial development. We find that during financial development, the land bubble first expands and then gradually contracts to zero; thus, the price-to-rent ratio first increases along the transition path and then decreases, consistent with the housing Kuznets curve observed over time for many countries. The underlying logic as follows: during the initial stages of credit loosening, as the loan-to-value ratio increases, each unit of land becomes more valuable because it can be used as collateral to borrow more funds. This increased the collateral value of the land and pushes up the housing price relative to its rental values. However, as the economy advances toward the frictionless steady state, the firm grows out of financial constraints, implying that the collateral premium diminishes to zero.

Throughout this transitional period, land prices and capital stocks initially experience rapid growth, which subsequently slows as they gradually approach the new steady state. Furthermore, we note a corresponding pattern in GDP growth rates: an initial increase followed by a decline during the financial development phase. This observation aligns with historical data, as in Japan where GDP growth was relatively modest in the 1960s, surged during the 1970s to 1980s, and later decelerated.

Our model can be used to think about the housing affordability issue in China. There has been concerns that with the past decades' strong economic and housing market growth the current housing prices in China are excessively high and not affordable to the working class. Our model suggests that the housing premium would diminish with further financial development, suggesting that liberalizing the financial market could be a viable approach to addressing the housing affordability challenge.

Literature Review There is a large literature on land or house price dynamics and macroeconomic fluctuations. Davis and Heathcote (2005) empirically demonstrates that the stock of residential land is large, and most price fluctuations in housing are due to land price fluctuations. Iacoviello (2005) and Iacoviello and Neri (2010) are early studies that introduce land or housing as collateral into traditional macroeconomic models to study the relationship between land prices and the business cycle. Davis and Heathcote (2007) emphasize the relationship between residential and nonresidential investment and develop a model to explain the fluctuations in house prices and the real economy. Liu et al. (2013) and Liu et al. (2016) estimate models with land price dynamics and real business cycles and find that a land demand shock can explain a considerable share of the fluctuations in macroeconomic variables such as output, investment, and employment. However, Miao et al. (2020) argues that the discount shock explains most of the price-to-rent dynamics and the linkage between house price dynamics and the real business cycle. Dong et al. (2022a) establish a model with heterogeneous beliefs and interpret the housing demand shock as a result of credit expansion. Kaplan et al. (2020) build a model with multiple aggregate shocks and credit constraints with respect to residential mortgages to study the dynamics around the Great Recession and show that the housing price collapse is mainly due to a shift in beliefs. Recently, more studies have focused on the relationship among housing prices, financial markets, labor markets, and the business cycle, including He et al. (2015), Justiniano et al. (2015), Piazzesi and Schneider (2016), Guerrieri and Iacoviello (2017), Favilukis et al. (2017), Berger et al. (2018) and Dong (2023). In these studies, housing or land can serve as collateral to generate a liquidity premium or as an input factor in production. Other studies regard housing or land as a safe asset to store wealth and investigate the relationship between the housing market and the real economy. Chen and Wen (2017) create an OLG model where houses serve as a store of wealth and show that China's housing boom is a rational bubble emerging naturally from its economic transition. Dong et al. (2021) and Dong et al. (2022b) focus on the housing market in China and show that agents may turn to holding houses when economic growth slows or policy uncertainty increases. The literature mainly focuses on the relationship between house or land price dynamics and short-term business cycles, while there is less research on the medium or long-term impact. Arifovic et al. (2018), Eggertsson et al. (2019), Biswas et al. (2020) and Cai (2021) present models with frictions such as financial frictions and nominal rigidity and show that shocks may drive the economy into long-lasting secular stagnation. Hirano and Toda (2023) present a model with rational housing bubbles and study the long-term relationships between house prices, house rents and income growth. As a supplement to the existing literature, our study focuses not only on the medium-term dynamics with asymmetric propagation but also on the effect of financial development in the long term on the housing or land market.

There is also a large body of literature that focuses on a speculative land or housing bubble and the real economy. Arce and López-Salido (2011) and Zhao (2015) introduce rational speculative housing bubbles into the OLG model framework, where agents use housing as a store of wealth. Kocherlakota (2009) and Miao et al. (2015) use infinite-horizon models with financial frictions to illustrate the relationship between rational housing bubbles and the real economy and its policy implications. Using an OLG framework, Jiang et al. (2022) investigate the crowding-in and crowding-out effects of the housing bubble in China, where infrastructure is important. Recently, Hirano and Stiglitz (2022) present a model with land speculation where there may exist multiple equilibria and wobbly dynamics, and Dong et al. (2024) focus on the competition for productive resources between the housing and non-housing sectors. Other studies focus on the relationship between bubbles and long-term economic growth (Martin and Ventura, 2012; Miao and Wang, 2014; Hirano and Yanagawa, 2016; Guerron-Quintana et al., 2023). Compared to the bubble model, our model features only one recursive equilibrium and multiple steady states, so we do not encounter the problem of equilibrium refinement when calculating the dynamics.

The remainder of the paper proceeds as follows. Section 2 presents the main time-series and cross-sectional evidence of the paper. Section 3 presents the model. Sections 4 and 5 describe the steady-state analysis and the transitional dynamics of our model. Section 6 concludes the paper. Technical proofs, the model solution algorithm, and additional figures are relegated to the appendices.

2 Empirical Evidence

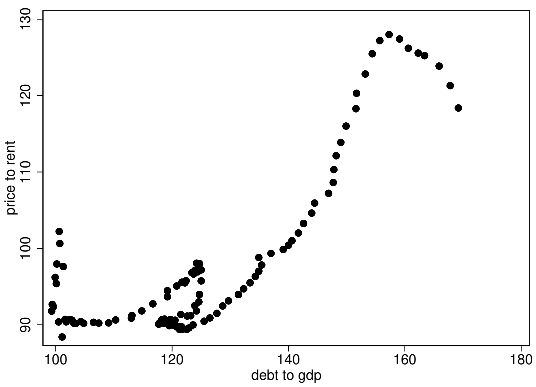
In this section, we present cross-sectional and time-series evidence that the price-to-rent ratio (or land/housing bubbles) may have a nonmonotonic relationship with the level of financial development.

Time-Series Evidence In this part, we investigate the relationship between financial development and the price-to-rent ratio for *one specific country*. Specifically, we use quarterly data on the price-to-rent ratio *index* from the OECD[1] and credit to the private nonfinancial sector from all sectors (% of GDP) from BIS total credit statistics. Using quarterly data, we draw the scatter plot of the debt-to-GDP ratio and price-to-rent index, and we find that for many prominent countries, there is a inverse U-shaped relationship between financial development and the price-to-rent ratio. We present some representative results in Figure 1. The three main economies in the Americas, Europe and East Asia, the US, Germany and Japan, feature this inverse U-shaped relationship between 1980 and 2008 before the Global Financial Crisis. As a medium-sized economy, Greece also features this inverse U-shaped relationship between 1997 and 2021. This feature occurs when the ratio of private debt to GDP exceeds 100%. As the level of financial development increases, which is proxied by a higher private debt-to-GDP ratio, the price-to-rent ratio first increases and then decreases.

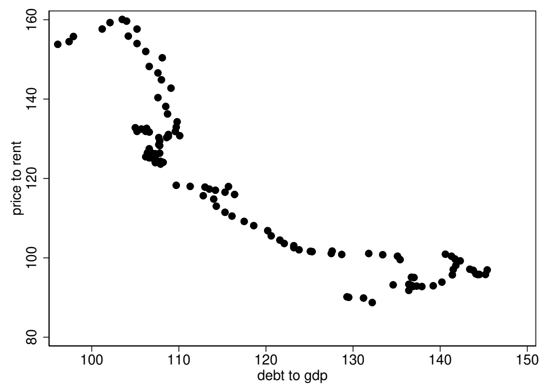
Cross-Sectional Evidence We next consider a cross-sectional analysis, where we compare the price-to-rent ratios of *different countries with different levels of financial development*. For the price-to-rent ratio, we use the annual price-to-rent ratio in the city centers of different countries from Numbeo[2]. As a proxy for the level of financial development, we use annual data on private credit from deposit banks and other financial institutions (% of GDP) from the World Financial

[1] Data source: https://data.oecd.org/price/housing-prices.htm.

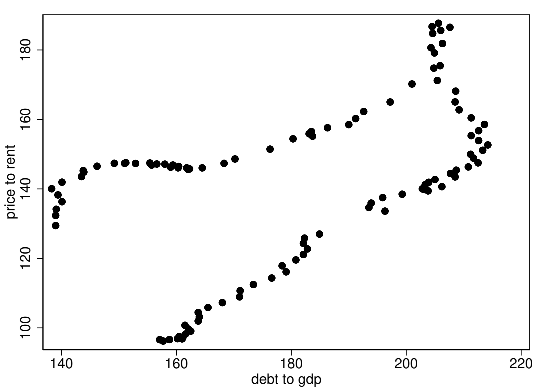
[2] Data source: https://www.numbeo.com/property-investment/rankings_by_country.jsp?title=2015&displayColumn=3. The price-to-rent ratio here is defined as follows: *"Price to Rent Ratio is the average cost of ownership divided by the received rent income (if buying to let) or the estimated rent that would be paid if renting (if buying to reside). Lower values suggest that it is better to buy rather than rent, and higher values suggest that it is better to rent rather than buy. Our formula to estimate rent per square meter assumes a 1 bedroom apt has 50 square meters and a 3 bedroom apartment has 110 square meters. It doesn't take into account taxes or maintenance fees"* (source: https://www.numbeo.com/property-investment/indicators_explained.jsp).



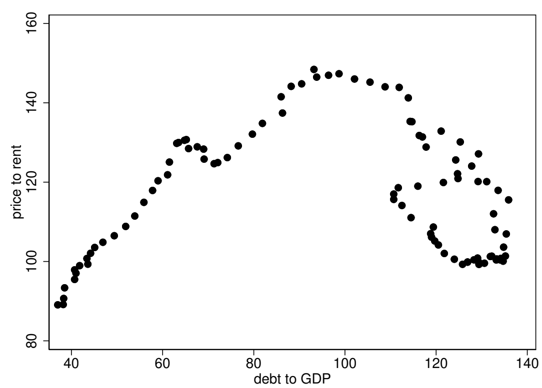
(a) USA: 1980-2008



(b) Germany: 1980-2008



(c) Japan: 1980-2008



(d) Greece: 1997-2021

Figure 1: Financial Development and Price-to-Rent Dynamics

Development Database of the World Bank. In this analysis, we focus on prominent economies, including the members and potential members of the Euro Zone (including the UK), China, the US, Hong Kong and Korea. In Appendix C, we list the countries used in our cross-sectional analysis, including the EU members and potential members, the US, Hong Kong, Korea, and China. First, we draw a bubble plot between the average price-to-rent ratio from 2015 to 2020 and the corresponding average financial development level indicator, using the average aggregate GDP as a weight. The results are shown in Figure 2, and the red dashed line is the fitted quadratic trend line. We find that there is a significant inverse U-shaped relationship between the financial development level and the price-to-rent ratio. When the level of financial development is relatively low (Eastern European countries such as Poland and Hungary), countries with a higher private debt-to-GDP ratio tend to have a higher price-to-rent ratio. However, when the financial development level is relatively high, say a private debt-to-GDP ratio above 150%, we find that countries with higher debt-to-GDP ratios instead tend to feature a lower price-to-rent ratio, such as the UK, Denmark and the US. In Appendix C, we provide additional cross-sectional empirical results without weights.

We also use panel regression to analyze the relationship between financial development and the price-to-rent ratio and obtain the same results. Specifically, we use panel data from 2013 to 2020 and run the following panel regression:

$$PTR_{i,t} = \beta_0 + \beta_1 DTG_{i,t} + \beta_2 DTG_{i,t}^2 + \gamma X_{i,t} + u_i + v_t + \epsilon_{i,t}, \tag{1}$$

where $PTR_{i,t}$ and $DTG_{i,t}$ denote the price-to-rent ratio and debt-to-GDP ratio for country i in year t, respectively. To exploit the inverse U-shaped relationship, we introduce the squared term of debt-to-GDP ratio. $X_{i,t}$ is the control variable, and in this investigation, we use GDP per capita as the main control. Since some studies also show that inequality may be correlated with the size of housing bubble and the price-to-rent ratio(Zhang et al., 2016; Zhang, 2016; Fang et al., 2016; Glaeser et al., 2017; Dong et al., 2021), we also consider including the Gini coefficient. Specifically, we use the Gini index from the World Development Indicators (WDI) database of the World Bank. We also control for country effects u_i and time fixed effects v_t. The regression results are shown in Table 1. We find that after controlling for country and time fixed effects and different control variables, the squared terms of the debt-to-GDP ratio always have a significant negative coefficient, which means that as the level of financial development increases, a more financially developed country will first feature a higher price-to-rent ratio and then feature a lower price-to-rent ratio, indicating a housing Kuznets curve.

