Labor Participation, Human Capital Accumulation, and the Business Cycle

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March 14, 2024

Abstract

We explore labor force participation in a directed search model where overlapping generations make college education decisions and experience on-the-job human capital growth. Our model closely matches U.S. labor force participation and its response to GDP growth across different age groups and education levels. We find that subsidizing job search may lower welfare, as it discourages college enrollment and negatively impacts skill development in the long run. However, an age-based subsidy can raise overall welfare.

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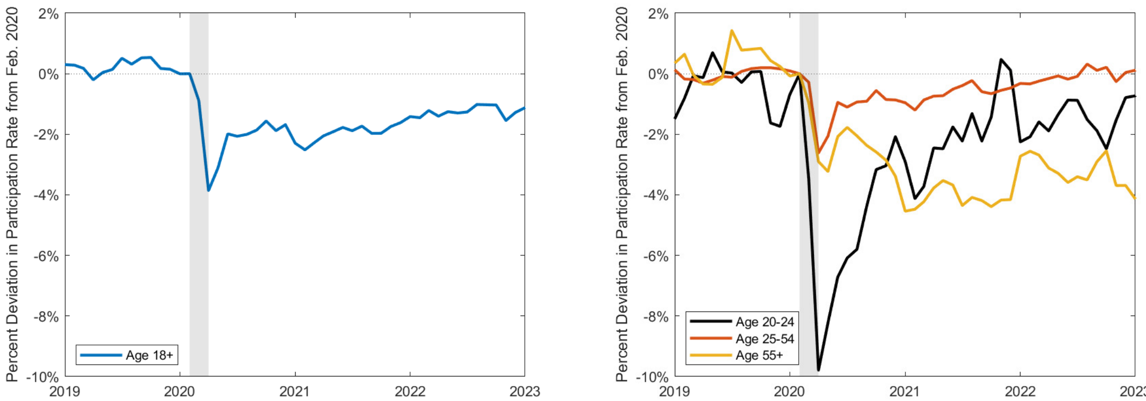
1 Introduction

Conventional labor market models primarily emphasize the dynamics between employment and unemployment, often disregarding the labor force participation margin. This limited focus may be innocuous if labor force participation is unresponsive to business cycles or policy changes. However, persistent declines in labor force participation are observed after economic recessions and the responses vary notably by age.[1] For example, *Figure 1* shows that both the young (20-24) and old (55+) populations exhibited a more significant and persistent decline in labor participation during the 2020 recession compared to prime-aged (25-54) individuals.

Figure 1: Overall and Age-specific Labor Participation Rates



Notes: This data was obtained from the monthly Current Population Survey (CPS) and was seasonally adjusted. Individuals who reported not participating due to illness, disability, or because they were caring for their home/family were excluded from the sample.

In line with recent research (Krusell et al., 2011, 2017, 2020), we explore endogenous labor force participation in a heterogeneous-agent model incorporating search and matching frictions. We focus on participation over the life cycle and its interactions with households' education and human capital accumulation choices. Empirically, we establish that the participation of younger individuals is most responsive to aggregate fluctuations, while the response of those aged 55+ is more persistent. Given these empirical findings, we seek to address the following questions: Why is the participation decision of certain age groups more susceptible to business cycle shocks? Can government intervention, such as workforce subsidies, improve overall welfare? If subsidies are considered, should they be contingent on age?

Our paper addresses these questions from both empirical and quantitative perspectives. We begin by documenting micro-level evidence on age-based labor force participation patterns using data from the Current Population Survey (CPS). Our findings reveal an inverted U-shape in the participation rates across various age groups, with the youngest

[1]See Hobijn and Şahin (2021), Cajner et al. (2021), and Cairo et al. (2022) for evidence on how labor participation responds to business cycle shocks.

and oldest populations exhibiting the lowest labor participation rates relative to prime-aged individuals. This pattern is consistent across different education levels (college vs. non-college), although college graduates generally display a higher propensity to participate in the labor force throughout the entire life cycle. We then investigate how labor force participation responds to shocks to aggregate GDP growth. Using vector autoregression (VAR) methods, we find that the responses exhibit a U-shape, with the youngest and oldest populations being the most responsive to changes in GDP growth rates. This pattern persists across different education levels, although individuals with college degrees generally demonstrate lower elasticity with respect to GDP growth changes. The U-shaped participation response aligns with previous research, which documents a U-shaped age profile of labor supply response to changes in the business cycle (Clark and Summers, 1981; Ríos-Rull, 1996; Gomme et al., 2005; Jaimovich and Siu, 2009; Erosa et al., 2016). Our study differs from those in that we examine how labor force participation reacts to business cycle shocks, while their focus is on labor supply responses.[2] Furthermore, our findings encompass more recent time periods, including the Great Recession and the Pandemic recession.

To explain these empirical findings, we develop an overlapping generations model that incorporates endogenous college choices and on-the-job human capital accumulation. In our model, attending college is costly but boosts expected lifetime income. Endogenizing the college enrollment decision enables us to match the relatively low labor participation among younger individuals aged 18-24, while on-the-job learning helps explain the high labor participation among prime-aged populations. In addition, the model predicts that older populations exhibit lower participation, due to a horizon effect - they are closer to retirement. Hence, the expected return from search is lower, making them less likely to continue participating in the labor force following a job loss.

