Self-fulfilling Resignation: Business Cyle and Policy Implications

Zhifeng Cai*

March 31, 2023

Abstract

Following the pandemic-induced recession, the US labor market witnessed rapid and dramatic surges in worker resignations, job vacancies, and wage growth. This paper explores the idea that these phenomena are driven by a self-fulfilling prophecy. By extending a standard labor search model to include replacement hiring by firms and voluntary departures by workers due to labor disutility shocks, I show that workers' decisions to quit become strategically complementary under specific conditions – namely, when productivity is sufficiently low and the disutility shock is not too small. This complementarity results in multiple equilibria characterized by varying levels of resignation rates.

Using global game techniques to refine equilibria, I find that a combination of productivity and labor disutility shocks can trigger a unique type of recession – a job-rich recession – marked by decreased output, elevated quit rates, and stronger wage growth than typically observed. The self-fulfilling resignation calls for intervention because private markets fail to internalize the externality imposed by invidual workers' quitting behavior on equilibrium market tightness.

*Cai: Department of Economics, Rutgers University, 75 Hamilton St, New Brunswick, NJ 08901, USA., (email: zhifeng.cai@rutgers.edu). Preliminary draft.

1 Introduction

Workers may leave their jobs due to various job-related factors such as inadequate compensation, limited opportunities for career progression, and unsatisfactory work conditions. They may also choose to resign due to non-job related reasons that stem from the broader economic environment. Specifically, in a tight labor market, workers are more inclined to quit their current positions in pursuit of higher-paying job opportunities. This has been especially evident during the Great Resignation period, where workers have cited low pay as the primary reason for their departures.

As quitting behaviors can impact the overall labor market, an individual's decision to leave their job can be influenced by their perceptions of other workers' propensity to quit. In essence, quitting behavior can be self-fulfilling: employees resign because they anticipate a tight labor market, and large number of workers quitting generates many job openings, thus validating the initial expectation of a tight labor market. When this self-fulfilling prophecy is in play, shifts in workers' expectations can lead to sudden and substantial increases in aggregate quit rates, accompanied by abrupt changes in other labor market indicators, consistent with the post-pandemic experience.

This paper explores this self-fulfilling channel of quitting in an extended version of a standard labor search model. In the model, workers have the option to quit (i.e., to voluntarily end an employment relationship without the employer's consent) due to labor disutility shocks. Additionally, vacancies may persist beyond the duration of the corresponding matches, allowing firms to engage in replacement hiring when a worker resigns. In this context, the workers' decision to quit can be viewed as a strategic game, as an individual worker's payoff from quitting is dependent on the number of other workers who also quit. If a large number of workers resign, firms need to post numerous vacancies to replace these workers, resulting in a tight labor market. This tight labor market subsequently makes quitting less costly, increasing individual payoffs for quitting. This creates *strategic complementarity* among all workers.

The first main result of the paper is to formalize this intuition, demonstrating that strategic complementarity results in multiple equilibria. There exists an equilibrium in which no workers quit because they expect others not to, and another equilibrium in which all workers resign because they anticipate everyone else doing the same.

Interestingly, such multiplicity is conditional and only emerges when a) the labor disutility shock is not too small and b) aggregate productivity is adequately low. The condition on labor disutility shock is straightforward: if labor disutility is very low (for instance, zero), quitting becomes a dominated strategy, eliminating strategic interactions among workers. Consequently, no worker would be willing to quit in the unique equilibrium.

The condition on aggregate productibvity is much less trivial and constitutes the paper's key insight, warranting a more detailed explanation. Recall that the strategic complementarity operates through equilibrium market tightness: specifically, market tightness increases with the aggregate number of quits, and a tighter market enhances workers' payoffs for quitting. However, when the free entry condition holds with equality, market tightness is determined by this condition and cannot potentially vary with quits. Consequently, multiplicity cannot emerge when the free entry condition holds with equality, as in the standard DMP (Diamond-Mortensen-Pissarides) model and all subsequent works.

In this model, I show that sufficiently low productivity would break the free entry condition. Vacancies can persist beyond the duration of a match in the model, which allows them to be conceptualized as a firm's "capital stocks" that can be created at a cost but may potentially yield future benefits that lasts beyond the duration of a match. The benefit of investing in vacancies is thus the future output if matched, and the cost is set at the vacancy posting expense. When productivity is sufficiently low, the benefit is strictly smaller than the cost, leading new entrant firms to strictly prefer not entering the market. This implies that the free entry condition (or equivalently, the firm's indifference condition) breaks down, paving the way for equilibrium multiplicity.

Given multiplicity, it is difficult to analyze equilibrium comparative statics. I therefore use global game techniques to refine the equilibrium by endowing each worker with an idiosyncratic realization of the labor disutility cost. This perturbation yields a unique equilibrium (Morris and Shin, 2003), under which all workers following a cutoff strategy in terms of their quitting behavior: workers quit if and only if their labor utility costs are greater than a threshold, determined in equilibrium.

The paper's second key finding is as follows: under the unique global game equilibrium, the economy can be segmented into two areas based on workers' quitting strategies: a normal region sustained by a high level of productivity where few workers resign, and a fragile region where a significantly larger proportion of workers might quit. Resignations occur due to the strategic complementarity in job departures, which only emerges when productivity is low, making it more likely to happen after a severe downturn such

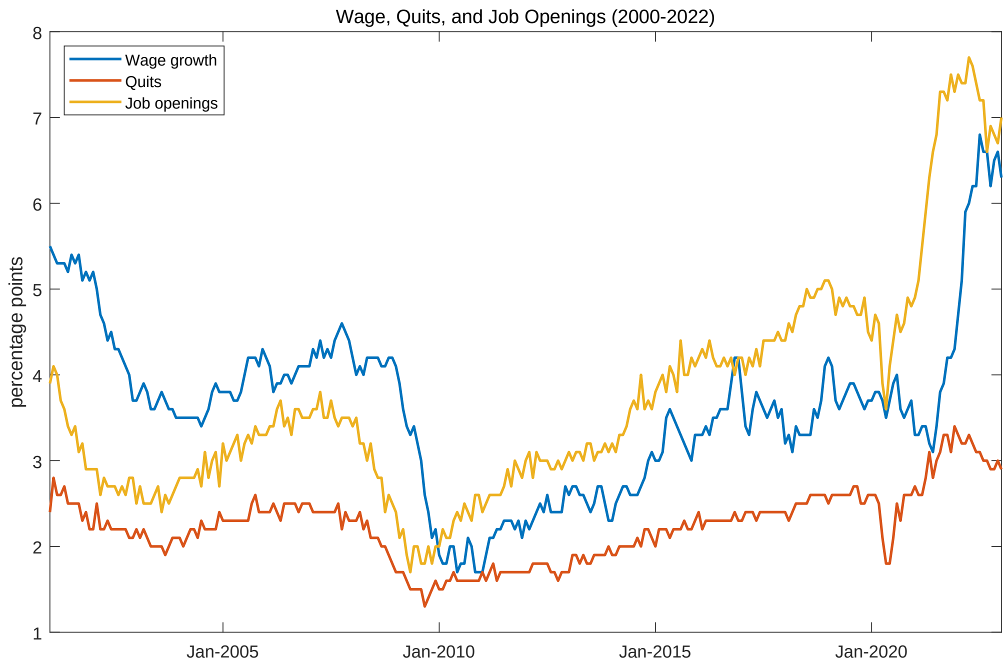


Figure 1: Jobful Recession

as the Covid crisis.

Under the unique global game equilibrium, we can derive model predictions concerning observables: when productivity is high, few workers, if matched, would choose to quit; when productivity is low, the economy enters a fragile region characterized by significantly more quits due to the stricter quitting threshold determined by the global game equilibrium. In this fragile region, the vacancy posting rate is also higher since firms need to engage in replacement hiring. Consequently, a negative productivity shock that pushes the economy into this fragile region results in a labor market that appears strong, with large amounts of unfilled jobs and vacancy postings. Wage growth is also strong in the fragile region due to an "efficiency wage channel," where firms aim to retain as many workers as possible and therefore increase wages to discourage workers from quitting. This model generates the surprising prediction that negative shocks lead to higher quit rates, more vacancy postings, and stronger wage growth, which is qualitatively consistent with recent experiences since the Covid crisis, including rapid and dramatic increases in quits, job openings, and wage growth (figure 1).

These predictions are derived under the condition that the labor disutility from working is not too low, making quitting a possibly desirable strategy for workers and allowing quitting concerns to be part of the equilibrium. If labor disutility costs are sufficiently small, the model would resemble a standard labor search model, where negative productivity shocks consistently reduce vacancy postings and depress wage growth. The model suggests that the key difference between the current "jobful" recession and previous recessions could be a shift in workers' preferences for work. The pandemic might have caused people to reassess their lives and consider how their careers align with their priorities, leading to a higher propensity to leave jobs. The model proposes that such a change in people's job preferences could have profound business cycle implications, significantly altering labor market dynamics.

The third main finding of this paper relates to policy implications. When self-fulfilling resignations occur, workers do not take into account the positive social externality that arises with their quitting behavior, which causes more vacancies to be reposted in the labor market. This, in turn, increases market tightness and the job-finding probability for other workers. The paper investigates this externality and concludes that providing a subsidy for workers' quitting behavior may lead to improved overall welfare.

Finally, the quantitative section of the paper incorporates the model into an infinite-horizon labor search framework and calibrates it to the US labor market. The model finds that combinations of productivity and preference shocks successfully generate substantial increases in quits, vacancy rates, unemployment rates, wages, and market tightness, consistent with the narrative of a jobful recession. In the model, increases in vacancies are driven by replacement hiring rather than new postings, representing a novel prediction that could potentially be validated with data.

Related Literature

This paper is connected to the literature examining the role of firm replacement hiring and its macroeconomic impacts. Two closely related papers are Mercan and Schoefer (2020) and Elsby et al. (2022), which identify vacancy chains using German data and US establishment-level data, respectively. They demonstrate that quits resulting from on-the-job search frictions can amplify business cycles when frictions make new vacancies and replacement vacancies imperfect substitutes.[1] Another related paper is Qiu (2022), which shows that quitting into non-participation can amplify business cycles. All of these papers operate under the assumption of a unique equilibrium. This paper identifies a

[1] In Mercan and Schoefer (2020) this is achieved through an adjustment cost of job creation. In Elsby et al. (2022) this is achieved through a sticky objective in employment-level.

novel mechanism that leads to the possibility of equilibrium multiplicity in such settings. It is also the first paper in this literature that discusses the inefficiency associated with worker's quitting behavior and how policies should be conducted to correct for such inefficiencies.

Several recent papers have focused on the possibility of quits to non-employment. Cai and Heathcote (2023) develops a quantitative model incorporating both quits to non-employment and quits to another job and discusses the optimal design of unemployment insurance. Blanco et al. (2023) consider an environment where workers quit due to productivity variations and wage rigidities, and explore the impact of monetary policy shocks. Unlike these works, this paper also investigates the role of both productivity and preference shocks. It highlights the role played by preference shocks.