From the above time-series and cross-sectional analysis, we consistently find that the finan-

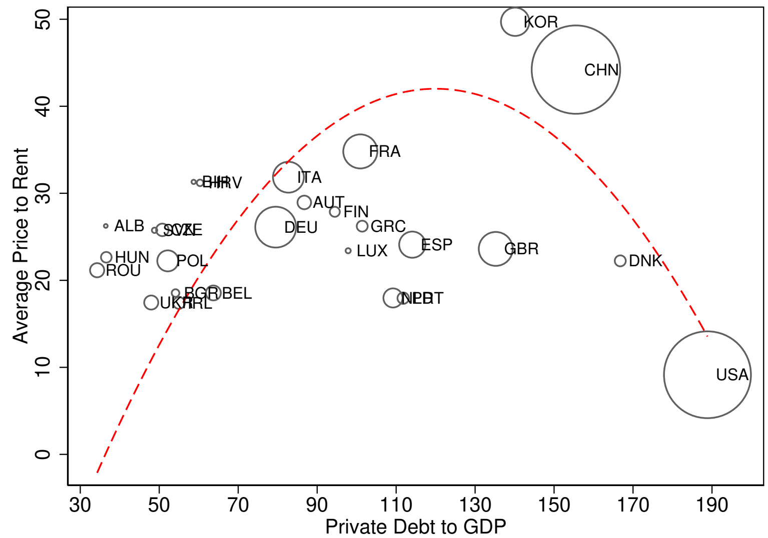
Table 1: Panel Regression Results

VARIABLES	(1) Price to Rent	(2) Price to Rent	(3) Price to Rent	(4) Price to Rent
debttoGDP	0.895***	0.876***	0.602***	0.507***
	(0.143)	(0.149)	(0.127)	(0.131)
debttoGDP2	-0.002***	-0.002***	-0.001***	-0.001**
	(0.000)	(0.000)	(0.000)	(0.000)
Country FE	YES	YES	YES	YES
Time FE	NO	NO	YES	YES
GDP per capita	YES	YES	YES	YES
Gini coefficient	No	YES	NO	YES

cial development level and housing (land) bubble have an inverse U-shaped relationship: when the financial development level is relatively low, a higher private debt-to-GDP ratio will lead to a higher price-to-rent ratio; when the financial system is well developed, a higher financial development level may lead to a smaller bubble and lower price-to-rent ratio. In the following part of the paper, we will present a simple macroeconomic model with financial frictions that can explain this housing Kuznets curve. We show that when the agents are financially constrained, a high price-to-rent ratio or land bubble is determined by the credit premium of land because it can be collateralized to obtain financing. As the financial market develops and the credit constraint becomes more relaxed, there are two competing effects: on the one hand, a loose credit constraint means that one unit of land can be collateralized to obtain more money, thus leading to a larger credit premium; on the other hand, if the credit constraint becomes loose enough, it is no longer tight in equilibrium, and the credit premium becomes zero. The two competing effects generate the inverse U-shaped credit premium of bubbly land: when the credit constraint is relatively tight, a looser constraint will lead to higher credit premium, while when the constraint is relatively loose, relaxing the credit constraint will crowd out the land bubble.

3 The Model

We consider the following version of a discrete-time, infinite-horizon model with a representative firm and a household. The key extension relative to a standard real business cycle model is that there are financial frictions in the model, and owning assets (such as capital and land) can help to relax the firm's borrowing constraint.



Note: In this figure, we draw a bubble plot of the average price-to-rent ratio from 2015 to 2020 against the average private debt-to-GDP ratio (which is a proxy for financial development in our paper). We use average aggregate GDP as the weight (the radius of the circles).

Figure 2: Financial Development and Price-to-Rent Ratio

Firm Sector There is a representative firm that combines capital and land to produce the final goods with the following production function:

$$F(z_t, k_{t-1}, l_t) = z_t \left[k_{t-1}^{\frac{\sigma-1}{\sigma}} + l_t^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{2}$$

where

$$\log z_t = (1 - \rho_z) \log z + \rho_z \log z_{t-1} + \varepsilon_{zt}. \tag{3}$$

Here, the parameter σ denotes the elasticity of substitution between land and capital. For simplicity, we assume that land and capital are the only two production inputs, and we demonstrate that the elasticity of substitution between the two are critical for our main results.

Capital accumulation follows the standard equation:

$$k_t = (1 - \delta) k_{t-1} + i_t. \tag{4}$$

We assume that the firm borrows to invest facing a credit constraint:

$$i_t \leq \xi_t p_t l_t + \eta_t k_t + \kappa_t, \tag{5}$$

where κ denotes the unsecured debt that does not need any collateral. We set the form of the borrowing constraint in the spirit of Kiyotaki and Moore (1997), He et al. (2015) and Cui et al. (2021)[3]. For simplicity, we assume that κ is an exogenous parameter. In the dynamic analysis, we assume that the credit constraint parameters may change and interpret these changes as credit shocks as in Jermann and Quadrini (2012).

As a result. the budget constraint for the representative firm is:

$$d_t + i_t + p_t(l_t - l_{t-1}) \leq F(z_t, k_{t-1}, l_t). \tag{6}$$

The firm chooses investments and landholding to maximize the discounted value of its profit flow. The value function of the firm can be recursively defined as:

$$V_{f,t}(k_{t-1}, l_{t-1}, \mathbf{s}_{f,t}) = \max_{d_t, k_t, l_t} d_t + \beta \mathbb{E}[\Lambda_{t,t+1} V_{f,t+1}(k_t, l_t, \mathbf{s}_{f,t+1})], \tag{7}$$

where $\mathbf{s}_{f,t} = \{z_t, \xi_t, \eta_t, \kappa_t\}$ is the vector of exogenous state variables for the firm, as we will consider changes in the aggregate borrowing capacity of the firm. The firm seeks to maximize its value subject to constraints 4, 5 and 6.

The Lagrangian function of the firms' optimization problem yields:

$$\begin{aligned} \mathcal{L} = \sum_{t=0}^{\infty} \beta^t \mathbb{E} \Lambda_t \left[F(z_t, k_{t-1}, l_t) - p_t(l_t - l_{t-1}) - i_t \right. \\ \left. + \mu_t \left(\xi p_t l_t + \eta k_t + \kappa - k_t + (1-\delta) k_{t-1} \right) \right], \end{aligned} \tag{8}$$

and the optimization conditions are:

$$\mu_t \left(\xi p_t l_t + \eta k_t + \kappa - i_t \right) = 0, \tag{9}$$

$$1 + (1-\eta)\mu_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[F_{k,t+1} + (1-\delta)(1+\mu_{t+1}) \right], \tag{10}$$

$$p_t = F_{l,t} + \mu_t \xi p_t + \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} p_{t+1}. \tag{11}$$

Household A representative household owns the firm and generates income from dividends and the firm's stock. The household chooses consumption c_t and share holding s_t to maximize its lifetime utility:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \quad u(c_t) = \begin{cases} \frac{c_t^{1-\gamma}}{1-\gamma}, & \text{if } \gamma \neq 1, \\ \log(c_t), & \text{if } \gamma = 1. \end{cases} \tag{12}$$

[3]Note that here we assume that this debt is intratemporal borrowing, and firms borrow only for investment. We can also extend this constraint to a working capital constraint, and the main results remain unchanged.

with the budget constraint:

$$c_t + s_t(V_{f,t} - d_t) \leq s_{t-1}V_{f,t}, \tag{13}$$

where $V_{f,t}$ is the market value of the firm.

The household's problem is given by:

$$V_{h,\tau}(s_{\tau-1}) = \max_{c_t, s_t} \mathbb{E}_\tau \sum_{t=\tau}^{\infty} \beta^{t-\tau} u(c_t), \tag{14}$$

subject to the sequential budget constraint 13, the optimization conditions are:

$$u'(c_t) = \lambda_t, \tag{15}$$

$$V_{f,t} = d_t + \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} V_{f,t+1}, \tag{16}$$

so we immediately obtain $\frac{\Lambda_{t+1}}{\Lambda_t} = \frac{\lambda_{t+1}}{\lambda_t}$, which means that the stochastic discount factor of firms equals the marginal utility of households.

Equilibrium Definition The recursive equilibrium of the model is defined as a sequence of endogenous variables $\{k_t, l_t, i_t, s_t\}$ and a price system $\{p_t, \mu_t, V_{f,t}\}$ such that:

1. Given the price system $\{p_t, \mu_t\}$, the firm maximizes its value subject to 4, 5 and 6;
2. Given the firm value $V_{f,t}$, the household maximizes its lifetime utility subject to 13;
3. Markets clear:
   (a) The stock market clears: $s_t = 1$;
   (b) The goods market clears: $z_t F(k_{t-1}, l_t) = c_t + i_t$.
4. The firm's discount factor is equal to the household's marginal utility.

The equilibrium system is given by:

$$\mu_t \left(\xi p_t l_t + \eta k_t + \kappa - i_t \right) = 0, \tag{17}$$

$$1 + (1-\eta)\mu_t = \beta \mathbb{E}_t \frac{u_{c,t+1}}{u_{c,t}} \left[z_t F_{k,t+1} + (1-\delta)(1+\mu_{t+1}) \right], \tag{18}$$

$$p_t = z_t F_{l,t} + \mu_t \xi p_t + \beta \mathbb{E}_t \frac{u_{c,t+1}}{u_{c,t}} p_{t+1}, \tag{19}$$

$$k_t = (1-\delta)k_{t-1} + i_t, \tag{20}$$

$$z_t F(k_{t-1}, l_t) = c_t + i_t. \tag{21}$$

4 Steady State Analysis

In this section, we study the properties of the steady states. In particular, we find that multiple steady states can exist: a bubbleless steady state where the credit constraint is slack, and a bubbly steady state where the constraint is binding and land provides a liquidity premium.

In steady state, we obtain $l = 1$, and the system is reduced to:

$$\mu\left(\xi p + \kappa + \eta k - i\right) = 0, \tag{22}$$

$$1 + (1-\eta)\mu = \beta[F_k + (1-\delta)(1+\mu)], \tag{23}$$

$$p = F_l + \mu\xi p + \beta p. \tag{24}$$

Bubbleless Land When the credit constraint does not bind, we have $\mu = 0$ so

$$\beta F_k = 1 - \beta(1-\delta) \Rightarrow k = k^f = \left[\left(\frac{1-\beta(1-\delta)}{\beta z}\right)^{\sigma-1} - 1\right]^{\frac{\sigma}{1-\sigma}}, \tag{25}$$

and the price of land is determined by

$$F_l - (1-\beta)p = 0. \tag{26}$$

Note that in this case, the price-to-rent ratio of land is just $\frac{1}{1-\beta}$, since the land rent denotes the marginal productivity of land in our model.

Bubbly Land When the credit constraint binds, we have $\mu > 0$, and the land pricing function becomes:

$$(1-\beta)p = F_l + \mu\xi p, \tag{27}$$

which implies that the price-to-rent ratio becomes $\frac{1}{1-\beta-\mu\xi} > \frac{1}{1-\beta}$. When the credit constraint is binding, the price-to-rent ratio can be higher than in the frictionless case since land can serve as collateral and generate a liquidity premium.

From the credit constraint and the fact that $\frac{b_f}{R} = 0$ in equilibrium, we have that $k = \frac{\xi p + \kappa}{\delta - \eta}$, and the corresponding multiplier is given by:

$$\mu = \frac{\beta(F_k + 1 - \delta) - 1}{1 - \eta - \beta(1-\delta)}. \tag{28}$$

From condition $\mu > 0$, we conclude that there exists a cutoff $p^* = \frac{(\delta-\eta)k^f - \kappa}{\xi}$, such that if and only if $p < p^*$ in steady state does the credit constraint bind, and when $p > p^*$, the constraint becomes slack with $k = k^f$.

From the FOC for land rental, we can define the land demand function as:

$$D(p) = \frac{1}{\beta}[(1-\eta)\mu(p) - \beta(1-\delta)(1+\mu(p)) + 1]k(p)^{\frac{1}{\sigma}} + \xi p\mu(p) - (1-\beta)p, \tag{29}$$

and any steady-state price must satisfy $D(p) = 0$. When $p \to 0$, it can be proven that $D(p) \to z\left[\left(\frac{\kappa}{\delta-\eta}\right)^{\frac{\sigma-1}{\sigma}} + 1\right]^{\frac{1}{\sigma-1}} > 0$ as long as $\kappa > 0$, and as $p \to \infty$, obviously $D(p) \to -\infty$, so there exists at least one steady state. We find that when the credit constraint parameters are in a plausible region, the model may feature multiple steady states.