Regarding the participation margin, we assume that each non-employed individual draws a stochastic cost of participation in each period and decides whether to participate in labor market search. The cost distribution is the key object to be disciplined in our quantitative exercise, and we calibrate it to match the overall participation-age profile in the data. Given that our model features an endogenous college sector, we match statistics on college enrollment, graduation, and the college wage premium. Finally, we replicate key labor market statistics, such as unemployment and vacancy posting rates. The calibration produces a relatively compressed cost distribution, indicating that older populations are almost indifferent regarding participation in the labor market. The identification comes from the sharp drop in the labor force participation rate among older populations.

[2] Erosa et al. (2016) studies labor supply responses at the intensive and extensive margins and identifies the response of groups of individuals that are non-employed. Their notion of non-employment is different from our notion of non-participation as our study excludes those who are actively searching for work.

We validate the quantitative model by confronting its empirical predictions with a set of untargeted, conditional moments. First, our model replicates the hump-shaped participation-age profile, *conditional on education*.[3] It predicts that, among older individuals, the labor participation rate decreases more rapidly for non-college workers compared to college graduates, which aligns well with the data. This sharper decline in participation as individuals approach retirement age among those without a college degree is due to different human capital dynamics among the two groups. Non-college workers typically experience higher unemployment rates, and due to fewer employment opportunities, their human capital grows at a slower rate. Hence their participation rate declines at a faster rate as they age because they do not benefit as much from employment.

Second, the model accounts well for the response of labor participation to GDP growth shocks, conditional on both age and education. The stochastic participation cost plays a key role here. Highly educated and prime-aged individuals have the highest expected values of work, leading to high participation thresholds. Hence their labor participation choices exhibit low sensitivity to variations in economic fundamentals. In contrast, non-college graduates, younger, and older populations have low relative values of work, which can be attributed to either less skill, the need to attend college, or being close to retirement. Consequently, their participation threshold is relatively low, indicating relatively greater elasticity in response to economic fundamentals.

Third, the model matches some key aspects of the college sector. It produces a realistic level of college wage premium and captures its evolution over the life cycle. It also generates an age profile of college students that closely aligns with data. For instance, although the model puts no age restriction on college enrollment, almost no individual over 30 enrolls in college, consistent with the data. Moreover, the model generates the right level of sensitivity of college enrollment with respect to business cycle shocks. In the model, a 1% positive productivity shock reduces college enrollment by 0.28% percent. We document, using VAR techniques, that the corresponding statistic in the data is 0.3%.

Overall, our micro-founded quantitative model successfully replicates important features of the labor market and college market. We thus use the model to examine the long-term welfare implications of introducing subsidies to incentivize job-seeking. These subsidies can be thought of as capturing unemployment insurance, job search assistance, or general workfare programs aimed at enhancing the job prospects of recipients (Pavoni and Violante, 2007).

Our first experiment studies a uniform subsidy given to all job-seekers regardless of age. Surprisingly, we find that such a policy reduces steady-state welfare, even if it narrows consumption gaps and enhances aggregate employment. This result is driven by the endogenous college attendance margin. In the model, income taxes are used

[3] In the calibration, we match the overall participation-age profile and college wage premium without directly targeting any moments from the conditional participation-age profile.

to provide insurance again unemployment risks but also generate a fiscal externality concerning college enrollment, as a portion of the benefits from attending college accrues to the government in the form of tax revenues. Job search subsidies exacerbate this externality by further discouraging college attendance among young people, reducing aggregate human capital, wages, and overall welfare.[4]

Considering that the college margin primarily impacts younger cohorts, we also explore age-based subsidies that target job-seekers aged 25-54 and 55+. We find that only the 55+ policy can improve steady-state welfare. Subsidizing the old is desirable because it is cost-effective, as our quantitative model suggests that older workers are often on the verge of whether to participate in the labor market. As a result, their labor force participation decisions are sensitive to even small increases in job search subsidies. In contrast, prime-aged workers (25-54) are mostly already participating in the labor market, and their participation decisions are less responsive to subsidies. Therefore, subsidizing only the older cohort achieves the most significant increase in labor participation with the least tax burden on the working population.

Our welfare findings offer new insights into the debate surrounding the desirability of workfare programs, such as those implemented in the US, UK, Canada, and other countries. In general, these programs either overlook the age aspect or target relatively young populations. For instance, the UK's Welfare-to-Work program allocates most of its funding to the New Deal for Young People (NDYP), a program targeting individuals aged 19 to 24 whose unemployment rate was high in the 1990s. Our findings suggest that such a policy could inadvertently discourage college attendance, negatively affecting the economy in the long run. In contrast, subsidizing older populations may be a cost-effective strategy for increasing long-term output and welfare.

Our final experiments focus on how the financial assistance to job seekers, or search subsidy, should vary over the business cycle. In particular, would providing a more generous search subsidy in recessions improve welfare? Our findings demonstrate that such a policy contributes to a negligible change in welfare. The main rationale for increasing the subsidy amount in recessions might be to smooth consumption over the business cycle. However, this benefit is relatively small in our model given the modest volatility of aggregate productivity. Furthermore, the decision to bolster search subsidies during recessions introduces an inefficiency by inadvertently urging workers to intensify their job search efforts during periods when vacancy posting is lower. Our findings suggest that while such a policy may temper fluctuations in output, it also results in a decline in average aggregate consumption due to the efficiency loss. This, in turn, leads to negligible, or potentially even negative, shifts in welfare.