The paper is also related to the literature on constrained efficiency of labor search models. Hosios (1990) identifies the potential inefficiency related to ex-post wage bargaining in random search models. Mangin and Julien (2021) generalizes the "Hosios condiiton" to environments where expected match output depends on market tightness. None of those paper discusses the inefficiency associated with worker's quitting behavior. The externality identified in this paper is novel and related to how worker's quitting behaivor affects equilibrium market tightness.

2 Model

The model is a two-period adaptation of the standard DMP (Diamond-Mortensen-Pissarides) model. A two-period model is the minimum necessary to capture the insights related to the possibility that a vacancy can outlive the duration of a match. This model involves two types of players: a continuum of initially unemployed workers and a potentially large pool of firms that can enter the labor market and post vacancies. Both workers and firms are risk-neutral and discount the future at a rate of β.

There are two dates, $t = 1, 2$. At the beginning of each date, workers and firms engage in search and matching; at the end of each date, employed workers produce z and earn wages w, while unemployed workers receive some value of leisure b (normalized to 0). Workers are subject to a labor disutility shock χ, and their per-period utility functions are given by $w - \chi$ if working and $b = 0$ if not.

At the search and match stage, both the worker's job finding probability $p(.)$ and the firm's job filling rate $q(.)$ depend on market tightness, which is the ratio of the number of vacancies available to the number of workers looking for jobs. $p(.)$ is an increasing function, while $q(.)$ is a decreasing function. $p(.)$ is assumed to be bounded between 0 and 1 as the worker's job finding probablity cannot exceed 100 percent.

Two novels elements, relative to the standard model, are as follows: first, between the search and match stage and the production stage, workers can choose to quit depending on the realization of the labor disutility shock χ; second, in the first period firms post long-lived vacancies, in the sense that if the firm matched with a worker and the worker quits the job, with some probability ζ the firm can re-post the vacancy at zero cost. With these key ingredients in mind, let's go through the timeline in detail (see figure 2).

In the beginning of period 1, both the value of productivity z and the value of labor disutility χ are realized. These values also represent the aggregate states of the economy. After that, period 1 search and match occurs: all workers look for jobs, and firms can post long-lived vacancies at cost ϕ_1. The search protocol we adopt is random search and therefore all workers search in an aggregate labor market. Equilibrium market tightness is governed by a firm's zero profit condition where new entrants must have no incentive to enter. After the search and match stage, worker and firm conduct Nash bargaining to determine the first period wage w_1. Then there is a separation stage where worker-firm pairs end at some exogenous rate γ. This γ shock intends to capture the layoff risks and is not crucial to the key results of the paper. After the γ shock workers can decide whether or not to quit. We denote matched worker's decision to quit to be a_i:

$$\begin{aligned} a_i &= 0 \text{ if the worker does not quit} \\ &= 1 \text{ if he quits} \end{aligned}$$

Hence, the aggregate share of matched workers who quit is A:

$$A = \int a_i di$$

Where the integration is taken over all matched workers. This A is a key equilibrium object that we will characterize later. After the separation stage, all remaining matched workers produce, while all unmatched workers receive the value of leisure. This concludes period 1.

In the beginning of period 2, the labor market opens and search and match occurs. At this stage, two types of firms post vacancies onto the market. First, new entrant firms

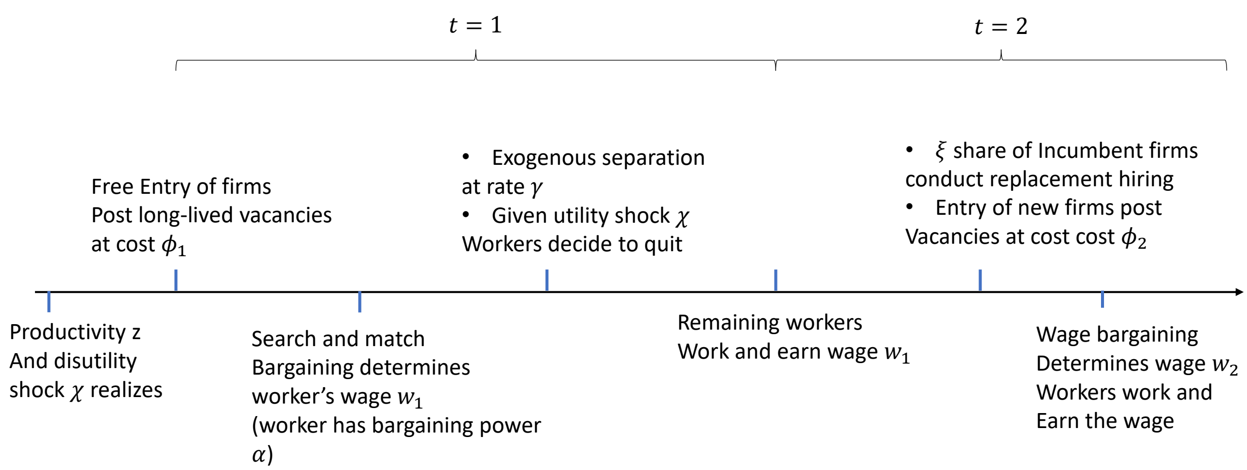


Figure 2: Timeline

can post vacancies at some cost ϕ_2. Second, there is *replacement hiring* meaning that firms that lose workers at the separation stage in period 1 can *repost* their vacancies at zero cost. The idea is that upfront investments for a vacancy has been incured for these firms. For example, office space might have already been rented and therefore no additional vacancy costs need to be incured in this case. We assume that ξ share of the incumbent firms can repost their vacancies. After the search and match stage, all pairs of matches conduct Nash bargaining to determine the second period wage w_2. Then the production stage occurs, with workers working and earning wage w_2. We assume that there is no utility cost in the second period.[2].

Note that this model nests the textbook version of the labor search model when there is no replacement hiring $\xi = 0$ and when there is no labor disutility shocks $\chi = 0$.

The equilibrium consists of the following objects: $(w_1, u_1, \theta_1, A, w_2, u_2, \theta_2)$ such that:

1. Wages are pinned down by Nash Bargaining

2. Market tightnesses are pinned down by free entry conditions

[2]This assumption is relaxed in the infinite-horizon quantitative model in section 7.

3. Unemployment is pinned down by search and matching

4. Quitting is optimal for the workers taking all other objects as given.

3 The Payoff to Quit and Strategic Complementarity

The model is solved backwards. The period 2 wage bargaining problem is straightforward given that worker's bargaining power is α and there is no labor disutility shocks in period 2:

$$w_2 = \alpha z$$

Now we get to the search and matching stage of period 2. There are three types of unmatched workers: workers that are initially unmatched in the first period, denoted by u_1; workers that are matched and exogenously separated $(1-u)(1-\gamma)$; and workers who quits: $(1-u)\gamma A$. There are also two types of firms posting vacancies: new entry of firms posting at cost ϕ_2, denote that by e_2; incumbent firms who post due to quit $(1-u_1)(1-\gamma+\gamma A)$ with probability ξ.

The second-period market tightness is given by:

$$\theta_2(A) = \frac{e_2 + \xi(1-u_1)(1-\gamma+\gamma A)}{u_1 + (1-u_1)(1-\gamma+\gamma A)} \tag{1}$$

An intuitive explanation of why strategic complementarity arise is as follows. A key observation here is that the second-period market tightness θ_2 varies with total quitting A. To highlight this we therefore write market tightness θ_2 as a function of A. A shows up both in the numerator and the denominator because more quitting implies both more non-employment and more vacancies. Hence, how the tightness varies with quitting depends on other equilibrium quantities. In particular, when new entrants are sufficiently scarce (e_2 is sufficiently low, 0 for example) , one can easily shown through differentiation that market tightness is increasing in quitting:

$$\theta_2'(A) > 0$$

If market tightness is increasing in A, so is the workers' job finding probabilities:

$$\frac{\partial p(\theta_2(A))}{\partial A} > 0$$

. Hence more workers quit leads to a tighter labor market and hence higher job finding probabilities for the remaining matched workers, increasing their incentive to quit. This introduces strategic complementarity in the worker's quitting decisions.

With this intuition in mind, let's get into the specifics of the workers' quitting game. Each worker's lifetime utility if not quit is

$$w_1 - \chi + \beta\alpha z$$

where $w_1 - \chi$ is the net gain from work in the first period and $\beta\alpha z$ is the discounted second period wage.

If the worker quits, he doesn't need to work (hence doesn't incur the labor disutility cost) in the first period and he needs to look for jobs in the second:

$$0 + \beta p\left(\theta_2\right)\alpha z$$

So the benefit of quit is that the worker can avoid the utility cost χ while the cost is that he gives up the first period wage w_1, and also lost job security in the sense that he will get re-employed only with probability $p\left(\theta_2\right)$ in the next period.

We can define a payoff function π to quitting which is the difference in utilities of the two cases:

$$\pi(\chi, A) = \beta\left(p\left(\theta_2\left(A\right)\right) - 1\right)\alpha z - w_1 + \chi \quad (2)$$

If π is positive, the worker would choose to quit, while otherwise, he would choose not to. The critical aspect of this payoff function is that it may depend on the aggregate share of quits A. In particular, as can be seen in equation 1, the market tightness θ_2 may be increasing in A, which makes individual payoff to quit increasing in the aggregate share of quits. Thus, when individual workers decide whether to quit, they need to forecast the quitting strategy of other workers. This makes workers' quitting decisions a strategic game.

To understand the nature of equilibrium of this game, we will proceed by discussing the following cases, divided by the values of the two key parameters (see table 1). We know that our model collapses to a standard textbook mode of labor search when there is no labor disutility ($\chi = 0$, hence no worker quits) and no replacement hiring ($\xi = 0$). In this case, therefore, the equilibrium is unique. We will discuss each of the two

Table 1: A Preview of Key Result

	$\zeta = 0$ (no replacement hiring)	$\zeta > 0$
$\chi = 0$ (no labor disutility)	Unique equilibrium (textbook DMP model)	Unique equilibrium
$\chi > 0$	Unique equilibrium	Multiple equilibria (possibly)

assumptions and show that multiplicity only arises when both $\chi > 0$ and $\xi > 0$.

Case 1: No Labor Disutility $\chi = 0$

From the expression of the payoff to quit (equation 2), one can see that if there is no labor disutility shocks $\chi = 0$, the payoff to quit is always nonpositive because job finding probability p cannot exceed 100 percent. Hence $\pi \leq 0$: quitting is always a dominated strategy and workers would never choose to quit.

Proposition 1. *If $\chi = 0$, there exists a unique equilibrium where workers never quit: $A = 0$.*

In general, when χ is sufficiently close to 0, quitting is never an optimal strategy.