Proposition 1. *As $\sigma \in (0,1)$, given the value of β, σ, z and δ, for each given η, there exist a cutoff value $\kappa^*(\eta)$ and three cutoff functions $\xi_1^*(\kappa)$, $\xi_2^*(\kappa)$ and $\xi_3^*(\kappa)$ such that for $\kappa < \kappa^*$:*

1. *if $\xi < \xi_1^*(\kappa)$, there only exists one bubbly steady state with $\mu > 0$;*
2. *if $\xi_1^*(\kappa) < \xi < \xi_2^*(\kappa)$, there exist multiple bubbly steady states with $\mu > 0$, two stable and one unstable;*
3. *if $\xi_2^*(\kappa) < \xi < \xi_3^*(\kappa)$, there exist multiple steady states, two bubbly with $\mu > 0$ and one unconstrained bubbleless steady state with $\mu = 0$ and $k = k^f$;*
4. *if $\xi > \xi_3^*(\kappa)$, there only exists one unconstrained bubbleless steady state with $\mu = 0$ and $k = k^f$.*

The three cutoff functions satisfy:

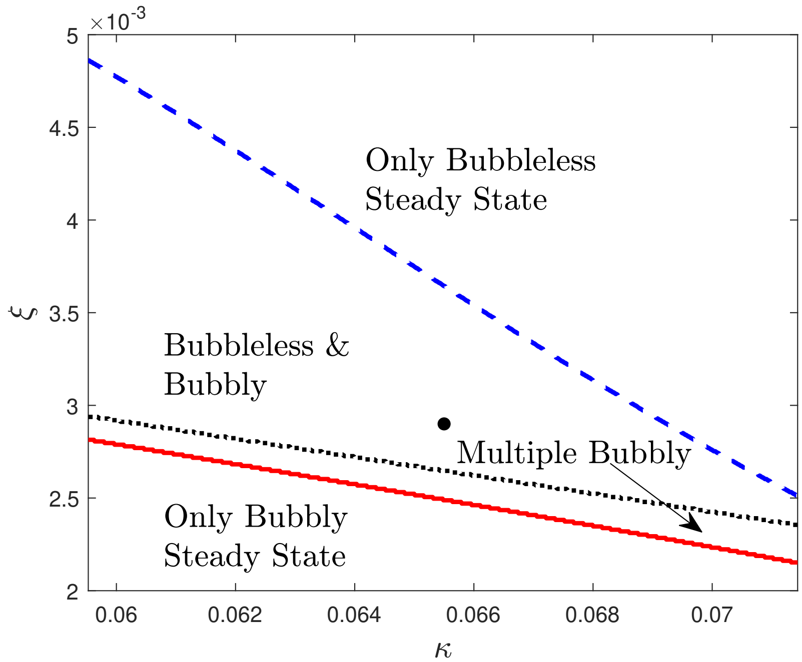
$$\xi_3^*(\kappa) \leq \frac{(1-\beta)[1-\beta(1-\delta)]}{\beta(z+1-\delta)-1}; \tag{30}$$

$$\xi_2^*(\kappa)p^f + \kappa \geq \delta k^f - k^f\eta; \tag{31}$$

and $0 < \xi_1^*(\kappa) < \xi_2^*(\kappa) < \xi_3^*(\kappa)$.

The proof of Proposition 1 is given in Appendix A. We depict the results of Proposition 1 in Figure 3 and take $\eta = 0$ as an example. In Appendix D, we take $\eta = 0.05$ and conduct the same exercise. The main results remain unchanged. Note that this holds if and only if the $\{\kappa, \xi\}$ pair is between the red line and the dashed blue line. For any given κ, if the credit constraint is too loose, it will never be binding, and there is only one bubbleless steady state. However, if the constraint is too tight, it will always bind, which means that the bubbleless steady state disappears.

Note that in Proposition 1, we assume that $\sigma \in (0,1)$, which means that capital and land are complementary. This is because when $\sigma > 1$ and p is close to zero, the marginal productivity of



Note: Here we keep $\eta = 0$, change the values of κ and ξ, and check the steady-state properties of our baseline model. The black dots denote the parameter values we use in the baseline model analysis as shown in Table 2.

Figure 3: Existence Region of Bubbly Steady State

capital $F_k = \left(k(p)^{\frac{1-\sigma}{\sigma}} + 1\right)^{\frac{1}{\sigma-1}}$ will diverge to infinity because the two factors are substitutes. As a result, $\mu = \frac{\beta(F_k+1-\delta)-1}{1-\eta-\beta(1-\delta)}$ diverges and the land demand function slopes upward when $p \to 0$, which means that the constrained steady states disappear. However, when $\sigma < 1$ and capital and land are complementary, the marginal productivity of capital is constrained by the total supply of capital $l = 1$, which means that the liquidity premium $\mu(p)$ is relatively constrained when p is near zero. As a result, the demand function still slopes downward when p is small, and there may be multiple steady states.

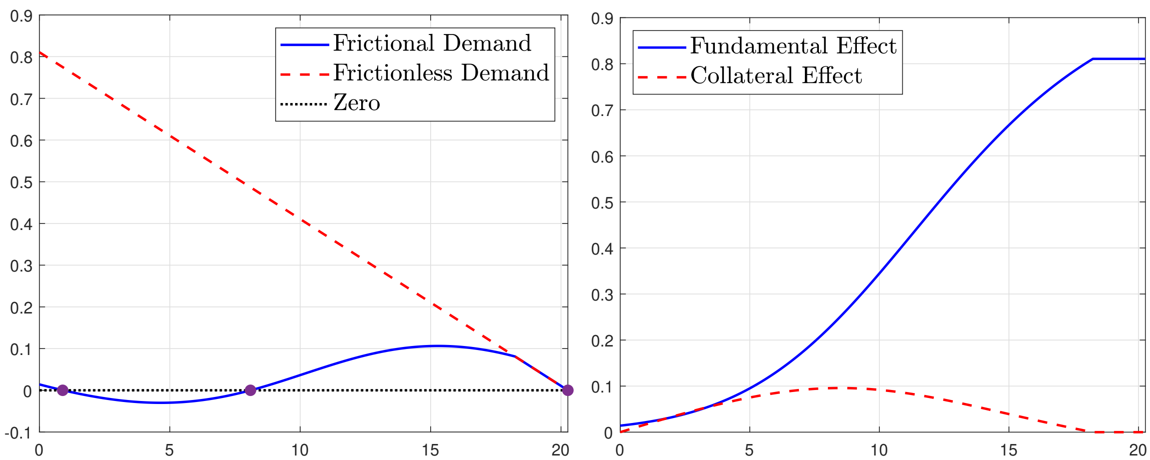
We can decompose $D(p)$ into three parts as follows:

$$D(p) = \underbrace{\frac{(1-\eta)\mu(p) - \beta(1-\delta)(1+\mu(p)) + 1}{\beta} k(p)^{\frac{1}{\sigma}}}_{\text{Fundamental effect}} + \underbrace{\xi p \mu(p)}_{\text{Collateral effect}} - \underbrace{(1-\beta)p}_{\text{Price effect}}, \tag{32}$$

and the steady state corresponds to $D(p) = 0$. When the credit constraint is binding, $\mu(p) > 0$, the fundamental effect always increases with p since a higher p leads to a higher capital stock k and higher land productivity. However, the collateral effect $\xi p \mu(p)$ is a nonmonotonic function of the land price p since $\mu(p)$ is smaller when p is higher, and when the price is high enough, the credit constraint is no longer binding, and hence $\mu(p) = 0$. When p is small, both the fundamental and collateral effects increase with p, while their slope may be small with a low ξ

and low η, and the decreasing price effect does not vary, which means that the demand function is downward sloping. As p increases, the fundamental effect increases rapidly and dominates, so that the demand function may become upward sloping: due to the complementarity of capital and land, a higher land price means a higher collateral value, and more capital leads to even higher land demand. However, when p is large enough, the credit constraint is not binding and the demand function is reduced to the frictionless case with $D(p) = F_l^f - (1-\beta)p$.

From the analysis above, the nonmonotonic land demand function may feature multiple steady states in one unique recursive equilibrium. We depict the demand function $D(p)$ and its decomposition in Figure 4:



(a) Demand Function and Steady States (b) Decomposition of Demand Function

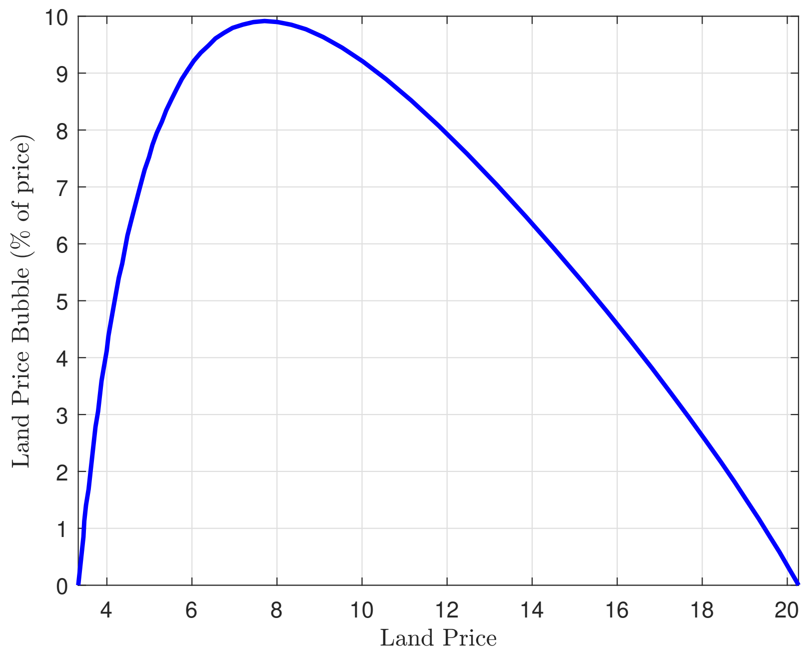
Note: Here, we use the parameters in Table 2 and calculate the land demand function as in Equation 32. The blue solid line denote the frictional demand function, and the red dashed line denotes the frictionless demand function $D(p) = F_l^f - (1-\beta)p$. The three dots denote three possible steady states, two bubbly (constrained) and one bubbleless (not constrained). In Panel (b) we depict the decomposition of the land demand function, where the blue solid line denotes the fundamental effect, and the red dashed line denote the collateral effect.

Figure 4: Demand Function with Three Steady States

From the land pricing function, it is natural to define the land bubble as the difference between the land price and its fundamental:

$$bubble = p - \frac{1}{1-\beta}F_l = \frac{1}{1-\beta}\mu\xi p, \tag{33}$$

for comparative statics analysis, we change the value of ξ and calculate the land price and land bubble in the steady states, as depicted in Figure 5 where we show the steady state bubble as a percentage of the land price $\frac{1}{1-\beta}\mu\xi$ and as a function of the steady-state land price p. During credit loosening, the land bubble first increases with the land price and then decreases to zero. This inverse U-shaped relationship is the result of two competing effects. On the one hand, a higher land price means that firms can raise more money given the credit constraint; thus land can generate a higher liquidity premium and support a larger land bubble. On the other hand, as the steady-state price increases, the credit constraint is less likely to be binding in the steady state, so the steady-state value of μ tends to decrease, and when the steady-state land price increases to p^f, the credit constraint never binds, and there is only one bubbleless steady state.



Note: In this figure, we change the value of ξ from 0 to 0.0041 and calculate the land price and land bubble in the steady state. Here, we choose the stable constrained steady state once it exists. All other parameters are the same as in Table 2.

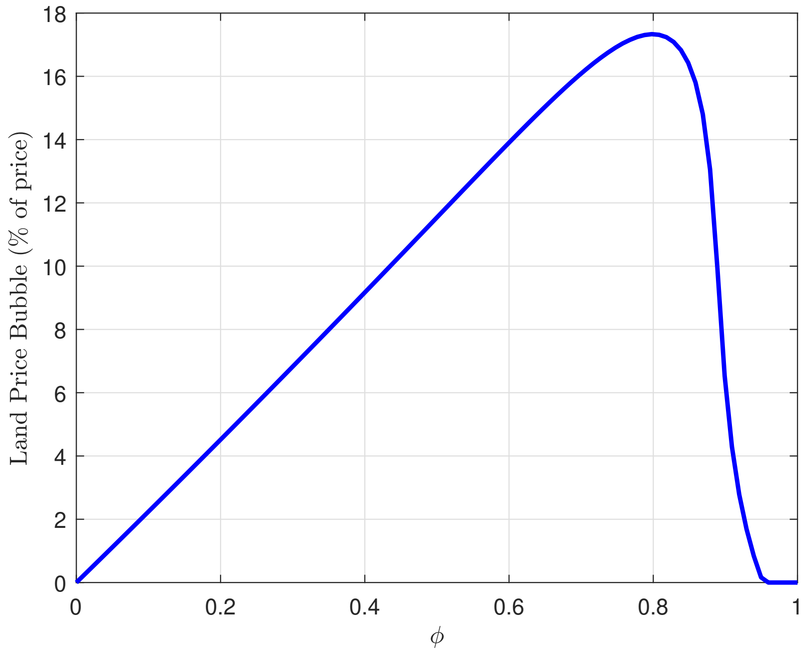
Figure 5: Land Price and Land Bubble

In the above analysis, we only change one parameter in the credit constraint, ξ. However, to capture the development of the financial system, the three parameters $\{\xi, \eta, \kappa\}$ must increase together. To conduct this exercise, we assume that the credit constraint is given by:

$$i_t \leq \phi(\xi_0 p_t l_t + \eta_0 k_t + \kappa_0), \tag{34}$$

where $(\xi_0, \eta_0, \kappa_0)$ implies a high-development stage with no land bubble (there only exists one bubbleless steady state with ξ_0, η_0 and κ_0), and ϕ is a proxy for overall financial development: as ϕ increases, private sector debt increases. We change the value of ϕ from 0 to 1 and calculate the land bubble in steady state. The result is shown in Figure 6, which implies that the land bubble first increases and then decreases with financial development, which is consistent with our empirical finding.