[4]To highlight the college enrollment channel, we explore a counterfactual experiment where college attendance choices do not respond to changes in government policy. That model predicts that subsidizing the search of all ages would increase welfare.

Literature Review The paper contributes to the literature on labor market participation over the business cycle. Early work, such as Tripier (2003) and Veracierto (2008), predict that standard RBC models extended with a frictional labor market and a participation margin generate counterfactual labor market outcomes. In contrast, work by Krusell et al. (2011), Elsby et al. (2015), Krusell et al. (2017, and 2020) focus on gross flows in the labor market and find that a model with wealth heterogeneity, along the line of Chang and Kim (2006), can match those flows well. Christiano, Trabandt, and Walentin (2021) find that introducing a labor participation margin into a standard monetary model enhances its performance against business cycle shocks. Unlike these previous studies, this paper emphasizes the role of age and human capital accumulation in driving labor participation decisions, providing a novel perspective in the field. Furthermore, our work generates new policy implications, while previous works primarily focus on positive analysis.[5]

Similar to our work, Goensche et al. (2022) examines the patterns of labor participation responses throughout the life cycle and explores design of labor market policies. In contrast to their approach, we integrate endogenous college choices and human capital accumulation into our life cycle framework. These new ingredients play a critical role in our policy experiments, particularly in understanding the potential advantages of age-dependent subsidies over uniform subsidies.[6] Additionally, we explore how labor force participation reacts to fluctuations in the business cycle, both empirically and within our model.

Our paper contributes to the empirical literature on "participation cycles" concerning the estimation of how participation responds to business cycle forces. This has been investigated by work including Elsby et al. (2019), Hobijn and Şahin (2021), and Cajner et al. (2021). Our approach is similar to that of Cairo et al. (2022), where they employ a VAR (Vector Autoregression) approach to estimate the impulse response function for labor participation. While Cairo et al. (2022) primarily focuses on aggregate patterns, our paper utilizes a disaggregated VAR to uncover how participation responses to business cycle shocks vary according to age and education.

This paper also connects to the literature on college enrollment and labor markets. By examining the sensitivity of college enrollment to business cycles and labor market policy changes, it aligns with empirical work by Dellas and Sakellaris (2003), Barrow and Davis (2012), Barr and Turner (2015), and Long (2015). The structural approach used in the paper allows us to explore how policy variations in the labor market could impact welfare, taking into account their impact on college enrollment decisions.

[5]For additional research on search models involving endogenous participation margins while excluding life-cycle considerations, see Haefke and Reiter (2011) and Griffy and Masters (2022).

[6]While Goensche et al. (2022) does not impose a government budget constraint, we incorporate a government budget balancing condition in our computation of equilibrium so that we obtain a well-defined notion of welfare comparison across different scenarios.

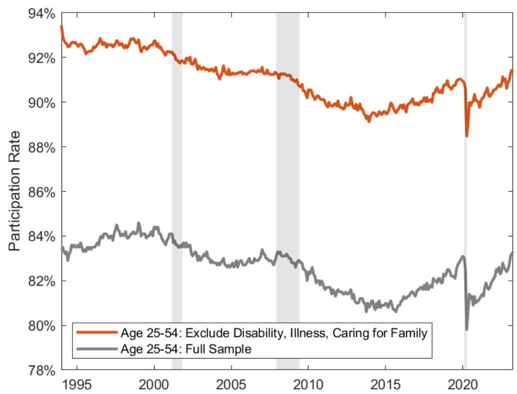
Finally, the paper's policy analysis is related to research on Welfare-to-Work programs, such as the work of Pavoni and Violante (2007), Pavoni et al. (2012), and Pavoni et al. (2016). Similar to those previous works, this paper investigates the influence of human capital on optimal labor-market policy design. However, the paper simplifies the analysis by considering a straightforward subsidy to join the labor force, allowing for a focused discussion on the novel aspects of the model, including life cycles, human capital growth, and endogenous college choices. Michelacci and Ruffo (2007) studies optimal age-dependent unemployment insurance in a life-cycle model with endogenous human capital accumulation. We also explore age-based subsidies, but in an equilibrium search model with a participation margin and an endogenous college enrollment channel.[7]

2 Empirical Motivation

2.1 Labor Participation Across Age and Education

We access data on labor force participation in the U.S. at a monthly frequency from the Current Population Survey (CPS) starting in January 1976.[8] However, information on the reason why an individual reports being out of the labor force, such as illness or disability, is available only from January 1994 onward. *Figure 2* displays prime-age labor force participation after excluding those who choose not to participate due to disability, illness, or because they report caring for their house or family. We will focus on

Figure 2: Participation Rate: Excluding Disability, Illness, or Caring for Family



Notes: This data is obtained from the monthly Current Population Survey (CPS) and was seasonally adjusted using the X-13ARIMA-SEATS seasonal adjustment software produced by the U.S. Census Bureau. Individuals who reported not participating in the labor force due to illness, disability, or because they were caring for their home or family were excluded from the sample.