Case 2: No replacement Hiring $\xi = 0$

When there are no replacement hirings, all vacancies are driven by new entrants. From equation 1, the market tightness expression becomes:

$$\theta_2 (A) = \frac{e_2}{u_1 + (1 - u_1)(1 - \gamma + \gamma A)}$$

Where in the numerator there is only new entry e_2. In this case, the job filling rate $q(\theta_2)$ closely tracks new entrant e_2: when $e_2 \to 0$, $q \to \infty$ and when $e_2 \to \infty$, $q \to 0$. By the continuity of the job filling function this implies that the free entry condition always holds with equality:

$$q(\theta_2)(1 - \alpha) z = \phi_2.$$

We obtain the following closed-form expression for θ_2:

$$\theta_2 = q^{-1} \left(\frac{\phi_2}{(1 - \alpha) z} \right)$$

and one can further show that e_2 is strictly positive by plugging in the expression of θ_2

into the above equation:

$$e_2 = q^{-1}\left(\frac{\phi_2}{(1-\alpha)\,z}\right)[u_1 + (1-u_1)(1-\gamma+\gamma A)] > 0$$

The important observation here is that, when the free entry condition holds, θ_2 can no longer depend on A. Hence the payoff to quit function becomes:

$$\pi(\chi) = \beta\left[p\left(q^{-1}\left(\frac{\phi_2}{(1-\alpha)\,z}\right)\right) - 1\right]\alpha z - w_1 + \chi$$

There is no strategic aspect of this model, and worker's quitting behavior only depends on economic fundamentals and their own individual cost of quitting. Hence there is a unique equilibrium:

Proposition 2. *If $\xi = 0$, there exists a unique equilibrium. Under this equilibrium, no workers quit as long as the value of χ is not too large.*

Case 3: Both $\chi > 0$ and $\xi > 0$

One useful lesson we learn from case 2 is that if the free entry condition holds with equality, individual workers' payoff to quitting can never depends on aggregate share of quits A, and hence there exists a unique equilibrium.

When there is replacement hiring, however, the free entry condition may not hold with equality and new entrants may strictly prefer not to enter. Consider the scenario where productivity z is very low. The free entry condition would call for very few vacancy postings. However, we know that existing vacancies can be re-posted at zero cost. If the amount of leftover vacancies exceeded the vacancy rate consistent with the free entry condition equal to zero, then new entrants would strictly prefer not to enter, i.e.

$$q(\theta_2)(1-\alpha)\,z < \phi_2$$

and there would be no new vacancy postings in equilibrium:

$$e_2 = 0$$

It would perhaps easier to understand the logic by drawing an analogy between vacancy creation and capital accumulation. Capital depreciates at a certain rate. One can invest in capital at a cost but this investment is irreversable: one cannot divest in capital. When the productivity of capital is not too low, one would invest in capital up to the point where the marginal benefit of investment is equal to the cost of invest. But when productivity is too low, existing capital stock would already imply that the marginal benefit of investment is below the marginal cost of invest, and hence agents would stay inactive. This is the same logic here. One could imagine long-lived vacancies as capital stocks for the firm. Vacancies depreciates at rate $1 - \xi$, but one can never destroy vacancies (or destroying such does not yield any value). Hence if productivity is sufficiently low, given the stock of vacancies, the marginal benefit of creating extra vacancies $\left(q\left(1-\alpha\right)z\right)$ would be lower than the marginal cost $\left(\phi_2\right)$, and there would be no new vacancies created $\left(e_2 = 0\right)$

We formalize this into the following proposition followed by a formal proof:

Proposition 3. *Suppose that $\xi > 0$. Define $\bar{\theta} = \frac{\xi(1-u_1)(1-\gamma)}{u_1+(1-u_1)(1-\gamma)}$. Define $\bar{z}$ such that*

$$q\left(\bar{\theta}\right)\alpha\bar{z} = \phi_1$$

Then for $z < \bar{z}$, there is no firm entry: $e_2 = 0$.

Proof. Fix a $z < \bar{z}$. Suppose otherwise, that there will be positive firm entry $e_2 > 0$. Denote equilibrium quitting to be $\tilde{A}$. Then equilibrium tightness is

$$\tilde{\theta} = \frac{e_2 + \xi\left(1-u_1\right)\left(1-\gamma+\gamma\tilde{A}\right)}{u_1 + \left(1-u_1\right)\left(1-\gamma+\gamma\tilde{A}\right)}$$

And

$$\tilde{\theta} > \frac{\xi\left(1-u_1\right)\left(1-\gamma+\gamma\tilde{A}\right)}{u_1+\left(1-u_1\right)\left(1-\gamma+\gamma\tilde{A}\right)} \geq \frac{\xi\left(1-u_1\right)\left(1-\gamma\right)}{u_1+\left(1-u_1\right)\left(1-\gamma\right)} = \bar{\theta}$$

so

$$q\left(\tilde{\theta}\right) < q\left(\bar{\theta}\right)$$

and hence

$$q\left(\tilde{\theta}\right)\alpha z < q\left(\bar{\theta}\right)\alpha\bar{z} = \phi_1$$

contradition. Hence it must be that $e_2 = 0$. □

The fact that when $z < \bar{z}$, the free entry condition no longer holds with equality opens the door for strategic interactions in the workers' quitting decisions. Because $e_2 = 0$, market tightness becomes:

$$\theta_2 = \frac{\xi (1 - u_1)(1 - \gamma + \gamma A)}{u_1 + (1 - u_1)(1 - \gamma + \gamma A)} \tag{3}$$

Provided that $\xi > 0$, market tightness θ_2 is always increasing in A. Plugging this expression into the payoff to quit function, we have:

$$\pi(\chi, A) = \beta (p(\theta_2(A)) - 1) \alpha z - w_1 + \chi \tag{4}$$

To the extent that θ_2 is increasing in A, the payoff function is also increasing in A. In other words, workers quitting decisions exhibit strategic complementarity. When $z > \bar{z}$, on the other hand, such complementarity disappears because θ_2 is tightly pinned down by the free entry condition and cannot depend on A. In other words, the complementarity is *conditional*, and worker's quitting decision becomes complements only when productivity is sufficiently low (that is, below $\bar{z}$). We summarize this into the following proposition:

Proposition 4. *For $z < \bar{z}$, market tightness is increasing in quits:*

$$\frac{\partial \theta_2(A)}{\partial A} > 0$$

This implies strategic complementarity among workers' quitting decisions, in the sense that the payoff to quitting increases with the share of quits:

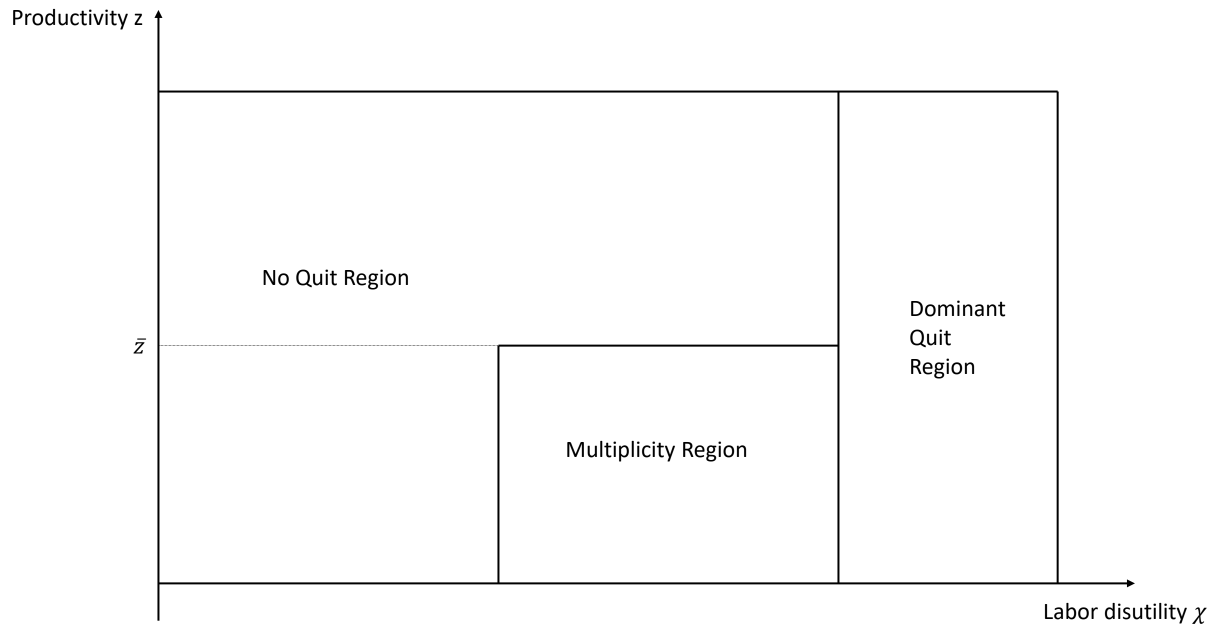
$$\frac{\partial \pi(\chi, A)}{\partial A} > 0$$

Given the strategic complementarity, we can show that there exists multiple equilibrium with drastically different quitting behaviors: in one, all workers quit; in the other, no workers quit:

Theorem 1. *Suppose that $\zeta > 0$ and $\chi > 0$ (within some interval). For $z < \bar{z}$, there exists multiple equilibria – at one no workers quit in equilibrium: $A = 0$; at the other all workers quit: $A = 1$.*

The theorem is illustrated in figure 3. For productivity sufficiently high, all workers behave non-strategically. Hence there is a cutoff which determines whether quitting is a dominating or dominated strategy. Below that threshold, all workers choose not to quit; while above that threshold, all workers choose to quit.

Figure 3: Illustration of Theorem 1



For productivity sufficiently low ($z < \bar{z}$), on the other hand, there exists three regions. When χ is sufficiently low, quitting is a dominated strategy and hence no one quits. When χ is extremely high, quitting becomes a dominant strategy and hence everyone quits. For χ within the intermediate range, workers may quit strategically: they would choose to quit if and only if they expect others to. This implies that when productivity is sufficiently low, belief can play an important role in shaping workers' quitting behavior: changes in workers belief would lead to an abrupt change in equilibrium level of quitting.

4 Global Game Refinement

The equilibrium multiplicity illustrates the fragility inherent in the model with endogenous quits and replacement hiring. But we want the model to produce empirical predictions, and equilibrium multiplicity makes it difficult to do. To proceed we will apply a global game refinement to the model as a minimum perturbation to achieve a unique equilibrium.