This inverse U-shaped relationship with the level of financial development ϕ is the result



Note: In this figure, we change the value of ϕ in Equation 34 from 0 to 1 with $\xi_0 = 0.0015$, $\eta_0 = 0.01$ and $\kappa_0 = 0.0833$. All other parameters are the same as in our baseline calibration (see the next section for details on how we calibrate the model). We choose $\{\xi_0, \kappa_0, \eta_0\}$ so that there is only one unconstrained steady state when $\phi = 1$. To draw this figure, we choose the stable constrained steady state when it exists.

Figure 6: Financial Development and Land Bubble

of two competing effects of financial development. On the one hand, a larger ϕ means that the firm can collateralize land to obtain more money for a given land price p, thus supporting a larger bubble. On the other hand, as ϕ increases, the credit constraint is less likely to bind in steady state. When the constraint is slack enough, there will only be one bubbleless steady state, and the bubble disappears.

5 Model Dynamics

To further illustrate the dynamics of our model, we consider the law of motion for capital. Note that the dynamic system of this model is given by (note that $l_t = 1$ in equilibrium):

$$\mu_t\left(\xi p_t l_t + \eta k_t + \kappa - i_t\right) = 0 \tag{35}$$

$$1 + (1-\eta)\mu_t = \beta \mathbb{E}_t \frac{u_{c,t+1}}{u_{c,t}}\left[z_t F_{k,t+1} + (1-\delta)(1+\mu_{t+1})\right] \tag{36}$$

$$p_t = z_t F_{l,t} + \mu_t \xi p_t + \beta \mathbb{E}_t \frac{u_{c,t+1}}{u_{c,t}} p_{t+1} \tag{37}$$

$$k_t = (1-\delta)k_{t-1} + i_t \tag{38}$$

$$z_t F(k_{t-1}, l_t) = c_t + i_t \tag{39}$$

where $F_{k,t} = \left[k_{t-1}^{\frac{1-\sigma}{\sigma}} + 1\right]^{\frac{1}{\sigma-1}}$ and $F_{l,t} = \left[k_{t-1}^{\frac{\sigma-1}{\sigma}} + 1\right]^{\frac{1}{\sigma-1}}$. These equations represent the complementary slackness condition for the collateral constraint, the Euler equations for capital, land, and bond investments, the law of motion for capital, and the resource constraint, respectively.

Note that for each firm, k_{t-1} is the state variable while k_t and l_t are the choice variables. We start by considering the case without aggregate uncertainty, i.e., how the economy evolves given an initial capital stock.

Note that there is an occasionally binding constraint problem because μ_t may become zero at some levels of state variables, and we need to use the global method to solve this model. Appendix B documents the algorithm to solve for the global dynamics of the model.

Calibration For the constrained steady state to exist, η must be sufficiently small. As the capital depreciation rate δ is generally small in macroeconomic models, we set $\eta = 0$ as our benchmark. Changing the value of η within a plausible range will not change our main results. In Appendix D, we consider $\eta = 0.05$ and conduct the same exercise. The main results remain unchanged. We set $\beta = 0.96$, $\delta = 0.1$, and $\gamma = 1$ as in standard macroeconomic models and normalize $z = 1$. We then choose the value of σ to match the frictionless steady-state capital-to-output ratio. To construct this ratio in the real US economy, we use the perpetual inventory method. We assume that the economy is on its balanced growth path, with capital growth rate g_K, and we have:

$$g_K = \frac{K_{t+1} - K_t}{K_t} = \frac{I_t}{K_t} - \delta \tag{40}$$

so we can use the investment-to-output ratio to back out the capital-to-output ratio. We use investment data from Federal Reserve Economic Data (FRED) and calculate the average compound annual growth rate of gross private domestic investment from 1980 to 2007 (after seasonal adjustment) before the crisis, which yields $g_I = 6.1\%$. We use g_I to approximate g_K, $\delta = 0.1$ as in our model, and $\frac{I_t}{Y_t} = 20\%$ as in the literature, which yields:

$$\frac{K_t}{Y_t} = \frac{I_t}{Y_t}\frac{1}{g + \delta} = 1.24, \tag{41}$$

and we set $\sigma = 0.1$, which leads to $\frac{K^f}{Y^f} = 1.22$ in our model.[4]

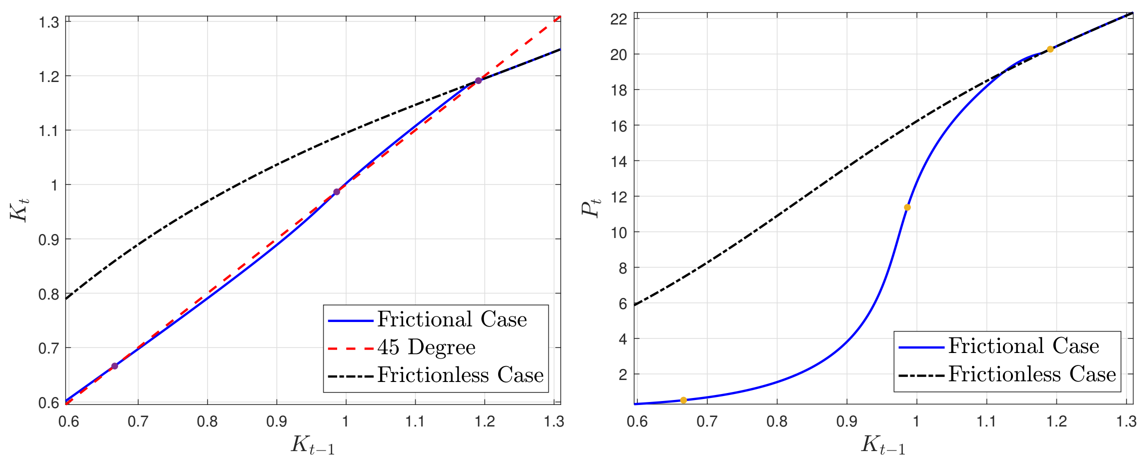
[4]There is a large literature that estimates the elasticity of substitution between capital and labor. Chirinko (1993) reviews the literature and shows that the estimation of σ is between 0 and 0.3 using the Euler equation method with investment data, and Chirinko (2008) summarizes different methods for estimating σ and shows that most of the literature suggests σ lie between 0 and 0.7. Regarding the elasticity of substitution between capital and land, the literature mainly focuses on the case of housing production. McDonald (1981) provides a review of the literature and states that the estimation of σ in housing production is between 0.36 and 1.2, while most values are less than 1. Batisani and Yarnal (2011) estimate the housing production function using both the

For the credit constraint, we set $\kappa = 0.0655$ such that the fraction of unsecured debt to total private debt in our model is 55%, consistent with the estimation of Azariadis (2018). We finally set $\xi = 0.0029$ so that the decline in investment between the constrained steady state (recession) and the frictionless steady state (before recession) is 44%, consistent with the data (see Cai, 2021). We summarize our calibration in Table 2:

Table 2: Parameters

Parameters	Meaning	Value	Target
β	Discount factor	0.96	Annual model
δ_k	Depreciation rate of capital	0.1	Standard
γ	Parameter in utility function	1	Standard
z	Total factor productivity	1	Normalization
σ	Complementarity of capital and land	0.1	Capital to output ratio
κ	Unsecured debt	0.0655	Share of unsecured debt for firms
ξ	Pledgeabililty of land	0.0029	Post-recession investment gap

Given the value of the parameters, we can solve for the policy function of the model. The policy functions of capital k_t and price p_t are shown in Figure 7:



(a) Capital Policy Function (b) Price Policy Function

Note: Here, we use the parameters in Table 2 and calculate the policy function based on the algorithm described in Appendix B. Here, we show both the capital and price policy functions. The blue solid lines denote the frictional policy functions, while the black dashed lines denote the frictionless policy functions. The dots denote the three possible steady states.

Figure 7: Policy Function

Under the baseline calibration, we obtain an S-shaped policy function with varying curvature and three steady states. The S-shaped capital policy function stems from the S-shaped

CES and VES function forms and show that σ is between 0.003 and 0.275 using the CES production function.

price policy function, which is a result of the credit constraint. Note that the Euler equations for land and capital are:

$$p_t = \underbrace{F_{l,t}}_{\text{Fundamental effect}} + \underbrace{\mu_t \xi p_t}_{\text{Collateral premium}} + \underbrace{\beta \mathbb{E}_t \frac{u_{c,t+1}}{u_{c,t}} p_{t+1}}_{\text{Expectation effect}}, \tag{42}$$

and

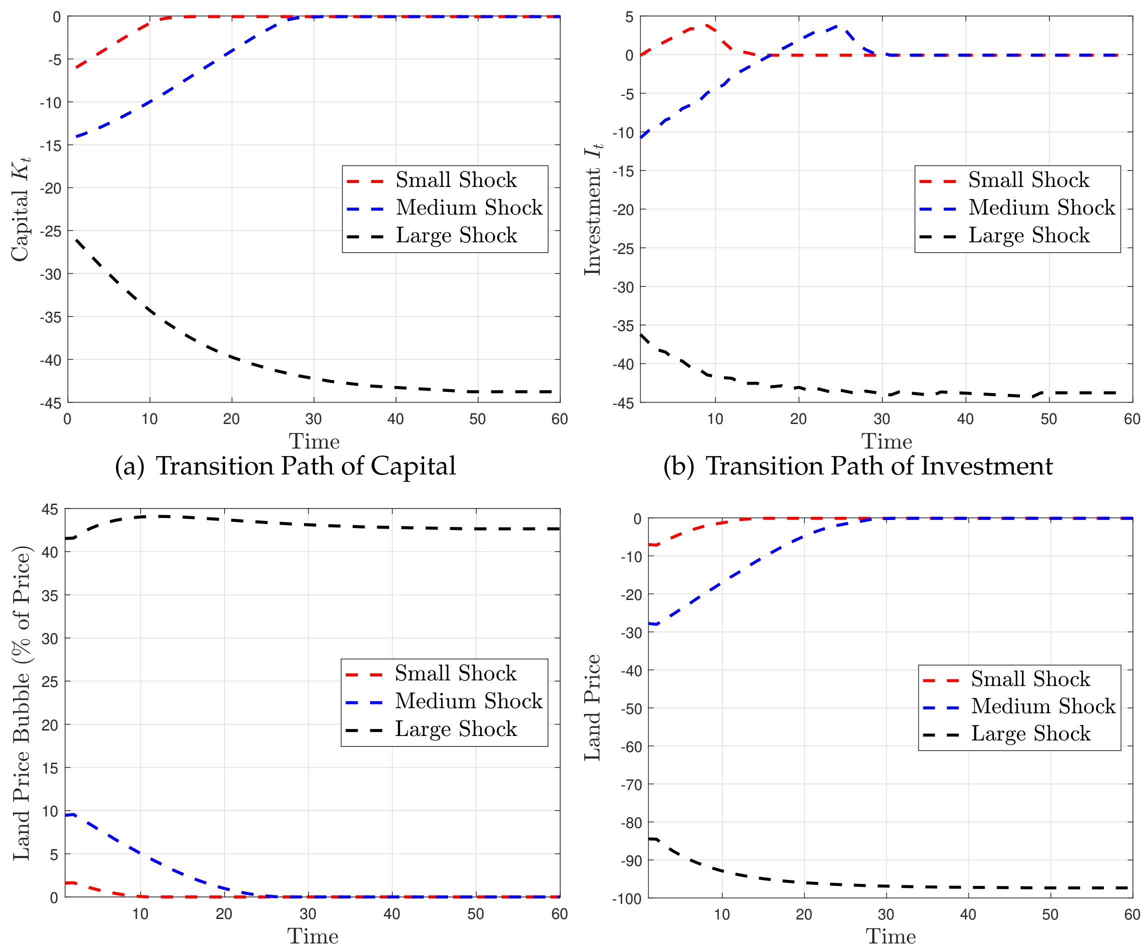
$$1 + (1-\eta)\mu_t = \beta \mathbb{E}_t \frac{u_{c,t+1}}{u_{c,t}} \left[F_{k,t+1} + (1-\delta)(1+\mu_{t+1}) \right]. \tag{43}$$

The above two equations determine the land price p_t given the expectation of p_{t+1}. In general, a higher expectation $\mathbb{E}_t \frac{u_{c,t+1}}{u_{c,t}} p_{t+1}$ will lead to a higher land price p_t. When the expectation of future housing price is high enough, the credit constraint is less likely to bind in the present, so a change in price expectation only affects the current land price through the traditional expectation effect $\mathbb{E}_t \frac{u_{c,t+1}}{u_{c,t}} p_{t+1}$. However, when the expectation is not large enough, μ_{t+1} is likely to be large, which means that the current-period credit constraint is also likely to be binding. As a result, the change in expectation has two other effects through the collateral premium: first, a high expectation leads to higher land price p_t and higher collateral premium, which leads to a convex pricing function; second, a high expectation means that the current-period credit constraint is less likely to be binding, so that μ_t is lower, inducing a concave pricing function. As price expectation increases, the first effect first dominates while the second dominates thereafter, so the pricing function is first convex and then concave. The role of external credit κ is to ensure that even if the firm does not have any assets, it can obtain some credit to invest; thus, the capital policy function is bounded away from zero.