this measure of labor force participation throughout both our empirical and quantitative analyses, as our attention will be on individuals' participation choices apart from ex-

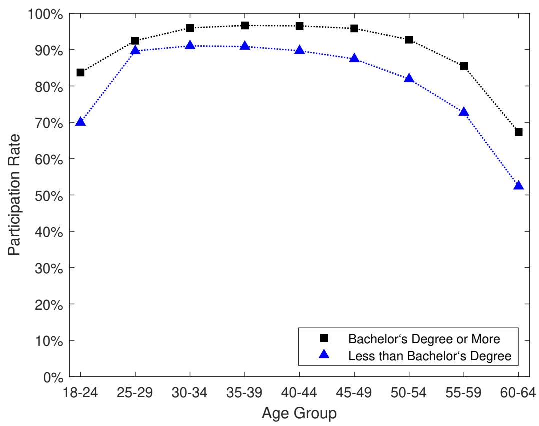
[7] For European-focused studies on age-dependent employment policies, see Chéron et al. (2011).

[8] This is the time frame for which this data is widely available online.

traneous circumstances related neither to education nor the perceived net benefit of job search. Even after excluding those who choose not to participate due to family or health concerns, labor force participation dropped notably during the 2020 pandemic recession and exhibits a possibly pro-cyclical behavior over other time-frames as well.

We now turn to study how labor participation varies over the life cycle. *Figure 3* plots the participation rate across different ages and educational attainment groups using CPS data, where we continue to exclude those not participating due to disability, illness, or because they were caring for their house or family. Throughout their entire life cycle, individuals with higher educational attainment demonstrate a higher likelihood of participating in the labor force. *Figure 3* illustrates that individuals holding a Bachelor's degree or higher are approximately 10 percent more likely to be in the labor force across various age groups. Labor participation begins to decline around the age of 40 for both education groups; however, the rate of decline is more pronounced for non-college graduates than college graduates. At ages 40-44, the difference in the participation rate between the two groups is under 10 percent, but this gap expands to around 15 percent by ages 60-64.

Figure 3: Participation by Educational Attainment over the Life-Cycle



Notes: This data is obtained from the monthly CPS, where the participation rate of each group reported is the average participation rate of that group over January 2000 - December 2022. Individuals who reported not participating in the labor force due to illness, disability, or because they were caring for their home or family were excluded from the sample.

2.2 Responsiveness of Labor Force Participation to Aggregate Shocks

This subsection investigates the relationship between participation and aggregate fluctuations. *Table 1* displays the results of regressing the participation rate of different age groups on GDP growth and the past four lags of both GDP growth and percentage change in the participation rate of that age group. The table shows that 1% higher GDP growth is associated with around a 0.5% immediate increase in the participation rate of those aged 20-24, while this same change in GDP growth is only associated with approximately

Table 1: Responsiveness of Participation Rate to Changes in Real GDP Growth

% Change in Participation	Age 20-24	Age 25-54	Age 55+
Real GDP Growth	**0.496******	**0.144******	**0.162******
	(0.062)	(0.0144)	(0.051)
Real GDP Growth (t-1)	**0.245******	**0.043*****	**0.091****
	(0.078)	(0.199)	(0.054)
Real GDP Growth (t-2)	0.002	-0.010	0.012
	(0.081)	(0.0192)	(0.053)
Real GDP Growth (t-3)	**0.052**	**-0.013**	**0.112*****
	(0.081)	(0.019)	(0.052)
Real GDP Growth (t-4)	0.079	-0.001	0.013
	(0.080)	(0.019)	(0.052)
% Change Participation (t-1)	-0.324***	-0.121	-0.245***
	(0.098)	(0.098)	(0.094)
% Change Participation (t-2)	-0.154	0.042	0.191*
	(0.103)	(0.097)	(0.088)
% Change Participation (t-3)	-0.028	0.284***	0.375***
	(0.104)	(0.099)	(0.089)
% Change Participation (t-4)	-0.088	0.103	0.141
	(0.099)	(0.096)	(0.091)
Constant	-0.661***	-0.103***	-0.119
	(0.145)	(0.030)	(0.106)
Observations	112	112	112
R^2	0.480	0.520	0.277

Notes: The participation data used here are quarterly averages of monthly, seasonally adjusted, data from the CPS. Individuals who reported not participating due to illness, disability, or because they were caring for their home/family were excluded from the sample. Real GDP data is from the U.S. Bureau of Economic Analysis, Real Gross Domestic Product [GDPC1]. The data spans 1994 Q1 to 2023 Q1. Stars denote statistical significance: $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$.

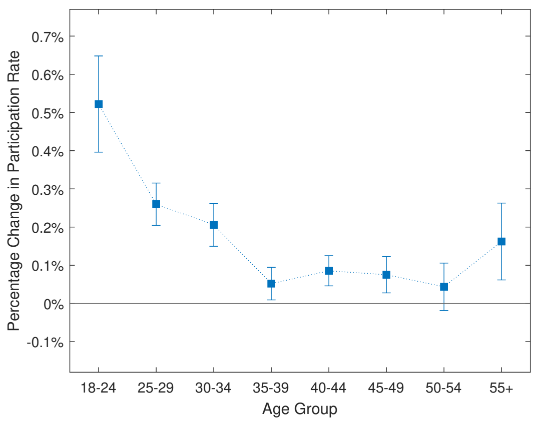
a 0.14% increase in the participation rate of prime age individuals.[9] For individuals aged 55 and older, 1% higher GDP growth is associated with an immediate 0.16% increase in their participation rate, with a lingering effect associated with a 0.11% increase even three quarters later, suggesting a more persistent impact on this age group. This pattern is consistent with the motivation in *Figure 1*.