We assume that there is some idiosyncratic noise in the utility shock

$$\chi_i = \chi + \varepsilon,$$

where the noise ε has zero mean and a variance of σ_χ^2. All workers observe their own realization of χ_i but no one observes the true value of χ, which they need to forcaste with their own signals. For the theoretical analsyis we will take the variance of the noise to be arbitrarily small $\sigma_\chi^2 \to 0$, representing minimal perturbation to the model's information structure. Although χ is realized at the very beginning of the period, it is unobservable to the worker. χ_i is only realized at the quitting stage and workers make their quitting decisions based on their individual state χ_i. Hence, each individual's decision problem is under the situation that $z < \bar{z}$:

$$\max_{a_i \in \{0,1\}} \left[a_i \beta p\left(\theta_2\right) \alpha z + \left(1 - a_i\right)\left(w_1 - \chi_i + \beta \alpha z\right) | \chi_i \right]$$

$$s.t.$$

$$\theta_2 = \frac{\xi\left(1 - u_1\right)\left(1 - \gamma + \gamma A\right)}{u_1 + \left(1 - u_1\right)\left(1 - \gamma + \gamma A\right)}$$

where the expectation is taken over A because the worker needs to forecast the population mean χ, which affects how many other workers would quit, and hence future market tightness. One could rearrange the objective function as:

$$\max_{a_i \in \{0,1\}} \left[a_i \pi\left(\chi, A\right) + \left(w_1 - \chi_i + \beta \alpha z\right) | \chi_i \right]$$

where $\pi\left(\chi, A\right)$ is given in equation 4. This expression can be further simplified into

$$\max_{a_i \in \{0,1\}} \left[a_i \pi\left(\chi, A\right) | \chi_i \right] \tag{5}$$

as $w_1 - \chi_i + \beta \alpha z$ is unaffected by the endogenous choice a_i. So we could directly work with the payoff function $\pi\left(\chi, A\right)$ without consulting the underlying structure of the economy.

The solution to a global game takes the form of a cutoff strategy: there exists a cutoff $\bar{\chi}$ such that the worker quits if and only if his own utility shock $\chi_i > \bar{\chi}$. All workers expects that other workers would follows this cutoff strategy, and given this, they also find such a cutoff strategy optimal.

Definition 1. *A global search equilibrium is the following pair of* $(w_1, u_1, \theta_1, \bar{\chi}, A, w_2, u_2, \theta_2)$ *such that:*

1. *Wages are pinned down by Nash Bargaining*
2. *Market tightnesses are pinned down by free entry conditions (possibly at boundary)*
3. *Unemployment is pinned down by search and matching*
4. *The quitting threshold is pinned down by individual workers solving the problem in equation 5, taking as given all the equilibrium objects. Aggregate quits* A *is pinned down by the mass of workers with* $\chi > \bar{\chi}$*.*

We now characterize the global search equilibrium. Using results from section 2 of Cai and Dong (2023), one can show that:

Proposition 5. *There exists a unqiue global search equilibrium under which the quitting threshold is pinned down by:*

$$\int_0^1 \pi\left(\bar{\chi}, A\right) dA = 0 \tag{6}$$

The intuition for why the cutoff is pinned down in this way is as follows. In this model there is very little fundamental uncertainty as the variance of the noise σ_χ is taken to be arbitrarily close to 0. However, there is large strategic uncertainty because agent need to forcaste χ and he is uncertain about the relative distance between χ and his own signal χ_i. This creates uncertainty regarding the share of quits A.to deal with this uncertainty agents take an expectation over all possible values of A from 0 to 100 percent. The worker at the threshold $\bar{\chi}$ should be indifferent between quitting or not hence the expectation evaluated at $\chi = \bar{\chi}$ is equal to 0. This equation pins down $\bar{\chi}$. The following proposition characterises the quitting game:

Proposition 6. *When* $z > \bar{z}$*, there exists a unique equilibrium where market tightness is pinned down by*

$$\theta_2 = q^{-1}\left(\frac{\phi_2}{(1-\alpha)\, z}\right)$$

and for an admissible range of χ*, worker quits if and only if* $\chi > \tilde{\chi}(z)$*, where* $\tilde{\chi}(z)$ *is pinned down by:*

$$\tilde{\chi}(z) = w_1 + \beta\alpha z - \beta\alpha z p\left(q^{-1}\left(\frac{\phi_1}{(1-\alpha)\, z}\right)\right)$$

when $z < \bar{z}$, there exists a unique global search equilibrium where all workers quit if and only if $\chi > \bar{\chi}(z)$, where $\bar{\chi}(z)$ is pinned down by:

$$\bar{\chi}(z) = w_1 + \beta\alpha z - \beta a z \int_0^1 p\left(\frac{\xi(1-u_1)(1-\gamma+\gamma A)}{u_1 + (1-u_1)(1-\gamma+\gamma A)}\right) dA$$

This proposition has two parts. In the first part, it characterizes the equilibrium when z is greater than $\bar{z}$. in this case we know that there is no strategic aspect in the worker's quitting behavior, as market tightness is pinned down by the free entry condition. Hence, the workers' decision to quit only depends on their idiosyncratic cost of work χ:

$$\pi(\chi) = \beta\left[p\left(q^{-1}\left(\frac{\phi_2}{(1-\alpha)z}\right)\right) - 1\right]\alpha z - w_1 + \chi$$

Hence the threshold determining whether a worker quit or not is pinned down by setting $\pi(\tilde{\chi}) = 0$. This gives the expression for $\tilde{\chi}(z)$.

The second part is more interesting. It characterizes part of the global search equilibrium when $z < \bar{z}$ and hence, when quitting becomes strategic complements. The threshold $\bar{\chi}(z)$ can be directly pinned down by substituting the expression of $\pi(\chi, A)$ into equation 6 and solve for $\bar{\chi}$.

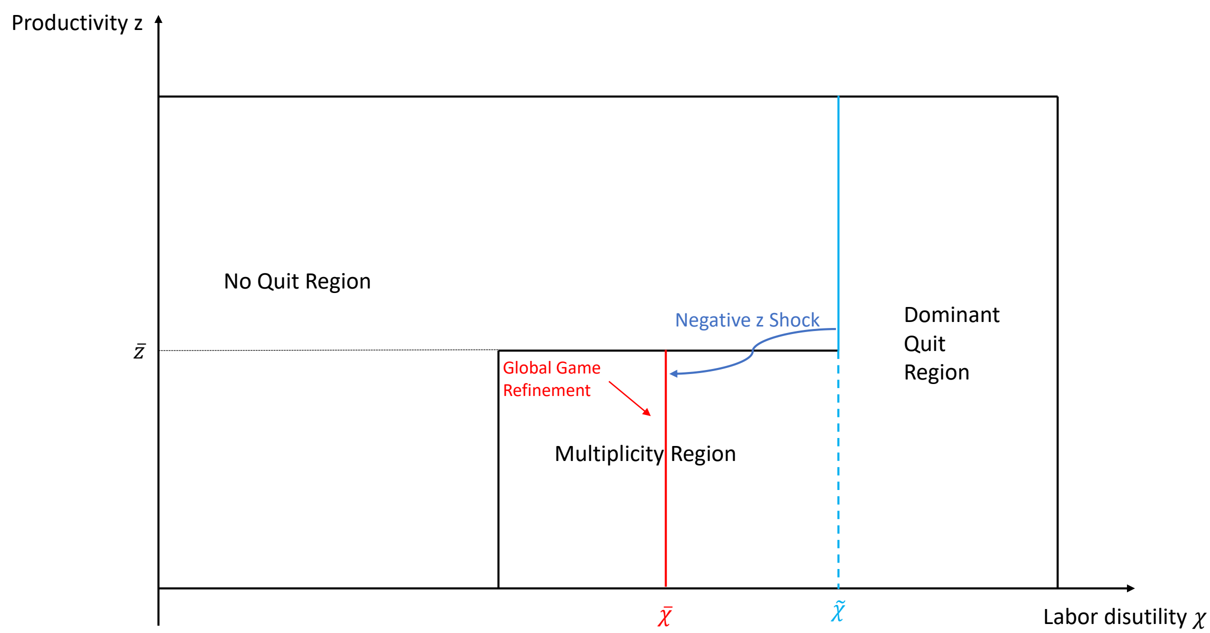
We now characterize the quitting thresholds. The following proposition shows that the quitting threshold becomes more stringent when productivity $z < \bar{z}$:

Proposition 7. *For any $z < \bar{z}$, $\bar{\chi}(z) < \tilde{\chi}(z)$. Hence there is a discrete downward jump in the quitting threshold $\bar{\chi}$ when productivity z decreases from $\bar{z}$ to $\bar{z} - \delta$, where δ is an arbitrarily small positive number.*

Figure 4 illustrates the proposition. The blue line depicts the $\tilde{\chi}$ threshold above which quitting becomes a dominant strategy, while the red line depicts the threshold under the global game refinement $\bar{\chi}$. The proposition states that the global game threshold is always below the dominant region threshold. This is because the refinement is with respect to the multiplicity region where the worker is not certain about the mass of worker who would quit. Hence they take an expectation over all possible cases ranging from none of the workers quit to all of the workers quit, and the threshold equilibrium lies in the middle of all these possible situations.

Thus, if there is a negative shock to productivity that pushes the value of z below the cutoff $\bar{z}$, this would induce a regime change and a downward shift of the quitting

Figure 4: Illustration of Proposition 7



threshold. As the value of the threshold becomes lower, workers are more likely to quit. This is the key mechanism of the model to generate quitting upon negative shocks.

5 Efficiency

Given the frictions in the worker's resignation behavior, one might wonder if there is any room for government intervention in this environment. To study this question, we need to first come up with an efficiency benchmark.

We will focus on the notion of constrained efficiency, under which a social planner cannot overcome the search friction, but can dictate payments to the workers, amounts of vacancies to post in both periods, and the share of workers that quit. To accord well with the timing of quitting in the previous section, we focus on an *interim* planner's problem where he takes as given the first period search and match and wage settings. A complete description of the planner's problem is the following:

$$\max_{v_2, w_2, A} p(\theta_1)\left(\gamma(1-A)(w_1 - \chi + \beta w_2) + (1-\gamma(1-A))(b + \beta p(\theta_2) w_2)\right)$$
$$+ (1 - p(\theta_1))(b + \beta p(\theta_2) w_2 + \beta(1 - p(\theta_2)) b)$$

$$s.t.$$

$$\theta_1 = v_1$$

$$\theta_2 = \frac{v_2 + \xi p(\theta_1)[1 - \gamma(1-A)]}{1 - p(\theta_1) + p(\theta_1)(1 - \gamma(1-A))}$$

$$q(\theta_2)(z - w_2) \le \phi_2, \text{ with inequality strict implying that } v_2 = 0$$

where v_1 and v_2 are the level of vacancy postings in period 1 and 2, and w_1 and w_2 are payments to the workers in both periods, and A which is the aggregate share of quitting. The objective function is the expected welfare of worker at the time of quitting. The first constraint says that market tightness in period 1 θ_1 is equal to v_1 because all workers are initially unemployed (note that neither v_1 or θ_1 is a choice variable in this interim social planner's problem). The second constraint describes the market tightness in period 2 and how it relates to vacancy postings and replacement hirings. The third constraint is the free entry conditions for the firm where the boundary situations are taken into account.