5.1 Asymmetric Propagation and the Bubbly Recession

In this subsection, we explore the business cycle property of the model. We focus on the economy that is initially in the high-level fundamental steady state, which is analogous to developed economies. We assume that they face different scales of negative shocks to the capital stock, which can be interpreted as a capital quality or quantity shock in Gertler and Karadi (2011) and Gourio (2013) or as a depreciation rate shock in Gomes et al. (2016) and Barro (2023). Similar to Cai (2021), different levels of initial capital stock imply different levels of recession. Given the policy function, we can solve for the transition paths after different scales of negative shocks, and the model exhibits asymmetric propagation in Figure 8.

When the recession is small, the economy does not fall into the low-level region and will eventually return to the high-level fundamental value. Along the transition path, the investment first increases and then decreases to the initial value, and the land price first drops



(a) Transition Path of Capital

(b) Transition Path of Investment

(c) Transition Path of Land Bubble

(d) Transition Path of Land Price

Figure 8: Transition Path After Different Levels of Capital Shocks

and then increases to the initial value. For the dynamic analysis, we define the bubble as $bubble = \frac{1}{1-\beta}\mu_t\xi_t p_t$ analogous to steady-state analysis, and the land bubble can be seen as the discounted value of the collateral premium of the land. We find that if the recession scale is small, the land bubble first emerges due to a binding constraint and then decreases to zero as the economy converges to the initial steady state.

However, when the recession is large, things are different: due to the S-shaped policy function, a large negative shock to the capital stock may drive the economy through the unstable steady state and into the low-level bubbly region, and the economy will eventually converge to the low-level constrained steady state with low capital, low investment, and low land price. Along the transition path, a land bubble emerges and never disappears, and the economy exhibits a bubbly recession.

After the Great Recession, authorities recognized that a loose credit constraint can lead to high financial risk, and an increasing number of countries implement credit-tightening macroprudential policies that restrict loan-to-value ratios in the economy. However, the S-shaped policy function and asymmetric propagation in our model imply that credit tightening should not be too large and itself may lead to bubbly recessions. When the scale of credit tightening is small, the economy will remain in its initial fundamental steady state and no bubble occurs. However, our S-shaped policy function implies that if the scale of credit tightening is large enough, the fundamental steady state disappears, credit tightening will lead to a recession and a land bubble will emerge, as shown in Figure 9.

5.2 Big Push and Credit Loosening

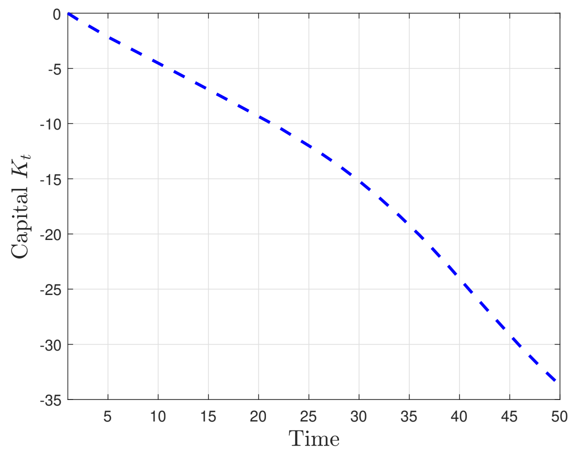
In this section, we consider an economy that starts at the low-level steady state, representing underdeveloped countries that are severely borrowing constrained. We then ask the following question: how can these countries move out of the poverty trap and reach the high-level fundamental steady state?

The first experiment we conduct is to consider a big push by giving a one-shot external credit $\Delta\kappa$, which means that the credit constraint becomes:

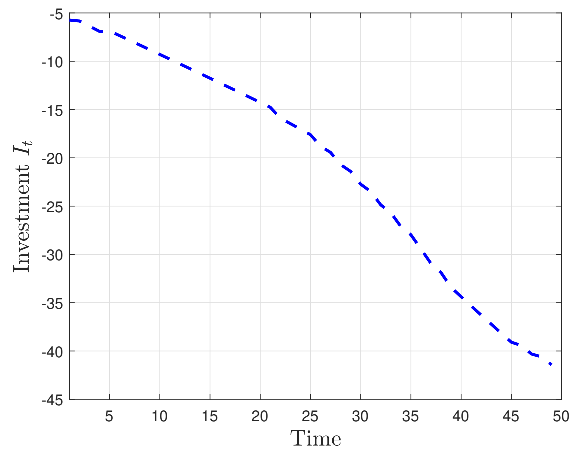
$$i_t \leq \xi p_t l_t + \eta k_t + \kappa + \Delta\kappa. \tag{44}$$

For simplicity, we assume that the credit constraint is binding even if after this one-time shock, the capital stock after this shock becomes:

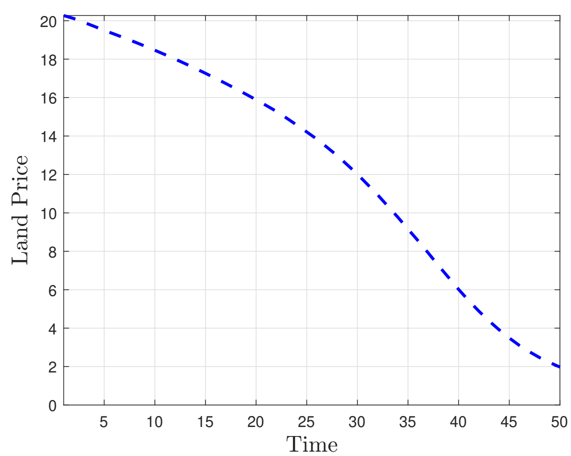
$$k_t' = (1-\delta)k_{t-1} + (\xi p_t l_t + \eta k_t' + \kappa + \Delta\kappa), \tag{45}$$



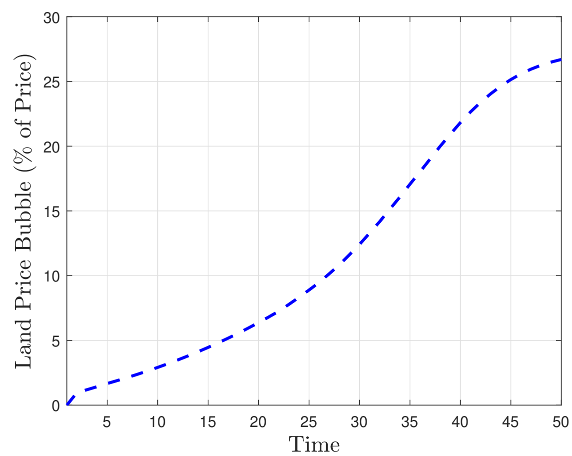
(a) Transition Path of Capital Stock



(b) Transition Path of Investment



(c) Transition Path of Land Price



(d) Transition Path of Land Bubble

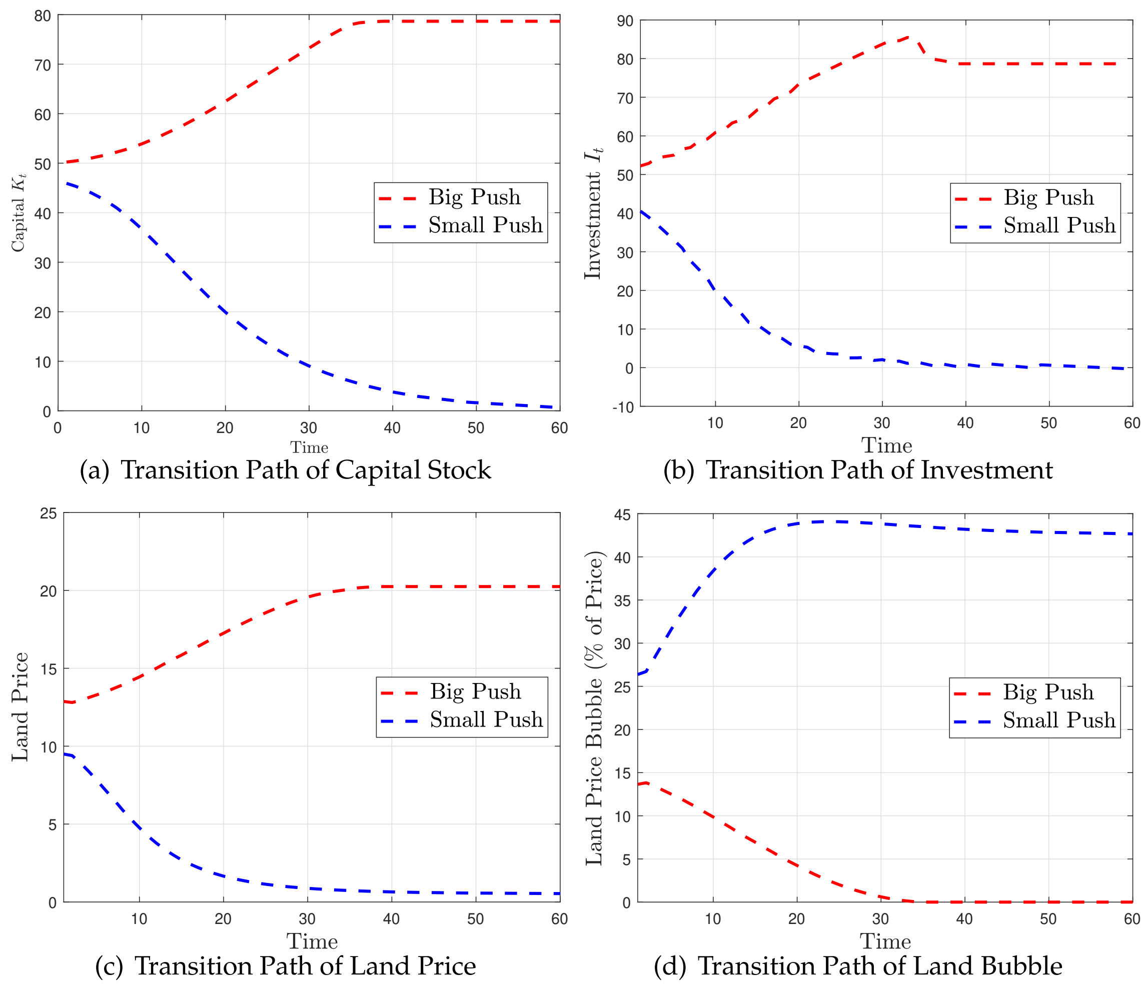
Note: Here, we decrease ξ by 20% and calculate the transition paths.

Figure 9: Credit Tightening and Bubbly Recession

so that

$$k_t' = \frac{(1-\delta)k_{t-1} + \xi p_t l_t + \kappa + \Delta\kappa}{1-\eta} = k_t + \frac{\Delta\kappa}{1-\eta}, \tag{46}$$

where $k_t = \frac{(1-\delta)k_{t-1}+\xi p_t l_t+\kappa}{1-\eta}$ denotes the capital stock without this credit shock. As a result, as long as the credit constraint is binding, this external credit is equivalent to a positive shock to the capital stock k_t. Similar to asymmetric propagation, we find that giving a small push by offer a small amount of external credit will lead to a temporary boom in the capital stock, output, investment, and land price. However, it is not enough for the economy to converge to the high-level steady state, and the boom eventually comes to an end. During the boom, the land price bubble first shrinks and then expands as the boom disappears, as shown in Figure 10.



(a) Transition Path of Capital Stock

(b) Transition Path of Investment

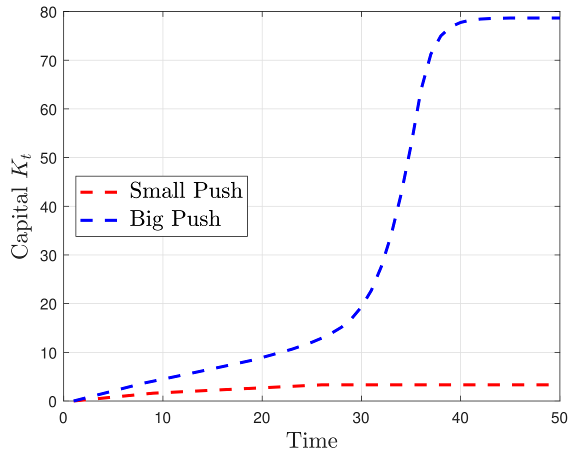
(c) Transition Path of Land Price

(d) Transition Path of Land Bubble

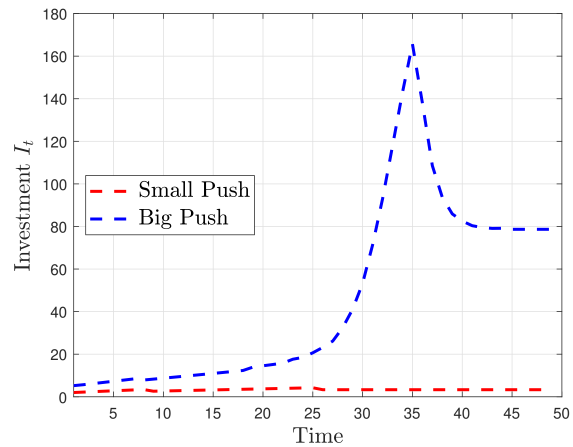
Note: For a small push, we assume that $\Delta\kappa = 0.48K_0 < K^* - K_0$, where K^* denotes the unstable medium steady state, and for a big push we assume that $\Delta\kappa = 0.5K_0 > K^* - K_0$ so that the economy will converge to the bubbleless steady state.