Labor force participation is responsive to aggregate fluctuations. Additionally, different groups of individuals have distinct levels of opportunity cost associated with labor force participation, which are likely differently affected by aggregate shocks. To further investigate how business cycle fluctuations affect different age groups' participation rates, we compute participation rates for eight age groups using seasonally adjusted CPS data from 1994 through the first quarter of 2023, where non-participants due to disability, illness, or caring for home/family are discluded. After estimating a VAR of the participation rate of each group on GDP growth and the past four lags of both GDP growth

[9] *Appendix A* shows that these results do not notably change when data regarding real GDP per capita, rather than real GDP were used. It also shows that the results presented in the section are robust when considering changes in TFP rather that real GDP growth.

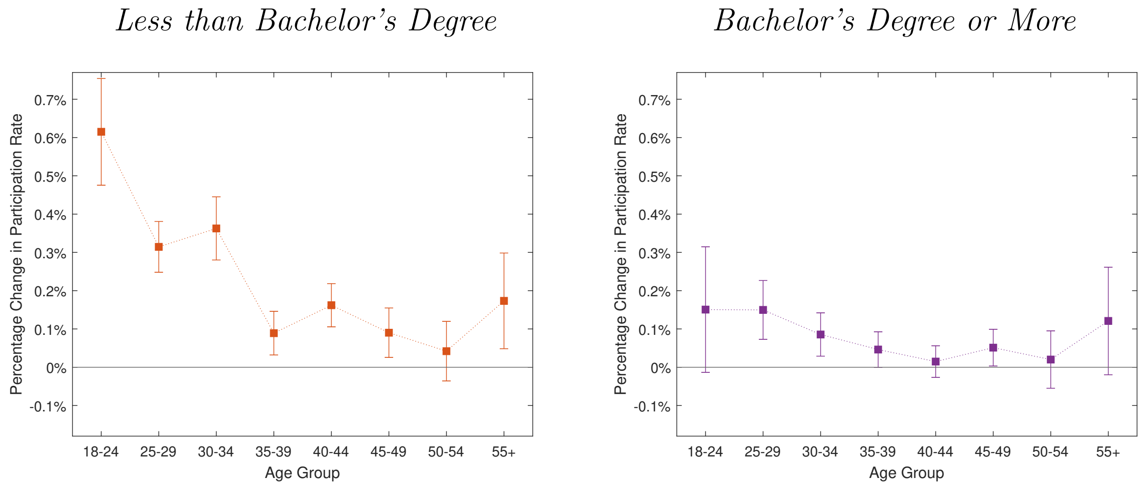
and participation, it is apparent that the participation of younger and older workers is more responsive to changes in GDP growth than that of middle-aged workers. *Figure 4* displays the percentage change in the participation rate of each group associated with 1% higher GDP growth in the same quarter. In other words, the figure displays the immediate (same-quarter) percentage change in participation associated with 1% higher GDP growth, while there are lagged responses to the change not displayed. These same-quarter responses are estimated using a VAR including four lags of both the percentage change in the seasonally-adjusted participation rate from the preceding quarter and the percentage change in real seasonally-adjusted GDP from the preceding quarter.

Figure 4: Same-Quarter Percentage Change in Participation Rate Associated with 1% Higher GDP Growth



Given that the value of participation varies significantly for individuals with different education levels, it is unsurprising that the participation of those without a Bachelor's degree is much more responsive to aggregate fluctuations than those with a Bachelor's or more. *Figure 5* shows the estimated same-quarter percentage change in the participation rate of individuals segmented both by age and education associated with 1% higher GDP growth. After controlling for education, we see that it is primarily younger individuals without a Bachelor's degree that account for much of the change in participation following aggregate fluctuations. Conditional on education, the participation profile displays an inverse U shape where the young and old are the most sensitive to GDP growth shocks.

Figure 5: Same-Quarter Percentage Change in Participation Rate Associated with 1% Higher GDP Growth by Education Group



When estimating the responsiveness of participation to changes in GDP growth, we may worry that changes in participation are perhaps instead affecting GDP growth. Additionally, some external factors, such as changes to the productivity of labor could affect both GDP and participation. To address this issue, in *Appendix A* we compare the response of participation to changes in business sector output and total factor productivity (TFP) estimated in Fernald (2014).[10] Our empirical results are robust to alternative measures of productivity, and are not driven by an external factor affecting both output and participation.