We will study the efficiency at the time of quitting, and how the planner would choose differently than the workers in terms of their quitting decisions.

We first assume that θ_2 does not vary with A, which is also the case when the free entry condition holds with equality. In this case, the planner would only ask the workers to quit if the payoff to quitting exceeds that from not quitting.

$$w_1 - \chi + \beta w_2 \le \beta p(\theta_2) w_2$$

Hence the quitting threshold $\chi^e(z)$ is given by

$$\chi^e(z) = w_1 + \beta w_2 - \beta p(\theta_2) w_2$$

When the free entry condition holds with equality, the quitting threshold becomes:

$$\chi^e = w_1 + \beta \alpha z - \beta p\left(q^{-1}\left(\frac{\phi_2}{(1-\alpha) z}\right)\right) w_2$$

This is exactly the same quitting threshold as $\tilde{\chi}(z)$ as shown in the proposition 6 (with the equilibrium wage $w_2 = \alpha z$), hence whenever $z \ge \bar{z}$ and hence the free entry condition holds with equality, quitting in the competitive equilibrium is socially efficient. The intuition is clear: the main source of externality in this model comes from the fact that individual workers do not take into account the impact of their actions on the equilibrium market tightness θ_2. This externality is muted, however, when the free entry condition holds with equality, in which case market tightness would be pinned down and would not change with aggregate amount of quits.

We now examine the case where the free entry condition holds with strictly sign, so that new entrant firms strictly prefer not to post vacancies $v_2 = 0$. In this case we know from the second constraint that market tightness would vary with A. Thus we can take first order condition with respect to A, and we obtain:

$$-\left(w_1 - \chi + \beta\alpha z\right) + \beta p\left(\theta_2\left(A\right)\right)\alpha z + \left[\left(1-\gamma\left(1-A\right)\right) + \frac{\left(1-p\left(\theta_1\right)\right)}{p\left(\theta_1\right)}\right]\beta p'\left(\theta_2\left(A\right)\right)\alpha z\theta_2'\left(A\right) = 0$$

Where the last term reflects the fact that changing aggregate share of quits A would have an impact on market tightness and hence worker's job finding probabilities in period 2.

Rearrange, we obtain that the quitting threshold in this case:

$$\chi^e\left(z\right) = w_1 + \beta\alpha z - \beta p\left(\theta_2\left(A\right)\right)\alpha z - \left(1-\gamma\left(1-A\right) + \frac{\left(1-p\left(\theta_1\right)\right)}{p\left(\theta_1\right)}\right)\beta p'\left(\theta_2\left(A\right)\right)\alpha z\theta_2'\left(A\right)$$

The last term are positive, reflecting the fact that the social planner would want more quitting instead of less. The reason is the following: when a worker quits, a job position is vacated and gets reposted onto the market, increasing the hiring probability of other workers. This externality is not internalized by the worker himself. Given the social benefit of quitting, the planner would like to encourage more quitting, and hence the quitting threshold is lower.

Comparing this quitting threshold to that in the global game equilibrium in the fragility region:

$$\bar{\chi}\left(z\right) = w_1 + \beta\alpha z - \beta\alpha z\int_0^1 p\left(\frac{\xi\left(1-u_1\right)\left(1-\gamma+\gamma A\right)}{u_1 + \left(1-u_1\right)\left(1-\gamma+\gamma A\right)}\right)dA$$

We see that the lower quitting threshold in the fragility region does not necessarily imply inefficiency. Although workers do not internalize the externality that works through market tightness, the self-fulfilling resignation may partially offset that externality by encourage more workers to quit. Generally, however, the government need to impose a wedge on the worker's quitting behavior to exactly replicate the socially efficient quitting threshold.

We summarize the discussion into the following proposition:

Proposition 8. *When $z \geq \bar{z}$, the free entry condition holds with equality, and hence no government intervention on the worker's quitting behavior is optimal. When $z < \bar{z}$, the government needs to impose the following tax on quitters to implement the efficient allocation:*

$$\tau(z) = \begin{array}{l} \beta\alpha z \int_0^1 p\left(\frac{\xi(1-u_1)(1-\gamma+\gamma A)}{u_1+(1-u_1)(1-\gamma+\gamma A)}\right) dA - \beta p\left(\theta_2(A)\right)\alpha z \\ -\left(1-\gamma(1-A)+\frac{(1-p(\theta_1))}{p(\theta_1)}\right)\beta p'\left(\theta_2(A)\right)\alpha z \theta_2'(A) \end{array}$$

Note that this tax rate can be negative, in which case it is optimal to subsidize quitting.

We will numerically explore this taxation scheme in the next section.

6 Countercyclical Wage Responses and Numerical Illustration

Note that all our previous results are derived at the quitting stage, taking first period wage w_1 and unemployment u_1 as given. In the general equilibrium, however, w_1 and u_1 would depend on fundamentals: z and χ. So to solve the full model we need to move to the very first steps of the model where initial wages and unemployment rates are determined.

Wage w_1 is assumed to be pinned down by Nash bargaining. This bargaining problem is non-trivial given the new ingredient of endogenous quitting. From proposition 6, we know that the quitting threshold depends on z, u_1, and w_1. Since for now we focus on wage determination, we omit other variables and write the quitting thresholds as

$$\bar{\chi}(w_1)$$

And we can also define the probability of retension for an individual worker:

$$R(w_1) = 1_{\{\chi<\bar{\chi}(w_1)\}}$$

which is an indicator function because the variance of the taste shock is taken to be arbitrarily to zero. The expected value to the worker if bargaining is successful is:

$$\gamma R(w_1)(w_1 + \beta\alpha z - \chi) + (1 - \gamma R(w_1))\beta p(\theta_2^*)\alpha z$$

where θ_2^* is the ex-post equilibrium market tightness, which individual workers take as given.

The corresponding expected profit to the firm is

$$\gamma R\left(w_1\right)\left(z-w_1+\beta\left(1-\alpha\right)z\right)+\left(1-\gamma R\left(w_1\right)\right)\xi\beta q\left(\theta_2^*\right)\left(1-\alpha\right)z$$

The threat point would be $\beta p\left(\theta_2^*\right)\alpha z$ for the worker and $\xi\beta q\left(\theta_2^*\right)\left(1-\alpha\right)z$ for the firm.

Proposition 9. *The bargaining set determining first-period wage w_1 is closed and convex.*

The important step would be to show that the bargaining set is convex. Shimer (2006) shows that if workers are allowed to conduct on-the-job search and quit to another job, then the bargaining set is no longer convex. This is not the case when workers can quit into non-employment, as I shown here that the convexity of the bargaining set is preserved.

The proof is illustrated in the following graph where in the top panel I plot worker's value as a function of wage and in the bottom panel I plot firm's value as a function of wage. When wage is sufficiently low, workers would decide to quit and hence both firm and worker receive the outside option which is flat in the wage. When wage is sufficiently high, worker's payoff would be increasing in the wage while the firm's payoff would be decreasing in it.

We need to show that for any value (say, point A and B), the linear combination of the two point still lies within the barganing set. Let us pick point C as an example. We need to find a level of wage such that the payoff to the firm and the worker is at least weakly higher than point C. To do so we can pick point D such that the worker would be indifferent between point D and point C, while the firm would strictly prefer point D. Hence we have shown that this linear combination lies within the feasible bargaining set.

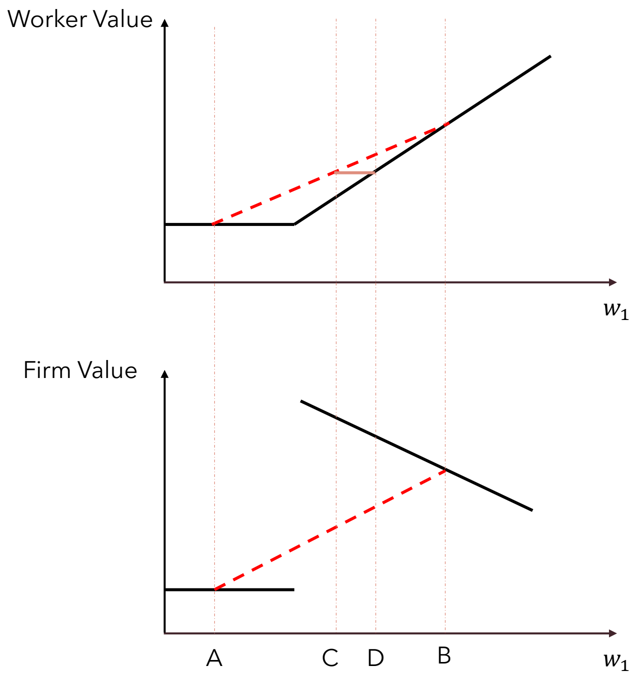
Hence w_1 solves the following Nash bargaining problem:

$$\max_{w_1^*}\left[\gamma R\left(w_1\right)\left(w_1+\beta\alpha z-\chi-\beta p\left(\theta^*\right)\alpha z\right)\right]^{\alpha}\left[\gamma R\left(w_1\right)\left(z-w_1+\beta\left(z-\alpha z\right)-\xi\beta q\left(\theta^*\right)\left(1-\alpha\right)z\right)\right]^{1-\alpha} \tag{7}$$

Given the bargained wage w_1^*, u_1 can be pinned down by the initial search and match problem. Denote firm profit as

$$\Pi\left(w_1^*\right)=\gamma R\left(w_1^*\right)\left(z-w_1^*+\beta\left(z-\alpha z\right)\right)+\left(1-\gamma R\left(w_1^*\right)\right)\xi\beta q\left(\theta^*\right)\left(1-\alpha\right)z$$

Figure 5: Convexity of Bargaining Set



, then free entry pins down market tightness in the initial period θ_1 :

$$q(\theta_1)\,\Pi(w_1^*) = \phi_1$$

This θ_1 then leads to $p(\theta_1)$, and hence we need to impose an equilibrium condition that second period initial unmatched workers