Figure 10: Results of One-Shot External Credit Push

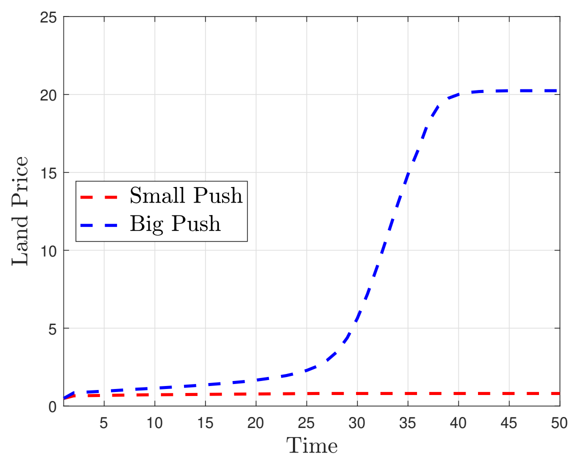
Another way to push the economy out of recession is to increase the colleteralizability of assets. On the one hand, when the credit constraint still binds, loosening credit by increasing ξ will help the firm obtain more financing for investment, which is equivalent to a positive shock to the capital stock. Furthermore, a higher level of ξ will lead to a higher land price, strengthening this effect. On the other hand, as ξ increases, the credit constraint is less likely to bind, and when ξ is large enough, there can only be one bubbleless steady state. In Figure 11, we show the results after small-scale and large-scale credit loosening.



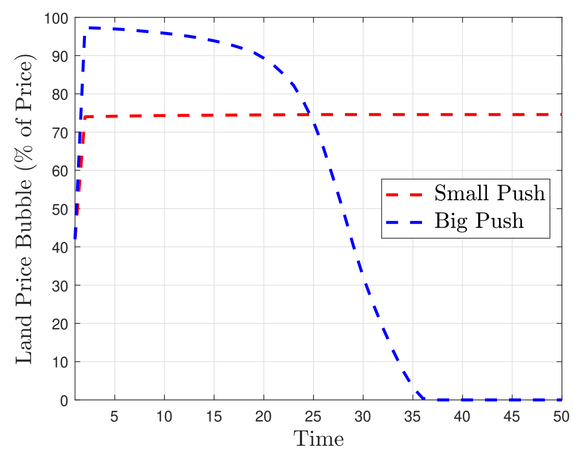
(a) Transition Path of Capital Stock



(b) Transition Path of Investment



(c) Transition Path of Land Price



(d) Transition Path of Land Bubble

Note: For a small push, we increase ξ to 0.0052, and for a big push, we increase ξ to 0.0073. We consider a one-time loosening of credit, and after that, ξ remains at the increased level.

Figure 11: Results of Increasing Collateralizability

After a small increase in the credit limit, ξ, the firm can borrow and invest more, which lead to higher investment, a higher capital level and a higher land price. However, a small credit loosening is insufficient to push the economy out of the constrained region, and the land bubble will increase to a higher level.

However, under sufficient credit loosening, the economy will move from the constrained region to the unconstrained high-level steady state. Along the transition path, the capital stock and land price first grow slowly. When the land price exceeds a critical level (approximately 4% higher than the initial steady state after approximately 25 periods), the economy leaves the low-level region, and the land price begins to increase rapidly. As a result, investment and capital stock also grow rapidly. Credit-driven high-speed growth lasts for approximately 10 periods, and then the economy is near the unconstrained high-level steady state, and the credit constraint becomes slack. Thereafter, investment gradually reduces to its new steady-state level, and the land price and capital stock grow slowly to the new steady state. Along the transition path, the land bubble first increases due to a higher ξ and then decreases to zero because the capital stock accumulates and the economy gradually leaves the constrained region.

5.3 Financial Development and Land Bubble

As the last part of our quantitative analysis, we examine the transition path with financial development, with a particular focus on economic growth and land bubbles during the financial development process. To conduct this exercise, we assume that the credit constraint is given by:

$$i_t \leq \phi_t(\xi_0 p_t l_t + \eta_0 k_t + \kappa_0), \tag{47}$$

where $(\xi_0, \eta_0, \kappa_0)$ implies an economy with only one bubbleless steady state and $\phi_t \in (0,1)$ denotes the level of financial system development. Given the parameters, we assume that the economy is initially in a steady state with a low level of financial development, say $\phi = 0.3$, and then allow ϕ_t to gradually increase to 1, which represents a financial development process. To solve the whole transition path, we first solve the policy function when $\phi = 1$ as the benchmark and then use backward induction to obtain the policy functions during the financial development process. Note that the period-t policy function is determined by the policies of period $t+1$, so we can use the equilibrium system to solve for the policy function on the grid of k_t given the next-period policy functions. We assume that ϕ_t grows linearly from 0.3 to 1 in 25 periods (corresponding to 25 years in the real world) and then remains at $\phi_t = 1$. We calculate the transition paths, and the results are shown in Figure 12. During the first 25 periods when ϕ_t grows from 0.3 to 1.0, the capital stock grows by over 150%, and the price of land almost triples. Then, after financial liberalization, the capital stock and land price continue to increase, albeit more slowly, to their new steady-state level.

Regarding the land bubble, or price-to-rent ratio, we find that during the process when ϕ_t increases and the financial market develops, the relative land bubble $\frac{1}{1-\beta}\mu_t\xi_t$ first increases and then decreases. To define the price-to-rent ratio, we always have the price decomposition:

$$House\ price\ =\ fundamental\ component\ +\ bubble\ component, \tag{48}$$

where the bubble component is defined as $\frac{1}{1-\beta}\mu_t\xi_t p_t$ and the fundamental component is defined as the discounted value of land rent, which means that price-to-rent ratio can be defined by:

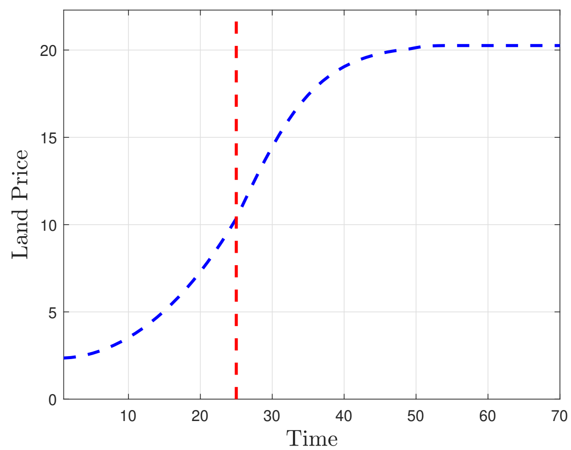
$$\frac{1}{1-\beta}\frac{p_t}{p_t - bubble_t}, \tag{49}$$

which also first increases and then decreases to its steady-state level, consistent with our time-series evidence in Figure 1. These inverse U-shaped transition paths are the result of two competing effects. On the one hand, as the financial market develops, land prices increase and the credit limit increases, which means that land can generate a higher liquidity premium and support a larger bubble or a higher price-to-rent ratio. On the other hand, as ϕ_t increases, the credit constraint is less likely to bind, so the shadow value of collateral μ_t tends to decrease. During financial liberalization, the capital stock and land price first increase relatively rapidly. Note that the land bubble begins to decrease before the end of financial liberalization circa period 20, and this is because we calculate the transition path using a perfect foresight equilibrium, and the agents will expect the fact that ϕ_t will eventually be high and the credit constraint will become slack.

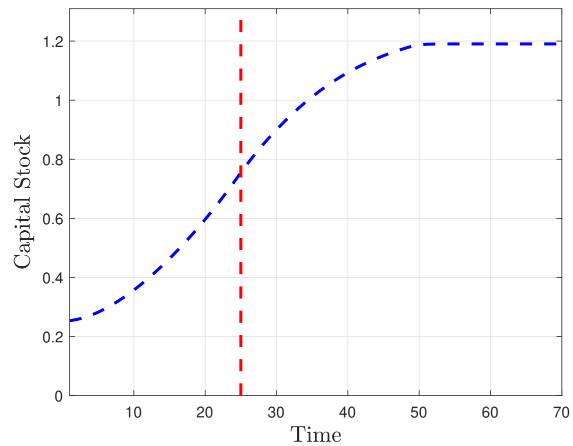
Another important exercise is to check the borrowing (investment) and the borrowing capacity-to-GDP ratio, and we show the results in Figure 13. From the borrowing capacity-to-GDP ratio, we find that there are three stages. During financial development, the borrowing capacity-to-GDP ratio first increases and then decreases. This is because along the transition path, the price first increases rapidly, and then price growth slows and GDP growth dominates. After period 25, financial development stops, and the borrowing capacity-to-GDP ratio begins to decrease and converges to its steady-state value circa period 35.

However, the borrowing-to-GDP ratio dynamics are more complex and feature four stages. Similarly, as the financial development level increases, the borrowing (investment) to GDP ratio first increases and then decreases. Then, after period 25, financial development ends, while the borrowing constraint still binds, so the borrowing-to-GDP ratio still moves with the borrowing capacity-to-GDP ratio. However, after circa period 49, the borrowing constraint no longer binds, the capital policy function comes to a kink, and the borrowing-to-GDP ratio diverges with the borrowing capacity, and gradually converges to its new fundamental steady-state level.

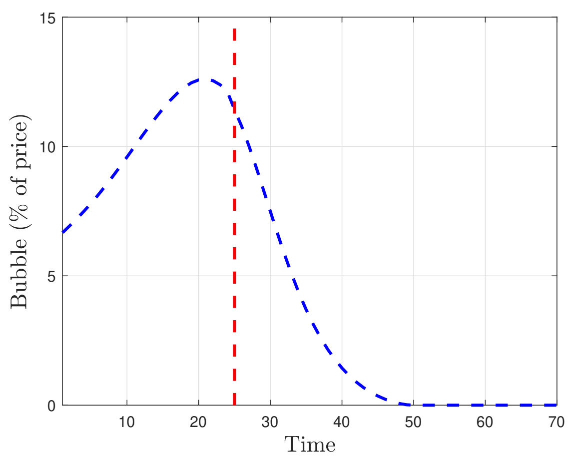
We also calculate the GDP growth rate during the transition path and depict the results in



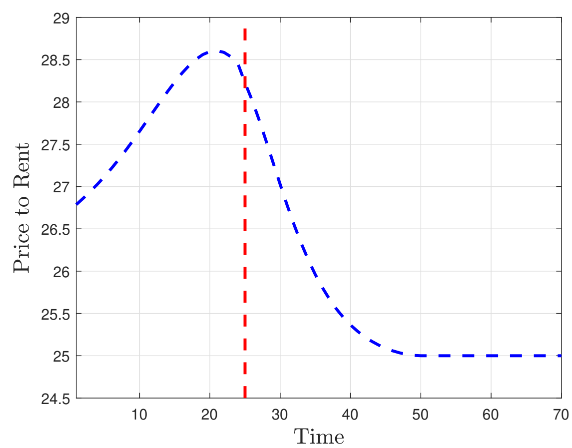
(a) Transition Path of Land Price



(b) Transition Path of Capital Stock



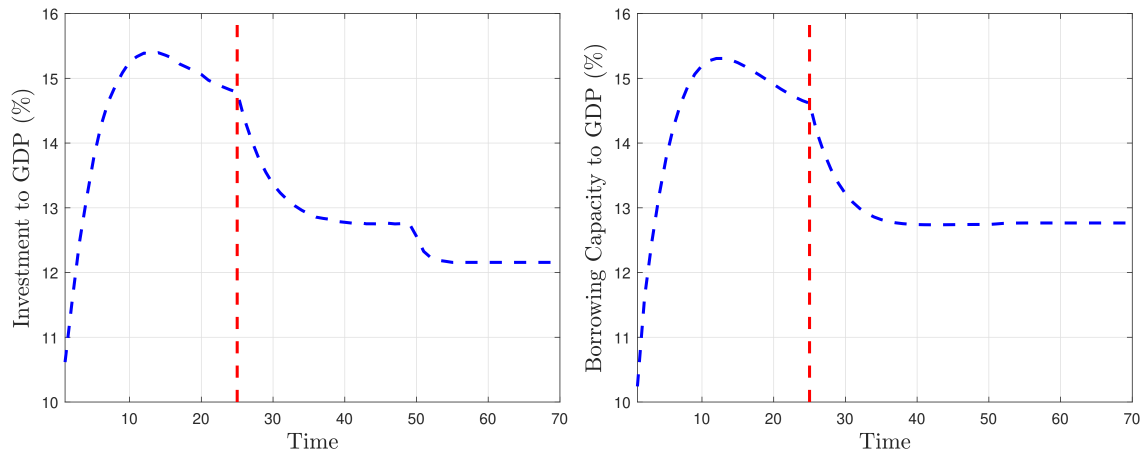
(c) Transition Path of Land Bubble



(d) Transition Path of Price-to-Rent

Note: Here, we use the parameters as in Figure 6. We assume that the financial development level ϕ_t grows from 0.3 to 1 over 25 periods and then stays at 1.