2.3 College Enrollment Over the Business Cycle

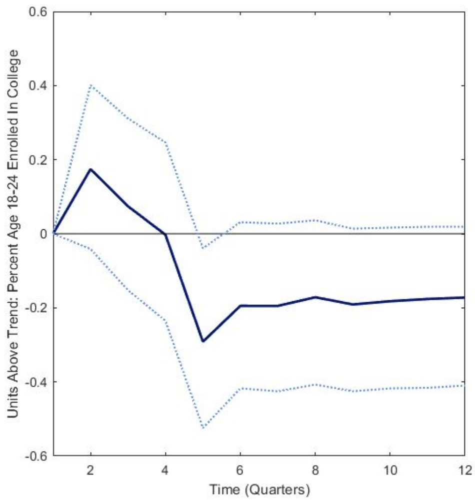
A modest literature exists estimating the change in college enrollment over the business cycle. The literature generally agrees that enrollment typically increases during recessions. To investigate the responsiveness of college enrollment to business cycle fluctuations, we take the quarterly average of the monthly enrollment rate and combine it with quarterly GDP data.[11] *Figure 6* shows an impulse response function (IRF) estimating the response of the college enrollment rate for individuals aged 18-24 following a 1% *increase* in GDP growth. To compute this impulse response function, we first estimate a bivariate VAR in GDP growth and the 18-24-year-old enrollment rate using data from

[10]This data was obtained from https://www.johnfernald.net/TFP, which provided an updated data set including estimates through the fourth quarter of 2022.

[11]Data on college enrollment (including both full- and part-time enrollment) is taken from CPS and spans from January 1990 through December 2022, and this data was seasonally adjusted using the X-13ARIMA-SEATS seasonal adjustment software produced by the U.S. Census Bureau. See census.gov/srd/www/x13as/ for details. Data on U.S. real GDP is from the U.S. Bureau of Economic Analysis GDPC1 series and was obtained from the Federal Reserve Bank of St. Louis FRED site.

1990 through 2019.[12] In estimating this VAR, a lag length of four was selected for the quarterly data. We then use the Cholesky decomposition to separate the variance of the enrollment rate due to changes in GDP growth.[13]

Figure 6: College Enrollment Response to a 1% GDP Growth Increase



The IRF shows that a 1% increase in GDP growth is expected to result in a decrease in the enrollment rate by about 0.3 percentage points after five quarters.[14] There is a lag between the change in GDP growth and the enrollment change, likely due to the time needed to apply before enrolling in college classes. Once it occurs, this estimated change in enrollment is persistent and remains around the same level even 12 quarters following the shock. These results suggest that there is usually a college enrollment channel present over the business cycle, whereby young individuals are more likely to enroll during recessions when the opportunity cost of enrollment is low. They are less likely to enroll during expansions when the opportunity cost is high.

[12]The response of college enrollment appeared quite different during the 2020 recession compared to the rest of the sample, possibly due to extraneous factors, including many classes having been moved online. We estimated the same IRF using data through the end of 2022 and found the same general pattern displayed in *Figure 6*, but with significantly wider confidence bands. Because of the extraneous factors that may have altered the enrollment response during the 2020 recession, we limit our data set to pre-pandemic observations when estimating the size of this response.

[13]In *Figure 6*, the dotted lines indicate the 95% confidence bands. These bands were computed using the VAR toolbox created by Ambrogio Cesa-Bianchi in Matlab. The code in the VAR toolbox computes the confidence bands by first using the bootstrap method to generate artificial data starting with observation 5 to the last observation, the first 4 observations (this is the number of lags used) remain as they are in the original data. A VAR is then estimated on this artificial data, and an impulse response is computed. This is done 10,000 times, and the confidence bands are then set to indicate the area that 95% of these 10,000 impulse responses fall within.

[14]We calculate the corresponding sensitivity in our model to be 0.28 percentage points.

3 Model

3.1 Model Environment

To investigate the labor force participation decision and college enrollment choice over the business cycle for individuals of different ages, we develop an overlapping generations model where agents can be in one of four states. In any period, an agent could be employed, unemployed, out of the labor force attending college, or out of the labor force engaging in no activity. Agents in the model acquire productive skills both through college attendance and through learning on-the-job, where the rate of learning on-the-job is allowed to vary by education group to match differences in wage growth observed in the data. Aggregate shocks in the economy influence individuals' participation and college enrollment decisions, and therefore can have persistent effects on worker productivity.

3.1.1 Setting

Agents with the option of labor force participation for 188 periods, where each period represents one quarter, populate the model. This setup portrays an environment where individuals can participate in the labor force from age 18 until age 65. Every period, a unit mass of agents enter the model one quarter before they turn 18 and decide if they want to search for a job, attend college, or do nothing in the next period when they are 18. Additionally, a unit mass of age 65 agents exit the model each period. In any period, an agent can be in one of four positions; employed, unemployed and searching for a job, attending college, or out of the labor force and not attending college. The model also contains an endogenous mass of firms determined via free entry.

3.1.2 Productivity and Skill Accumulation

Agents in the model are heterogeneous in terms of their age (a), skill level (z), and education level (τ). Each agent's education level affects their skill accumulation on the job. When employed, the output of a worker with skill z_t at time t is $f(Z_t, z_t)$, where Z_t is the level of aggregate productivity. Aggregate productivity follows an AR(1) process where $\ln Z_t = \rho \ln Z_{t-1} + \epsilon_t$ and $\epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2)$. A worker who is not employed enjoys leisure benefit $b(z_t)$.