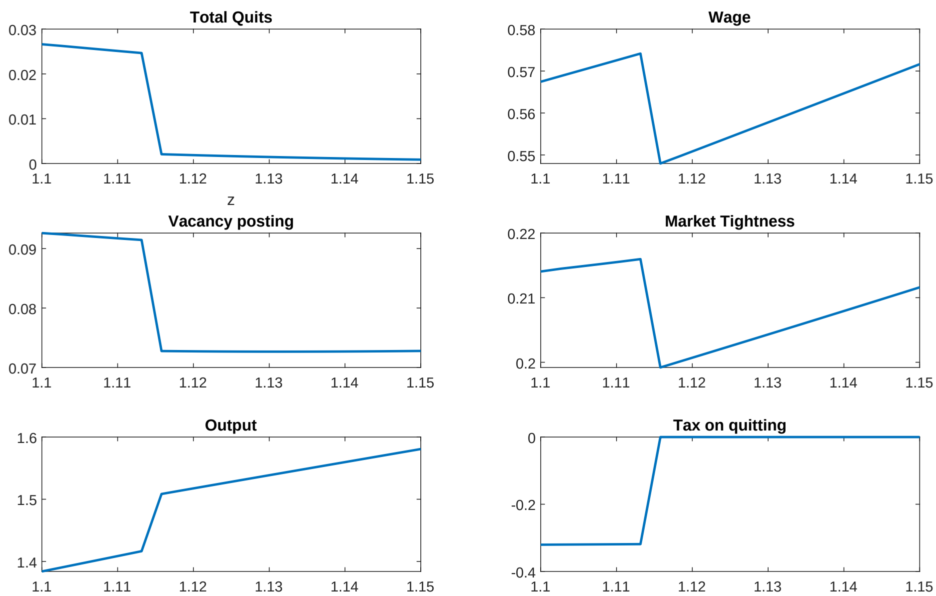
$$u_1 = p(\theta_1)$$

These equations pin down all the objects. The following theorem shows that the model could feature countercyclical wage fluctuations:

Proposition 10. *A reduction in productivity from $\bar{z}$ to $\bar{z} - \delta$ where δ is some positive and arbitrarily small number can lead to a rise in the equilibrium wage w_1^*.*

The intuition for the countercyclical wage fluctuation is as follows. In the model, quitting burns value because it would imply that all party receive a value of zero. So conditional on the realization of χ not too large, the firm and worker would strive to negotiate a wage such that quitting do not happen ex-post. From proposition 7 and figure 4 we

Figure 6: Numerical Example



know that the quitting threshold becomes lower if productivity decreases to below $\bar{z}$. A lower quitting threshold calls for a higher wage to deter quitting. This results in higher wages when productivity declines.

As the model features strong non-linearity, we are not able to solve the full model analytically. We thus resort to numerical simulation. Note that in this numerical simulation we relax the assumption that the noise in the taste shock is arbitrarily close to zero. A strictly positive noise level enables us to produce a more smooth version of how all equilibrium variables change as functions of economic fundamentals.

Figure 6 plots the model response to changes in labor productivity z.[3] As the value of z decreases, there is an abrupt regime shift around $z = 1.115$. This reflects that the model enters into the strategic quitting region. In this region, quit rate increases abruptly (top left panel), together with the amount of vacancy postings (middle left panel). Vacancy posting rate goes up because workers quit their job and therefore many vacancies get reposted in the beginning of period 2. As a result of more vacancy postings, labor market becomes tighter (middle right panel). More quitting also leads to higher wages (top right panel) because firms face a more stringent quitting constraint, and to preserve value the

[3]Parameter values used: $\xi = 1, \phi_1 = 1.5, \phi_2 = 1, \gamma = 0.9, \alpha = 0.6, \beta = 0.9, \chi = 0.5, \sigma_\chi = 0.1$.

negotiated wage needs to go up to address the excess quitting problem. All of these happen when total output declines due to the deterioration in productivity. Hence the model generates a jobful recession in which output declines but labor market becomes tighter with stronger wage growth (see figure 1).

The bottom right panel of the figure 1 plots the tax on quitting to implement the efficient allocation (see Proposition 8). When productivity is sufficiently high, the free entry condition with equality and hence the tax rate is zero – no intervention is optimal. When productivity is sufficiently low, however, the optimal subsidy is *negative*, implying that the positive externality in the worker's quitting behavior calls for the government to subsidize it, in the event that self-fulfilling resignation happens.

A successful theory must not only explain the current job-rich recession but also why such a recession did not occur in previous downturns where productivity significantly declined, such as the Great Financial Crisis. This paper suggests that there might be a shift in workers' attitudes towards labor. To illustrate this, figure 7 shows the alternative case where the labor disutility shock is set very close to zero,[4] with all other parameters remaining the same as in figure 6. In this scenario, the decline in productivity does not trigger strategic quitting, resulting in minimal changes in quit rates. Consequently, both wage and vacancy postings closely follow the output decline, aligning with previous jobless recessions. The theory proposes that workers' preferences for work appear to have shifted during this recession, making them more likely to quit, which is the key underlying structural change that explains the ongoing job-rich recession. In this case, there will be no government intervention as there is no surges in resignations in equilibrium.

7 Quantitative Exploration in an Infinite Horizon Model

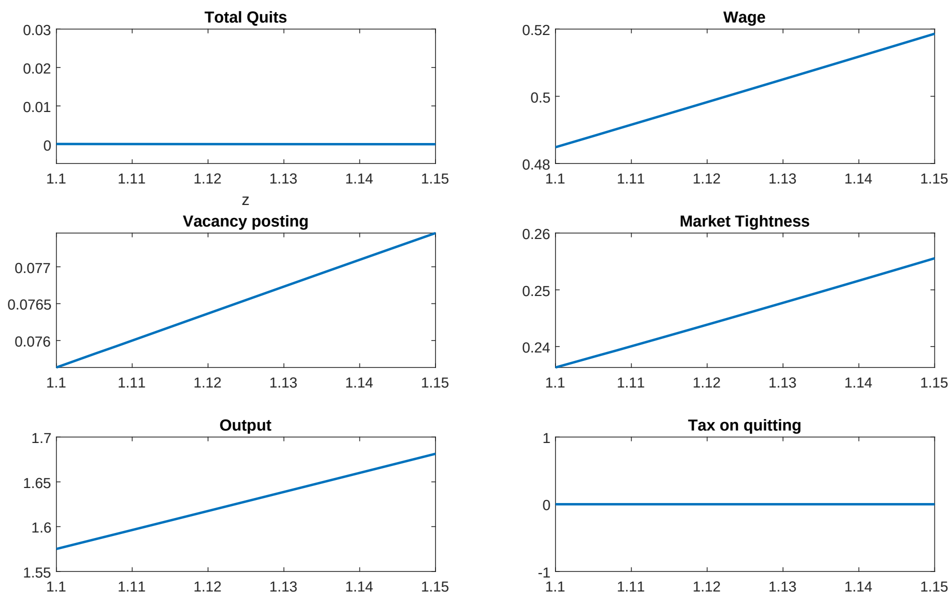
We now embed this mechanism into an infinite horizon model and assess its quantitative potentials. Time runs from 0 to infinity. Both firms and workers are risk neutral and discount future at rate β.

In the beginning of each period t, there are four state variables: first u_t the mass of unmatched workers; second v_t the amount of unfulfilled vacancies inherited from the past; third the productivity z_t and fourth the taste shock $\mu_{\chi t}$.

The first event is wage negotiation. All existing pairs of workers negotiate the current period wage w_t, where the worker's bargaining power is given by α.

[4]In figure 7, χ is set to 0.1.

Figure 7: Numerical Example with Low Labor Disutility



Then separation occurs. First $1-\gamma$ fraction of all the matches are destroyed exogenously. Then the private utility shock χ realizes and all remaining workers decide on whether to quit. The χ shock follows a distribution $F(\mu_{\chi t}, \sigma_\chi)$. Index all the remaining workers by $i \in [0,1]$. Denote their individual quitting decisions by

$$a_{it} = \begin{cases} 1 \text{ if quits} \\ 0 \text{ if not} \end{cases}$$

Hence

$$A_t = \int_0^1 a_{it} di$$

is the percentage of remaining workers who quits.

Then we get to the production stage. The share of workers are given by

$$\gamma (1-u_t)(1-A_t)$$

These workers work and earn w_t while the firms earn $z_t - w_t$.

Now the new productivity z_{t+1} as well as the new taste shock $\mu_{\chi t+1}$ realize. Then we get to the search and matching stage. The unmatched workers are given by

$$u_t + (1 - u_t)(1 - \gamma + \gamma A_t)$$

Also due to the worker quitting, there are unfulfilled vacancies:

$$v_t + (1 - u_t)\,\gamma A_t$$

Thus after the decay shock ξ there are

$$\xi\left[v_t + (1 - u_t)\,\gamma A_t\right]$$

remaing total vacancies.

Given that there are new firm entry e_t, there is random matching among workers and vacancies with market tightness given by

$$\theta_t = \frac{e_t + \xi\left[v_t + (1 - u_t)(1 - \gamma + \gamma A_t)\right]}{u_t + (1 - u_t)(1 - \gamma + \gamma A_t)}$$

The job finding rate would be $p(\theta_t)$ and the vacancy filling rate would be $q(\theta_t)$. Hence the next period state variables u_{t+1} and v_{t+1} are given by:

$$u_{t+1} = (1 - p(\theta_t))\,(u_t + (1 - u_t)(1 - \gamma + \gamma A_t))$$

$$v_{t+1} = (1 - q(\theta_t))\left[e_t + \xi\left(v_t + (1 - u_t)(1 - \gamma + \gamma A_t)\right)\right]$$

The key equilibrium variables to figure out are w_t, e_t, and A_t, θ_t.

7.1 Model Characterization

We work out the model through recursive induction. To figure out what new entrant e_t is, denote the future value for the firm to be π_{t+1} and the value of the workers to be V^e_{t+1}, and V^u_{t+1} respectively for employed and unemployed workers.

The free entry condition is given by:

$$q(\theta_t)\,\pi_{t+1} = \phi$$

where ϕ is the vacancy posting cost. And if the future profit of vacancy posting is too low, then the free entry conditon could hit the zero lower bound:

$$q(\theta_t)\,\pi_{t+1} < \phi, \text{ and } e_t = 0.$$

This means that there is no new vacancies being posted in the current period. Following results from the static model, we can set a threshold future productivity $\bar{\pi}_{t+1}$ below which the no new entrant constraint binds in period t.

For any $\pi_{t+1} < \bar{\pi}_{t+1}$, there will be no firm entry $e_t = 0$. Otherwise there will be positive firm entry, and the equilibrium market tightness is pinned down by the free entry condition equaling to zero.

Now move to the individual quitting decision. The payoff to quitting is given by

$$\pi(A_t,\chi) = b + \beta\left[p(\theta_t)\,V_{et+1} + (1-p(\theta_t))\,V_{ut+1}\right] - \left[w_t - \chi + \beta V_{et+1}\right]$$

where b is the value of leisure, V_{et+1} and V_{ut+1} are the values of future employment and unemployment respectively. Thus the first and the second term $b + \beta\left[p(\theta_t)\,V_{et+1} + (1-p(\theta_t))\,V_{ut+1}\right]$ measures expected utility for a worker that quits. The third term measures the value of working, where w_t is the current period wage and χ is the current realization of worker disutility shock.