Figure 12: Transition Paths During Financial Development



(a) Borrowing to GDP (b) Borrowing Capacity to GDP

Note: Here, we use the parameters as in Figure 6. We assume that the financial development level ϕ_t grows from 0.3 to 1 over 25 periods and then stays at 1.

Figure 13: Transition Paths of Borrowing and Borrowing Capacity

Figure 14. We find that during the first approximately 15 periods of financial development, the GDP growth rate increases from approximately 1.2% to near 5.5%. However, as the financial liberalization process ends and ϕ_t stays at 1, the GDP growth rate gradually decreases to 0 after approximately 20 periods after financial development. Note that the GDP growth rate also begins to decrease before the financial liberalization process ends, and this is also the result of rational expectations about the development of the financial system.

We argue that our quantitative results can partly match the trend of economic growth and price-to-rent levels for some economies during financial liberalization. As an example, we consider the case of Japan from 1975 to 2000, which is shown in Figure 15. During the 25-year period, the private debt-to-GDP ratio of Japan increases from approximately 110% to over 180%, and during this period, its price-to-rent ratio index first increases by approximately 80% and then decreases back to the initial level. We also find that there is an inverse U-shaped dynamic of real GDP growth: during the first approximately 15 years, the annual real GDP growth rate increases from 3% to around 7% then decreases to approximately 0 and fluctuates around 0. Additionally, similar to our quantitative results, the decrease in the price-to-rent ratio and GDP growth rate both begin before the debt-to-GDP ratio reaches its maximum level, and the GDP growth rate begins to decrease earlier than the land bubble indicator.

6 Conclusion

In this paper, we investigate the nonmonotonic relationship between financial development and the price-to-rent ratio. Using time-series data, we find that for many countries, there is

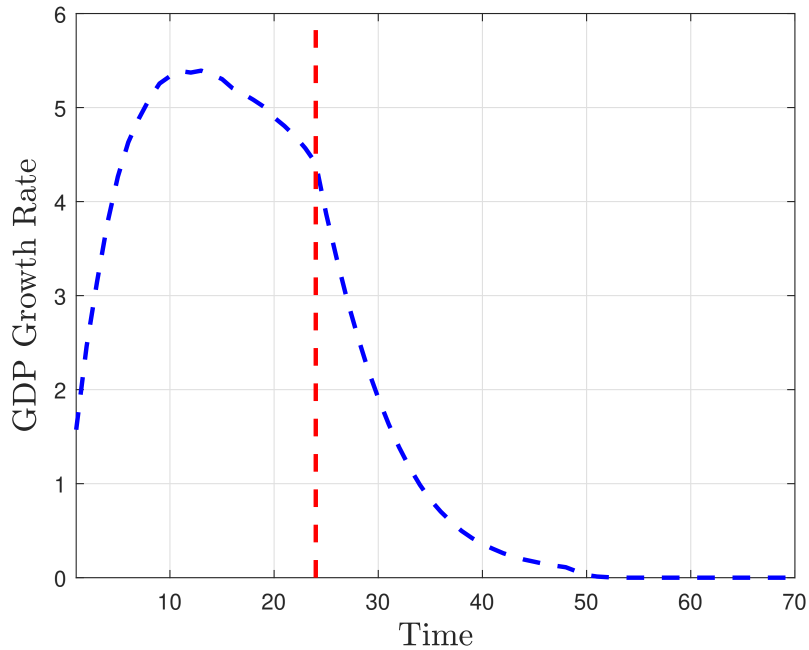
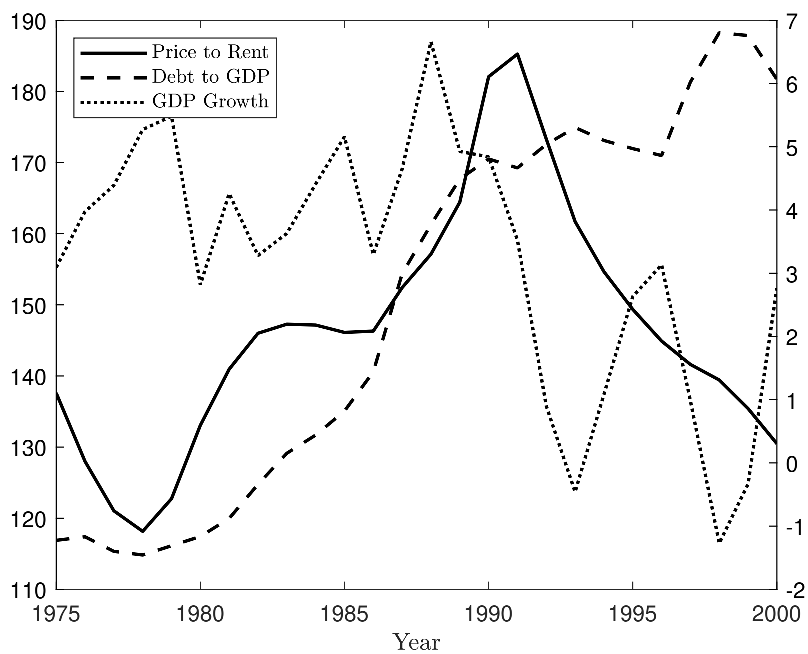


Figure 14: GDP Growth During Financial Development



Note: In this figure, we plot the price-to-rent ratio index, private debt-to-GDP ratio and real GDP growth rate for Japan from 1975 to 2000. The price-to-rent ratio index and debt-to-GDP ratio are plotted on the left axis, and the GDP growth rate (%) is on the right axis.

Figure 15: Financial Development and Economic Growth

an inverse U-shaped relationship between the private debt-to-GDP ratio and price-to-rent ratio in some stage of their financial development. Using cross-sectional data about financial development and price-to-rent ratio in the city center, we use panel regression to study the nonmonotonic relationship for (potential) EU members, the US and the largest countries in East Asia. We find that, after controlling for GDP per capita and country fixed effects, there is an inverse U-shaped relationship between the debt-to-GDP ratio, which is a proxy for financial development, and the price-to-rent ratio for our sample in recent years, which is the housing Kuznets curve.

We then propose a simple macroeconomic model with financial frictions to explain this relationship. In our model, firms borrow to invest, and they face credit constraints, which can be relaxed by using capital and land as collateral. We find that when capital and land are complementary for firm production, the occasionally binding credit constraint leads to S-shaped policy functions, and there may be multiple steady states. Loosening credit will have two competing effects in our model: on the one hand, a higher credit limit will help firms to raise more money and expand production, leading to a higher land price and higher liquidity premium; on the other hand, when the credit constraint is loose enough, it will not bind in equilibrium, which means that land bubble disappears and the price-to-rent ratio falls back to its fundamental level. Under the calibrated model, we find that the transition path after credit loosening can temporarily lead to a higher house price-to-rent ratio compared to both steady states, consistent with the empirical housing Kuznets curve.

Our model can be extended in various ways and can serve as a tool for addressing policy questions. For instance, it prompts consideration of potential government interventions along the transition path, especially when housing prices are substantially higher relative to rents. Indeed, there have been concerns that the housing price in China is excessively high and not affordable for the working class. Our model suggests that the housing premium would diminish with further financial development, suggesting that liberalizing the financial market could be a viable approach to addressing the housing affordability challenge.

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Appendix

A Proof of Proposition 1

Recall that the land demand function is given by:

$$\begin{aligned}D(p) &= \frac{1}{\beta}[(1-\eta)\mu(p)-\beta(1-\delta)(1+\mu(p))+1]k(p)^{\frac{1}{\sigma}}+\xi p\mu(p)-(1-\beta)p \\ &= z\left[\left(\frac{\xi p+\kappa}{\delta-\eta}\right)^{\frac{\sigma-1}{\sigma}}+1\right]^{\frac{1}{\sigma-1}}+\frac{\xi}{1-\eta-\beta(1-\delta)}p\left\{z\beta\left[\left(\frac{\xi p+\kappa}{\delta-\eta}\right)^{\frac{1-\sigma}{\sigma}}+1\right]^{\frac{1}{\sigma-1}}-1+\beta(1-\delta)\right\}-(1-\beta)p,\end{aligned} \tag{A.1}$$

and any steady state is correspond to a price p with $D(p)=0$, so we can just study the properties of $D(p)$ and check the potential of multiple steady states.

By taking derivative of the demand function with respect to p we get:

$$\begin{aligned}D'(p) = &\frac{1}{\sigma}\frac{\xi z}{\delta-\eta}\left[\left(\frac{\xi p+\kappa}{\delta-\eta}\right)^{\frac{\sigma-1}{\sigma}}+1\right]^{\frac{2-\sigma}{\sigma-1}}\left(\frac{\xi p+\kappa}{\delta-\eta}\right)^{-\frac{1}{\sigma}}+\frac{\xi z\beta}{1-\eta-\beta(1-\delta)}\left[\left(\frac{\xi p+\kappa}{\delta-\eta}\right)^{\frac{1-\sigma}{\sigma}}+1\right]^{\frac{1}{\sigma-1}} \\ &-\frac{1}{\sigma}\frac{\xi z\beta p}{1-\eta-\beta(1-\delta)}\frac{\xi}{\delta-\eta}\left[\left(\frac{\xi p+\kappa}{\delta-\eta}\right)^{\frac{1-\sigma}{\sigma}}+1\right]^{\frac{2-\sigma}{\sigma-1}}\left(\frac{\xi p+\kappa}{\delta-\eta}\right)^{\frac{1-2\sigma}{\sigma}}-\frac{1-\beta(1-\delta)}{1-\eta-\beta(1-\delta)}\xi-(1-\beta).\end{aligned} \tag{A.2}$$

Consider a special case where $p\to 0$:[5]

$$D'(0)=\frac{\xi z}{\sigma\delta}\left(x^{\frac{\sigma-1}{\sigma}}+1\right)^{\frac{2-\sigma}{\sigma-1}}x^{-\frac{1}{\sigma}}+\frac{\xi z\beta}{1-\beta(1-\delta)}\left(x^{\frac{1-\sigma}{\sigma}}+1\right)^{\frac{1}{\sigma-1}}-(\xi+1-\beta)=f(x), \tag{A.3}$$

where $x=\frac{\kappa}{\delta-\eta}>0$. Note that

$$f(x)=\frac{\xi z}{\sigma\delta}\left(x^{\frac{\sigma-1}{\sigma}}+1\right)^{\frac{2-\sigma}{\sigma-1}}x^{-\frac{1}{\sigma}}\left[\sigma\delta+\frac{\beta\sigma\delta}{1-\beta(1-\delta)}\left(x^{\frac{\sigma-1}{\sigma}}+1\right)\right]-(\xi+1-\beta). \tag{A.4}$$

For any given η, when $\sigma\in(0,1)$ and $x\to 0$ as $\kappa\to 0$, we get

$$f(0)\to\frac{\xi z\beta}{1-\beta(1-\delta)}-(\xi+1-\beta), \tag{A.5}$$

so given η, when $\xi<\frac{(1-\beta)[1-\beta(1-\delta)]}{\beta(z+1-\delta)-1}$, we have $\lim_{x\to 0}f(x)<0$. As a result, there exists a cutoff $\kappa^*(\eta)$ such that when $\kappa<\kappa^*(\eta)$ we get $D'(0)=f(x)<0$, which means that the demand

[5] Note that $\delta>\eta$ is a required condition, so the value of η is small given a small δ.

function is downward sloping at $p = 0$.[6] Note that when p is near zero, $D'(p)$ is an increasing function of p, and as $p \to \infty$, $D'(p) \to -\frac{1-\beta(1-\delta)}{1-\eta-\beta(1-\delta)}\xi - (1-\beta) < 0$, so there may exist three regions: $D(p)$ first decreases, then increases and then decreases again. The analysis above is done given that $\mu(p) > 0$, and when $\mu(p) = 0$, $D(p) = F_{k,f} - (1-\beta)p$ is a decreasing function of p. Combining the two potential parts, we know that there may be three regions for the land demand function: the land demand function first decreases (region I), then increases (region II), and finally decreases again (region III). As a result, there may exist multiple steady steady states, if: (i) in region I, $\min D(p) < 0$, and that (ii) in region II, $\max D(p) > 0$, and that (iii) in region III the demand function satisfies $\min \mu(p) = 0$ such that the bubbleless steady state exists. To ensure that condition (i) is satisfied, the credit constraint must be tight enough, say ξ should be small. Intuitively, the steady state in region I is a constrained steady state where $\mu > 0$ and the credit constraint is binding. If ξ is too large given the values of η and κ, the credit constraint need not bind and the constrained steady state disappears. Moreover, to ensure (iii) is satisfied, ξ should not be too small, or there will be multiple bubbly steady states.

However, to ensure that condition ii is satisfied, κ and ξ cannot be too small. If κ or ξ is too small, it is likely that for any $p < p^f$ we will have $\mu(p) > 0$, which means that there does not exist a frictionless steady state. To ensure that a frictionless steady state exists, we need:

$$p^* = \frac{(\delta-\eta)k^f - \kappa}{\xi} < p^f \Rightarrow \xi p^f + \kappa > \delta k^f - k^f\eta, \tag{A.6}$$

which means that $\xi > \frac{\delta k^f - k^f\eta - \kappa}{p^f}$ given any η and κ to ensure that the frictionless steady state exists.