Let τ denote the education level of each agent, where $\tau = 1$ represents an agent without a college degree and $\tau = 2$ signifies an agent who has graduated from college. All agents enter the model without a college degree ($\tau = 1$) and with $z = 0$. Workers can acquire skill on-the-job, and the rate of skill acquisition depends on their education level. Specifically, if a worker with education τ is employed, their skill level evolves according to the following process.[15]

$$z_{t+1} = \begin{cases} z_t & \text{with probability } 1 - \pi_\tau \\ z_t + \Delta_z & \text{with probability } \pi_\tau. \end{cases} \tag{1}$$

3.1.3 Endogenous College Attendance and Labor Force Participation Decisions

The decision to attend college is endogenous, and agents can attend college at any age. A non-employed worker may pay a fixed cost κ to attend college in the next period. With probability g per period, an agent attending college graduates. Upon graduation, an agent's education level becomes $\tau = 2$, and they receive a one-time skill increase of Δ_g. At the same time, workers also make an endogenous labor force participation decision. A non-employed worker can pay c_w to search in the next period, where c_w is drawn independently log-normal$(\mu_{cw}, \sigma_{cw}^2)$. An agent may not simultaneously attend college and search for a job.

3.1.4 Search

Non-employed agents who paid cost c_w in the preceding period direct their search to a job offering them fraction μ of production (this fraction is often referred to as a "piece-rate"). Firms may post a vacancy at cost c_f. In this environment, a sub-market is a collection of vacancies offering the same piece-rate to workers with the same characteristics. Define sub-market tightness as $\theta \equiv \frac{v}{u}$, where v is the mass of vacancies in a sub-market and u is the mass of agents searching in that sub-market. Free entry of firms decides the tightness of every sub-market in the model. Matches occur according to a constant returns to scale matching function $M(u, v) = \frac{uv}{\left(u^\ell + v^\ell\right)^{1/\ell}}$ as in Den Haan, Ramey, and Watson (2000). Define the probability that a worker meets a firm as $\frac{M(u,v)}{u} \equiv p(\theta)$ and the probability that a firm meets a worker as $\frac{M(v,u)}{v} \equiv q(\theta)$. Shimer (2005) reports that while the job-finding rate is strongly pro-cyclical, the separation rate is not as responsive to aggregate fluctuations and is just weakly counter-cyclical. Therefore, we assume that once a match is formed, it continues unless destroyed with exogenous probability $\delta_a(t)$, which we allow to depend on the age and education level of the worker.

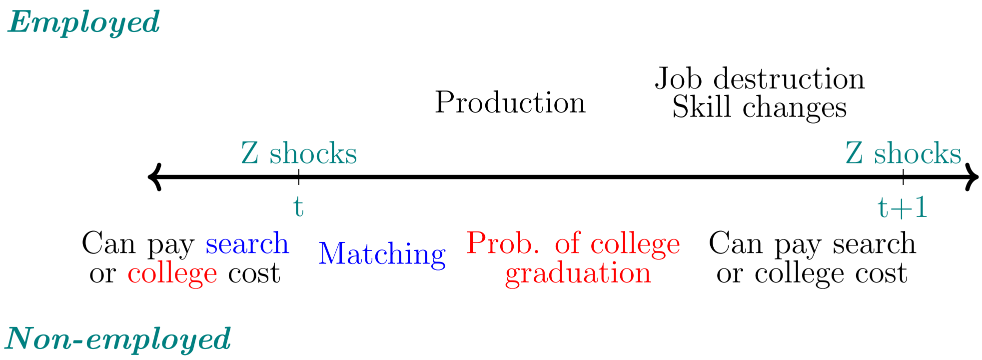
3.1.5 Timing

In summary, *Figure 7* illustrates the timing of the model. The top of the timeline lists events as they occur for employed agents, while the bottom lists events as they occur

[15] We create a grid of possible z values, where the lower bound of the grid equals 0 and the upper bound equals 4.5. We allow for 150 grid points so that $\Delta_z = \frac{4.5-0}{150-1} = 0.0302$. Note that the parameter estimates π_τ respond to the choice of Δ_z.

for non-employed agents. Both employed and non-employed agents observe any changes to aggregate productivity Z at the beginning of each period. When non-employed, an agent chooses at the end of the period whether they want to pay κ to attend college in the next period or pay their realization of the random search cost c_w for the chance of being matched with a firm at the beginning of the next period.[16] If the non-employed agent paid the college attendance cost, they graduate from college with probability p_c. Graduating from college gives the agent an immediate skill increase Δ_g and allows them to acquire skill faster on the job. If the agent paid the search cost, they could be matched with a firm and produce in the next period. If the agent chooses not to pay the search or college cost, they remain non-employed and can make this decision again at the end of the next period.