Note that we already characterized θ_t: if $\pi_{t+1} \geq \bar{\pi}_{t+1}$, θ_t is pinned down by

$$q(\theta_t)\,\pi_{t+1} = \phi$$

Otherwise the no new entrant condition binds and

$$\theta_t = \frac{\xi\left[v_t + (1-u_t)(1-\gamma+\gamma A_t)\right]}{u_t + (1-u_t)(1-\gamma+\gamma A_t)}$$

One can show that if vacancy rate v_t is less than unemployment rate u_t, there would be strategic complementarity, whereby θ_t is increasing in the aggregate share of quits A_t. In this case, we can refine the equilibrium using global game techniques as in the static model, and the quitting threshold is given by

$$\int \pi(A_t,\bar{\chi})\,dA_t = 0$$

Hence we can solve for the quitting threshold:

$$\bar{\chi} = w_t - \beta \left[\int p \left(\frac{\xi \left[v_t + (1-u_t)(1-\gamma+\gamma A_t) \right]}{u_t + (1-u_t)(1-\gamma+\gamma A_t)} \right) dA_t - 1 \right] (V_{et+1} - V_{ut+1}) - b.$$

While if future profits are sufficiently high, then the quitting threshold is given by

$$\bar{\chi} = w_t - \beta (p(\theta_t) - 1)(V_{et+1} - V_{ut+1}) - b$$

where θ_t is given by the free entry condition.

With these quitting thresholds, we can obtain wages through Nash bargaining solution. With wages and all other equilibrium variables at hand, we can define the value functions for the current period:

$$V_{et} = \gamma R(w_t)(w_t - E(\chi|\chi < \bar{\chi}) + \beta V_{et+1}) + (1 - \gamma R(w_t)) \left(b + \beta p\left(\bar{\theta}_t\right) V_{et+1} + \beta \left(1 - p\left(\bar{\theta}_t\right)\right) V_{ut+1} \right)$$

$$V_{ut} = b + \beta p\left(\bar{\theta}_t\right) V_{et+1} + \beta \left(1 - p\left(\bar{\theta}_t\right)\right) V_{ut+1}$$

$$\pi_t = \gamma R(w_t)(z_t - w_t + \beta \pi_{t+1}) + (1 - \gamma R(w_t)) \left(0 + \beta \xi q\left(\bar{\theta}_t\right) \pi_{t+1} \right)$$

Note that all the equilibrium variables needs to be solved simultaneously within a period. Therefore in the algorithm we first assume that the no free entrant condition is slack and solve for all equilibrium variables in perido t. We then check if new entrant is indeed strictly positive. If yes, we are done. If not, we switch to the regime where $e_t = 0$.

Proposition 11. *In a steady state, the no new entrant condition never binds: $e_t > 0$.*

The intuition for this is clear: in an economy there is an aggregate amount of vacancies (whether it is filled or unfilled). Given that aggregate amount of vacancies depreciates, there needs to be new vacancies created every period to keep the total amount of vacancy the same. Hence the no new entrant condition can never be binding in a steady state.

We therefore study the model behavior out of the steady state. To do so, we need to first calibrate the model. The calibration is summaried in table 2. We adopt a monthly model. The matching function is given by $m = Au^{0.5}v^{0.5}$ where parameter A is calibrated to match a steady state unemployment rate. The taste distribution is picked to be a log normal with μ_χ being the mean of the associated normal distribution and σ_χ being its standard deviation. The quit rate is considerably lower than the quit rate series shown in figure 1 to reflect that fact that this is a model about just quit to non-employment and

Parameters	Value	Calibration
Discount rate	$\beta = 0.9968$	Monthly model
Worker bargain power	$\alpha = 0.6$	Standard
Value of leisure	$b = 0.9$	Standard
Separation rate	$\gamma = 0.98\%$	2% exogenous separation
Vacancy repost rate	$\zeta = 0.8$	Literature
Matching efficiency	$A = 0.8$	Unemployment rate 5.16%
Vacancy posting cost	$\phi = 2.5$	steady state vacancy rate 2.3%
Taste Distribution (mean)	$\mu_\chi = -4.3$	Quit rate 1%
Taste Distribution (std)	$\sigma_\chi = 0.5$	Literature

Table 2: Calibration

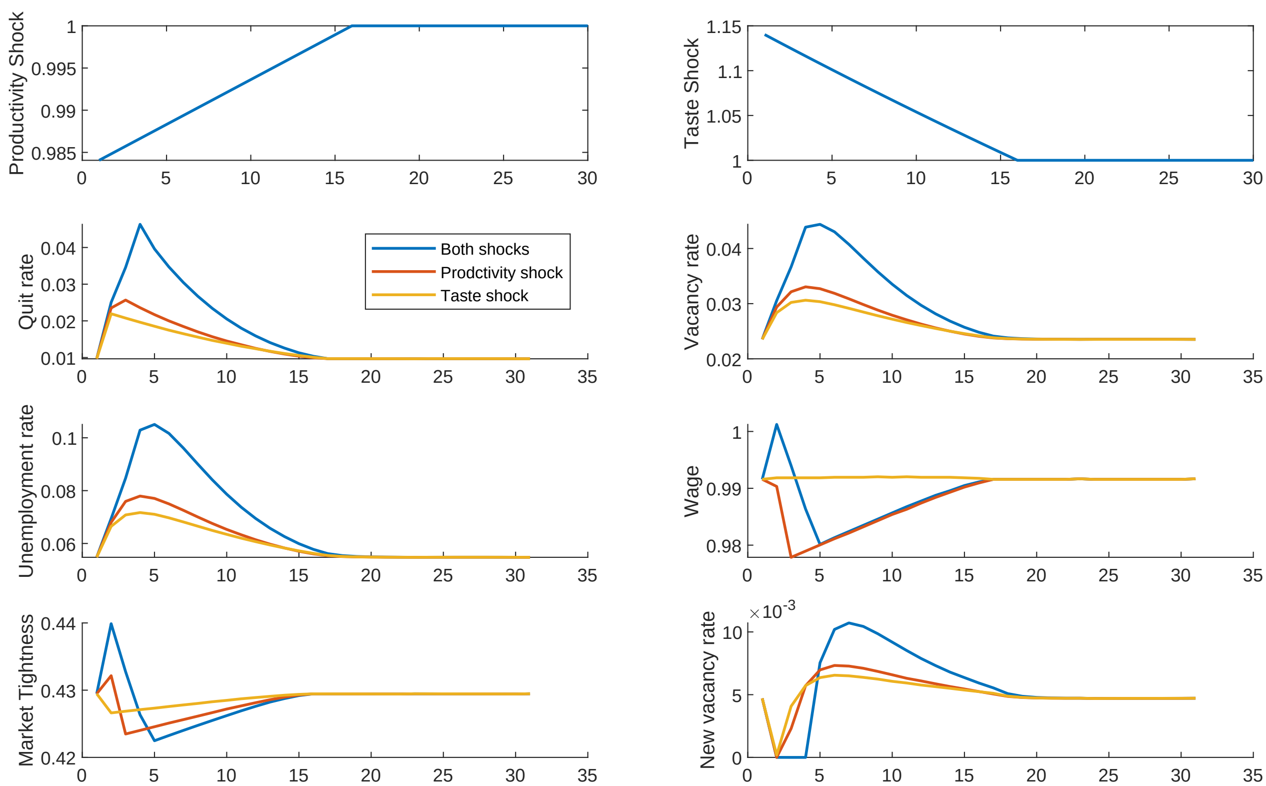
therefore quit due to job-to-job transition needs to be teased out. The vacancy reposting rate is roughly consistent with estiamtes and inferences by Mercan and Schoefer (2020) and Elsby et al. (2022).

The main quantitative result is summarized in figure 8 where I show the model's impulse response to productivity shocks and labor disutility shocks. I study a negative productivity shock of 1.5% and/or positive labor disutility shock of 15% (meaning that workers find it 15% more costly to work). Those shocks decay at constant rate as shown in the top two panels and last 16 months. The red line depicts the model's impulse response to just the productivity shocks, the yellow line just the taste shcoks, and the blue line a combination of both.

Relative to each shock alone, a combination of shocks generate much larger rise in the quit rate, vacancy rate, and unemployment rate. For example, with each shock along, the quit rate can be increased by just 1%, while a combination of both can increase the quit rate by nearly 4%. This is due to the fact that the no new entrant constraint is binding for relatively long periods in the case where both shocks are operative (bottom right panel).

Especially interesting is the behavior of wage (third right panel). With just productivity shocks, wage declines largely following the path of productivity – the wage is procyclical. With just the taste shocks, wage barely moves. Surprisingly, a combination of both shocks produce a *countercyclical* wage responses. The mechanism at work here is suggested by theorem 10: negative productivity shocks push the economy into a quit region and firms raise wages in order to curb inefficiently high level of quitting. Thus the model successfully generates strong wage growth in response to recessions, consistent with the narrative of a jobful recession.

Figure 8: Impulse Response



8 Conclusion

This paper presents a model of jobful recession: in which the key mechanism is the self-fulfilling nature of quitting and vacancy posting, but only in periods with low productivity. I demonstrate that multiple equilibria can arise in this environment and utilize global game techniques to refine these equilibria. In the unique equilibrium, I find that negative productivity shocks, combined with labor disutility shocks, can successfully generate a job-rich recession characterized by high quit rates, high vacancy rates, and strong wage growth.

There are several potential directions for future work to extend this paper. Currently, the model only features quitting into non-employment. It would be interesting to combine the mechanism proposed in this paper with another type of quitting due to on-the-job-search, and explore the role played by preference shocks, for example. As the model features a counter-cyclical wage response, examining optimal monetary policy in this setting could be interesting: how should monetary policy be set during a recession with a tight labor market? Additionally, it would be intriguing to investigate how the mechanism discussed in this paper interacts with workers' decisions to participate in the labor force. Lastly, since this paper is theoretical in nature, exploring a fully quantitative version of the model and examining the role of changing labor disutility in shaping the job-rich recession would be valuable. These questions are left for future research.