Taking the analysis above, we can get the proposition 1.

B Algorithm to Solve for Policy Function

In this section, we describe the algorithm to solve for the policy function $k_{t+1}(k_t)$, $p_t(k_t)$ and $\mu_t(k_t)$ over a discrete grid on k_t

1. Guess a policy function $k_t^{(0)}(k_{t-1})$, $p_t^{(0)}(k_{t-1})$, and $\mu_t^{(0)}(k_{t-1})$. For the next-period variables p_{t+1}, k_{t+1} and μ_{t+1}, we can directly get them by interpolation $p(k_t(k_{t-1}))$, $k_{t+1}(k_t(k_{t-1}))$ and $\mu_{t+1}(k_t(k_{t-1}))$;

2. Assume that collateral constraint binds, that is $\xi p_t + \eta k_t + \kappa = i_t$, then solve for $k_t(k_{t-1})$

[6] However, if $\sigma > 1$, we will have $f(0) \to \infty$ and the demand function is upward sloping at $p = 0$.

and $p_t(k_{t-1})$ using the following system:

$$1+(1-\eta)\mu_t=\beta\frac{u_{c,t+1}}{u_{c,t}}\left[zF_{k,t+1}+(1-\delta)(1+\mu^*_{t+1})\right] \tag{B.1}$$

$$p_t=z_tF_{l,t}+\mu_t\xi p_t+\beta\frac{u_{c,t+1}}{u_{c,t}}p^*_{t+1} \tag{B.2}$$

where we use the credit constraint and capital accumulation rule to substitute out $i_t = \xi p_t + \eta k_t + \kappa$ and $k_t = \frac{(1-\delta)k_{t-1}+\xi p+\kappa}{1-\eta}$, and use the budget constraint to substitute out $c_t = zF(k_{t-1}, l_t) - i_t$, $i_{t+1} = k^*_{t+1} - (1-\delta)k_t$ and $c_{t+1} = zF(k_t, l_{t+1}) - i_{t+1}$, and k^*_{t+1}, p^*_{t+1} and μ^*_{t+1} are from interpolation in step 1;

3. Check if $\mu_t(k_{t-1}) > 0$ (considering the computation error, we actually check if $\mu > 10^{-5}$), if yes, go to next grid; if no, go to step 4;

4. Assume that the credit constraint is not binding, $\mu_t(k_{t-1}) = 0$, then solve for $k_t(k_{t-1})$ and $p_t(k_{t-1})$ using the following system:

$$1=\beta\frac{u_{c,t+1}}{u_{c,t}}[zF_{k,t+1}+(1-\delta)(1+\mu^*_{t+1})]$$
$$p_t=zF_{l,t}+\beta\frac{u_{c,t+1}}{u_{c,t}}p^*_{t+1}$$

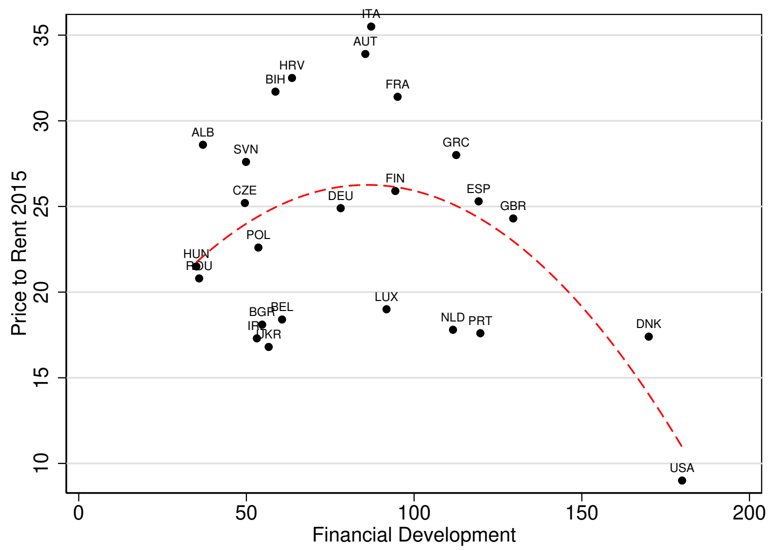
where we use the budget constraint to substitute out $c_t = zF(k_{t-1}, l_t) - i_t$, $i_{t+1} = k^*_{t+1} - (1-\delta)k_t$ and $c_{t+1} = zF(k_t, l_{t+1}) - i_{t+1}$, and k^*_{t+1}, p^*_{t+1} and μ^*_{t+1} are from interpolation in step 1;

5. Now we have $k_t^{(1)}(k_{t-1})$, $p_t^{(1)}(k_{t-1})$ and $\mu_t^{(1)}(k_{t-1})$. Check if the policy functions are close to the initial ones. If so, we have solved the policy functions; if not, go back to Step 1 and update the initial guess.

C Additional Empirical Evidence

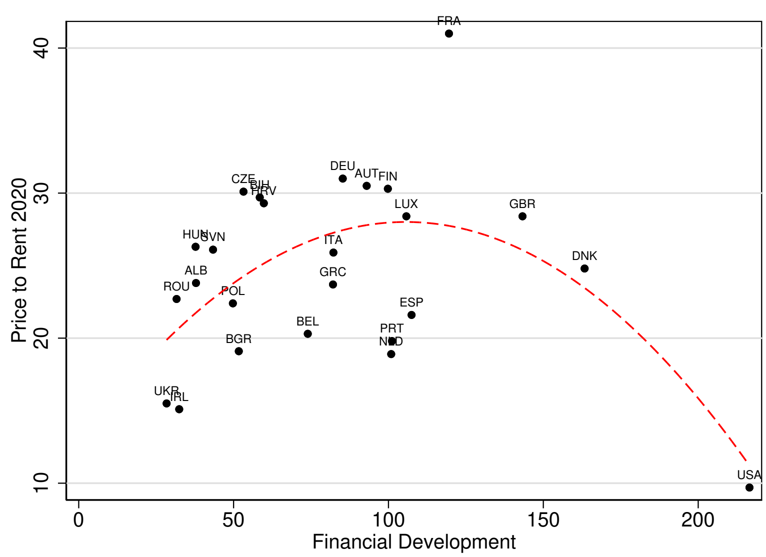
C.1 Countries Use in Cross-Sectional Analysis

Albania, Austria, Belgium, Bulgaria, Bosnia And Herzegovina, China, Czech Republic, Germany, Denmark, Spain, Finland, France, United Kingdom, Greece, Hong Kong, Croatia, Hungary, Ireland, Italy, South Korea, Luxembourg, Netherlands, Poland, Portugal, Romania, Slovenia, Ukraine, United States.



Note: In this figure we show the scatter plot of price to rent ratio in 2015 against financial development (private debt to GDP ratio) for the (potential) EU member countries and the US.

Figure C.1: Financial Development in Eurozone and America: 2015

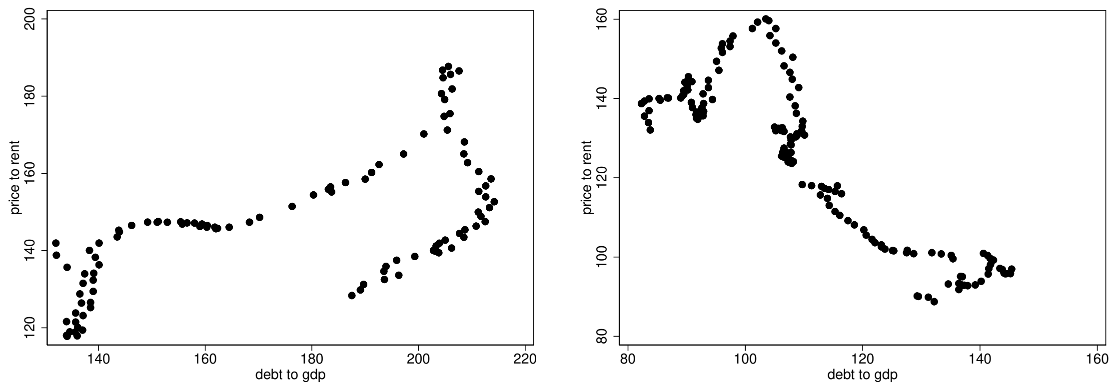


Note: In this figure we show the scatter plot of price to rent ratio in 2020 against financial development (private debt to GDP ratio) for the (potential) EU member countries and the US.

Figure C.2: Financial Development in Eurozone and America: 2020

C.2 Additional Cross-Sectional Empirical Evidence without Weights

C.3 Additional Time-Series Empirical Evidence

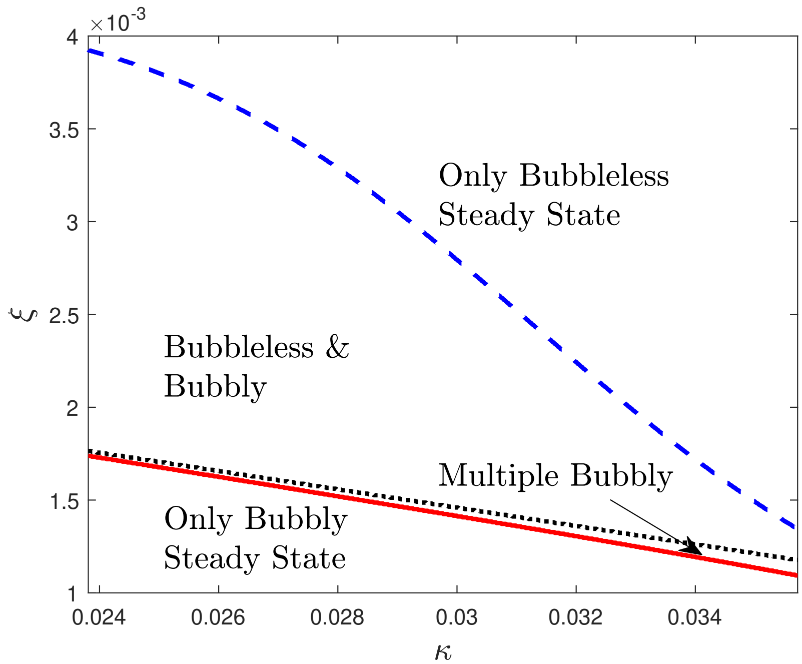


(a) Japan: 1975-2000 (b) Germany: 1970-2008

Figure C.3: Some Additional Time-Series Evidence with Different Episodes

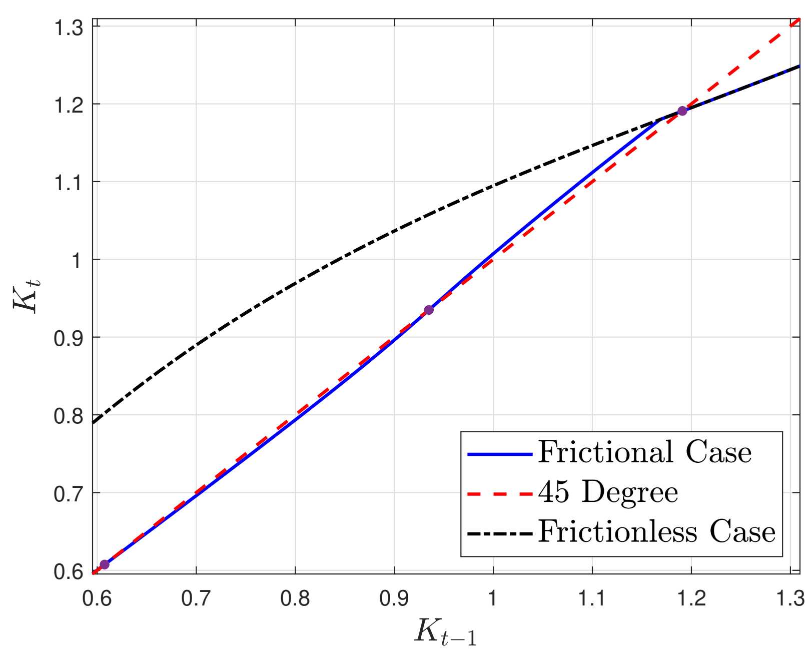
D Robustness Check

In this section we do some robustness check of our baseline model. Especially, we take $\eta = 0.5\delta$, and do the same exercise as in Proposition 1. The steady-state analysis results are given in Figure D.1. We find that there exists four regions: for each given κ, there are three cutoffs. As ξ increases, there are first only bubbly steady states, then multiple bubbly steady states, then bubbly and bubbleless steady states, then only one bubbleless steady state. In Figure D.2, we calculate the policy function with $\eta = 0.05$, $\xi = 0.002$ and $\kappa = 0.03$. We can see that we still get a S-shaped policy function with three steady states, and main results remains.



Note: Here we keep $\eta = 0.05$, change the values of κ and ξ, and check the steady-state properties of our baseline model.

Figure D.1: Existence Region of Bubbly Steady State



Note: Here we take $\eta = 0.05$, $\xi = 0.002$ and $\kappa = 0.03$ and calculate the policy function. The dots denotes three steady states.

Figure D.2: Policy Function of K_t