References

Blanco, Julio, Andres Drenik, Christian Moser, and Emilio Zaratiegui, "A Theory of Non-Coasean Labor Markets," 2023. 1

Cai, Zhifeng and Feng Dong, "Public Disclosure and Private Information Acquisition: A Global Game Approach," 2023. 4

_ **and Jonathan Heathcote**, "The Great Resignation and Optimal Unemployment Insurance," *Working paper*, 2023. 1

Elsby, Michael, Axel Gottfries, Ryan Michaels, and David Ratner, "Vacancy Chains," August 2022, (22-23). 1, 1, 7.1

Hosios, Arthur J., "On the Efficiency of Matching and Related Models of Search and Unemployment," *The Review of Economic Studies*, 1990, *57* (2), 279–298. 1

Mangin, Sephorah and Benoit Julien, "Efficiency in search and matching models: A generalized Hosios condition," *Journal of Economic Theory*, 2021, *193*, 105208. 1

Mercan, Yusuf and Benjamin Schoefer, "Jobs and Matches: Quits, Replacement Hiring, and Vacancy Chains," *American Economic Review: Insights*, March 2020, *2* (1), 101–24. 1, 1, 7.1

Morris, Stephen and Hyun Song Shin, *Global Games: Theory and Applications*, Vol. 1 of *Econometric Society Monographs*, Cambridge University Press, 1

Qiu, Xincheng, "Vacant Jobs," *Working paper*, 2022. 1

Shimer, Robert, "On-the-Job Search and Strategic Bargaining," *European Economic Review*, 2006, *50* (4), 811–830. 6

Appendix

Proof of Proposition 1

Workers' payoff function is given by

$$\pi\left(\chi, A\right)=\beta\left[p\left(\frac{\xi\left(1-u_{1}\right)\left(1-\gamma+\gamma A\right)}{u_{1}+\left(1-u_{1}\right)\left(1-\gamma+\gamma A\right)}\right)-1\right]\alpha z-w_{1}+\chi$$

The first two terms are negative:

$$\beta\left[p\left(\frac{\xi\left(1-u_{1}\right)\left(1-\gamma+\gamma A\right)}{u_{1}+\left(1-u_{1}\right)\left(1-\gamma+\gamma A\right)}\right)-1\right]\alpha z-w_{1}<0$$

because $p(.) \leq 1$ by assumption and equilibrium wage w_1 is strictly positive given a non-zero bargaining weight. Hence for χ sufficiently close to 0,

$$\pi\left(\chi, A\right)<0$$

regardless of the value of A. Hence in equilibrium no worker would quit.

Proof of Proposition 2

Worker's payoff function is given by

$$\pi\left(\chi\right)=\beta\left[p\left(q^{-1}\left(\frac{\phi_{2}}{\left(1-\alpha\right)z}\right)\right)-1\right]\alpha z-w_{1}+\chi$$

Thus the decision rule only depends on the wage w_1 and χ and the quitting threhshold for a worker to quit is that

$$\bar{\chi}=w_{1}-\beta\left[p\left(q^{-1}\left(\frac{\phi_{2}}{\left(1-\alpha\right)z}\right)\right)-1\right]\alpha z>0$$

Hence there exists a unique equilibrium under which workers following this quitting rule.

For χ sufficiently small $\chi<\bar{\chi}$, in this equlibrium no workers would quit.

Proof of Proposition 3

In text.

Proof of Proposition 4

We first show that market tightness is increasing in A. Take derivative with respect to market tightness expression:

$$\theta_2(A) = \frac{\xi (1-u_1)(1-\gamma+\gamma A)}{u_1 + (1-u_1)(1-\gamma+\gamma A)}$$

$$\begin{aligned}
\frac{d\theta_2}{dA} &= \frac{[\xi (1-u_1)\gamma](u_1 + (1-u_1)(1-\gamma+\gamma A)) - [\xi (1-u_1)(1-\gamma+\gamma A)](1-u_1)\gamma}{(u_1 + (1-u_1)(1-\gamma+\gamma A))^2} \\
&= \frac{\xi (1-u_1)\gamma u_1}{(u_1 + (1-u_1)(1-\gamma+\gamma A))^2} > 0
\end{aligned}$$

Given that θ_2 is strictly increasing in A, the job finding probability p is increasing in A too. Hence worker's payoff function

$$\pi(\chi, A) = \beta \left[p\left(\frac{\xi (1-u_1)(1-\gamma+\gamma A)}{u_1 + (1-u_1)(1-\gamma+\gamma A)} \right) - 1 \right] \alpha z - w_1 + \chi$$

is increasing in A.

Proof of Theorem 1

Given that worker's payoff function is increasing in A, it must be the case that $\pi(\chi, 1) > \pi(\chi, 0)$. Pick any $\chi \in (a, b)$ where

$$a = \beta \left[p\left(\frac{\xi (1-u_1)(1-\gamma)}{u_1 + (1-u_1)(1-\gamma)} \right) - 1 \right] \alpha z - w_1$$

$$b = \beta \left[p\left(\frac{\xi (1-u_1)}{u_1 + (1-u_1)} \right) - 1 \right] \alpha z - w_1$$

Then $\pi(\chi, 1) > 0$ and $\pi(\chi, 0) < 0$.

Since $\pi(\chi, 1) > 0$, all worker quit is an equilibrium in which all workers strictly prefer to quit. Since $\pi(\chi, 0) < 0$, $A = 0$ is also an equilibrium in which all workers strictly prefer not to quit.

Proof of Proposition 5

There are five requirements that needs to be satisfied in order for the unique equilibrium to exist.

1. Action monotonicity: $\pi(\chi, A)$ is non-decreasing in A

2. State monotonicity: $\pi(\chi, A)$ is non-increasing in χ

3. Strictly Laplacian state monotonicity: there exists a unique χ such that

$$\int_0^1 \pi(\chi, A)\, dA = 0$$

4. Limit Dominance

5. Continuity

One can verify that all conditions are satisfied with our specified payoff function when $z < \bar{z}$, hence a unique equilibrium exists. And the equilibrium cutoff is given by

$$\int_0^1 \pi(\bar{\chi}, A)\, dA = 0$$

See Cai and Dong (2023), section 2 for details of the derivations.

Proof of Proposition 6

For $z > \bar{z}$, workers' payoff functions are given by:

$$\pi(\chi) = \beta \left[p\left(q^{-1}\left(\frac{\phi_2}{(1-\alpha) z} \right) \right) - 1 \right] \alpha z - w_1 + \chi$$

setting worker's payoff to 0, we obtain a cutoff $\tilde{\chi}(z)$

$$\beta \left[p\left(q^{-1}\left(\frac{\phi_2}{(1-\alpha) z} \right) \right) - 1 \right] \alpha z - w_1 + \tilde{\chi}(z) = 0$$

Hence

$$\tilde{\chi}(z) = w_1 + \beta\alpha z - \beta\alpha z p\left(q^{-1}\left(\frac{\phi_2}{(1-\alpha) z} \right) \right)$$

For $z < \bar{z}$, the quitting threshold is pinned down by

$$\int_0^1 \pi(\bar{\chi}, A)\, dA = 0$$

where the payoff function is given by

$$\pi(\chi, A) = \beta\left[p\left(\frac{\xi(1-u_1)(1-\gamma+\gamma A)}{u_1+(1-u_1)(1-\gamma+\gamma A)}\right)-1\right]\alpha z - w_1 + \chi$$

Hence the cutoff $\bar{\chi}(z)$ is given by

$$\bar{\chi}(z) = w_1 + \beta\alpha z - \beta\alpha z \int_0^1 p\left(\frac{\xi(1-u_1)(1-\gamma+\gamma A)}{u_1+(1-u_1)(1-\gamma+\gamma A)}\right) dA$$

Proof of Proposition 7

We need to compare the expression of $\tilde{\chi}(z)$ and $\bar{\chi}(z)$ in the proof of Proposition:

$$\tilde{\chi}(z) = w_1 + \beta\alpha z - \beta\alpha z p\left(q^{-1}\left(\frac{\phi_2}{(1-\alpha)z}\right)\right)$$

and

$$\bar{\chi}(z) = w_1 + \beta\alpha z - \beta\alpha z \int_0^1 p\left(\frac{\xi(1-u_1)(1-\gamma+\gamma A)}{u_1+(1-u_1)(1-\gamma+\gamma A)}\right) dA$$

when $z < \bar{z}$, we know that new entrants strictly prefers not to enter:

$$q(\theta_2)(1-\alpha)z < \phi_2$$

where

$$\theta_2 = \frac{\xi(1-u_1)(1-\gamma)}{u_1+(1-u_1)(1-\gamma)}$$

Rearrange:

$$\frac{\xi(1-u_1)(1-\gamma)}{u_1+(1-u_1)(1-\gamma)} > q^{-1}\left(\frac{\phi_2}{(1-\alpha)z}\right)$$

For z sufficiently low, this condition will be met. Under this condition:

$$p\left(\frac{\xi(1-u_1)(1-\gamma)}{u_1+(1-u_1)(1-\gamma)}\right) > p\left(q^{-1}\left(\frac{\phi_2}{(1-\alpha)z}\right)\right)$$

and hence

$$\int_0^1 p\left(\frac{\xi\left(1-u_1\right)\left(1-\gamma+\gamma A\right)}{u_1+\left(1-u_1\right)\left(1-\gamma+\gamma A\right)}\right)dA > p\left(\frac{\xi\left(1-u_1\right)\left(1-\gamma\right)}{u_1+\left(1-u_1\right)\left(1-\gamma\right)}\right) > p\left(q^{-1}\left(\frac{\phi_2}{\left(1-\alpha\right)z}\right)\right)$$

It follows that

$$\bar{\chi}(z) < \tilde{\chi}(z)$$

Proof of Proposition 10

In the Nash bargaining problem, the firm-worker pair solves:

$$\max_{w_1^*}\left[\gamma R\left(w_1\right)\left(w_1+\beta\alpha z-\chi-\beta p\left(\theta^*\right)\alpha z\right)\right]^{\alpha}\left[\gamma R\left(w_1\right)\left(z-w_1+\beta\left(z-\alpha z\right)-\xi\beta q\left(\theta^*\right)\left(1-\alpha\right)z\right)\right]^{1-\alpha}$$

Note that if the worker quits, the joint value is equal to zero. So the wage must be set so that workers do not quit in equilibrium. This no-quitting constraint would be binding if α is sufficiently low (so that the unconstrained level of wage would always violate the no-quitting constraint).

When $z=\bar{z}$, the wage is set so that

$$\tilde{\chi}\left(\bar{z}\right)=\chi$$

Where χ is the level of taste shock. Plug in the expression

$$w_1+\beta\alpha\bar{z}-\beta\alpha z p\left(q^{-1}\left(\frac{\phi_2}{\left(1-\alpha\right)\bar{z}}\right)\right)=\chi$$

$$\tilde{w}_1=\chi-\beta\alpha\bar{z}+\beta\alpha z p\left(q^{-1}\left(\frac{\phi_2}{\left(1-\alpha\right)\bar{z}}\right)\right)$$

for $z=\bar{z}-\delta$,the wage is given by

$$\bar{\chi}(\bar{z}-\delta)=\chi$$

and w_1 is given by:

$$\bar{w}_1=\chi-\beta\alpha\left(\bar{z}-\delta\right)+\int_0^1 p\left(\frac{\xi\left(1-u_1\right)\left(1-\gamma+\gamma A\right)}{u_1+\left(1-u_1\right)\left(1-\gamma+\gamma A\right)}\right)dA$$

Given that

$$\beta\alpha\left(\bar{z}-\delta\right) > \beta\alpha\bar{z}$$

and

$$\int_0^1 p\left(\frac{\xi\left(1-u_1\right)\left(1-\gamma+\gamma A\right)}{u_1+\left(1-u_1\right)\left(1-\gamma+\gamma A\right)}\right)dA > p\left(q^{-1}\left(\frac{\phi_2}{\left(1-\alpha\right)\bar{z}}\right)\right)$$

We know that

$$\bar{w}_1 > \tilde{w}_1$$