Figure 7: Model Timing



When employed, an agent does not search on the job. However, job-to-job transitions still occur. At the end of each period, agents realize any changes to their skill from learning on the job, and exogenous separations occur with probability $\delta_a(\tau)$. An employed agent who loses their job can immediately search and find new employment at the beginning of the next period. If not matched, they follow the same timeline as any other non-employed agent. This assumption that employed agents who lose their job can immediately search and match with a new firm at the start of the next period without paying a search cost simplifies the model in that we only solve for the maximum search cost that an agent is willing to pay when they are non-employed. Recently employed agents may retain more contacts and networking capital from their previous employment and receive a match due to this. This assumption aligns with observations that recently employed workers are more likely to search and remain part of the labor force than those who have been

[16]This timing assumption that agents do not bear the search cost c_w and make their directed search decision in the same period greatly simplifies the model. Suppose instead that agents pay c_w and make their search decision in the same period. If they were successfully matched, the utility they receive in the current period would be $u(\mu f(Z,z) - c_w)$, and if they were unsuccessful, they would receive $u(b(z) - c_w)$. The relative value of finding a job then would depend on the amount c_w that the worker must pay so that the agent's directed search decision would depend not only on their skill, age, and the aggregate state, it would also depend on their randomly drawn value of c_w.

non-employed for more extended periods of time.

3.2 Equilibrium

3.2.1 Value Functions

Before formally defining an equilibrium in this model, we must introduce a few additional pieces of notation. Let $N_a^S(\tau, Z, z)$ denote the value of non-employment when the current aggregate state is Z to an agent of age a, with education τ, and with individual productivity z who is paying cost c_w to be able to search at the start of the next period. The non-employed agent receives leisure/home production benefit $b(z)$, and if after observing the value of c_w drawn independently from log-normal$(\mu_{cw}, \sigma_{cw}^2)$ they decide to pay the cost, they can search for a job at the start of the next period. The value $\widehat{U}_a(\tau, Z, z)$ denotes the value of searching, which is defined by (6).

$$N_a^S(\tau, Z, z) = u(b(z) - c_w) + \beta \mathbb{E}[\widehat{U}_{a+1}(\tau, Z', z')] \tag{2}$$

$N_a^C(\tau, Z, z)$ is the value of non-employment to an agent choosing to pay cost κ to attend college in the next period when current aggregate productivity is Z, and the agent is age a with current education τ and productivity z.[17] The cost κ is constant over time. An agent who attends college graduates with probability g per period. The agent's education type affects how quickly they accumulate skill when employed, and upon graduation, the agent's type becomes $\tau = 2$. Agents also enjoy an immediate increase of Δ_g in their productivity upon graduation. After graduation, the agent is non-employed at the start of the next period, a state where they may choose to search for a job, attend college, or do neither.

$$N_a^C(\tau, Z, z) = u(b(z) - \kappa) + \beta \mathbb{E}\left[(1-g) N_{a+1}(\tau, Z', z) + g N_{a+1}(2, Z', z + \Delta_g)\right] \tag{3}$$

Let $N_a^N(\tau, Z, z)$ denote the value of non-employment for an agent who has decided to neither search for a job nor attend college. The non-employed agent obtains benefit $b(z)$ and is generally non-employed at the start of the next period, where they again face the decision to search for a job, attend college, or do neither.

$$N_a^N(\tau, Z, z) = u(b) + \beta \mathbb{E}[N_{a+1}(\tau, Z', z')] \tag{4}$$

Allow $N_a(\tau, Z, z)$ to denote the value of non-employment in aggregate state Z for an

[17] Notice that all agents may attend college regardless of age or education. An agent who has already attended college enjoys an increase in their skill level of Δ_g upon graduation. However, unlike an agent who has yet to graduate from college, a previous graduate does not experience an increased skill accumulation rate when employed. This makes the value of college attendance significantly higher for agents who have not already graduated from college.

agent of age a, education-type τ, and skill-level z. At this stage, the agent chooses, in the same period, between the values presented by equations (2), (3), and (4).

$$N_a(\tau, Z, z) = \max\left\{N_a^N(\tau, Z, z), N_a^S(\tau, Z, z), N_a^C(\tau, Z, z)\right\} \tag{5}$$

The non-employed agent choosing to pay cost c_w is able to search in the next period, giving them value $\widehat{U}_a(\tau, Z, z)$. Based on the aggregate state and the worker's characteristics, a worker searching for piece-rate μ (the fraction of production they receive) will be successfully matched with a job offering that wage with probability $p\left(\theta_a(\tau, Z, z, \mu)\right)$. If they are not matched with a job, they remain non-employed.

$$\widehat{U}_a(\tau, Z, z) = \max_{\mu}\left\{p\left(\theta_a(\tau, Z, z, \mu)\right) W_a(\tau, Z, z, \mu) + \left(1 - p\left(\theta_a(\tau, Z, z, \mu)\right)\right) N_a(\tau, Z, z)\right\} \tag{6}$$

Notice that an unemployed agent choosing the optimal piece-rate to search for faces a trade-off: increasing the piece-rate they search for increases their employment value if they are successful but decreases their probability of success. Because the value of posting a vacancy declines for the firm when the piece-rate increases, firms post fewer vacancies in sub-markets offering higher piece-rates, given the same mass of searching workers and worker characteristics.

Define $W_a(\tau, Z, z, \mu)$ as the value of employment to an age a worker with education type τ, skill z, and piece-rate μ when aggregate productivity is Z. The worker gets utility $u(\mu f(Z, z))$ from their wage $\mu f(Z, z)$, and they are separated from their job with probability $\delta_a(\tau)$ at the end of the period. If they are not separated, they remain employed at the same piece-rate and may experience changes to their skill or to aggregate productivity. Notice that the wage the worker earns, $\mu f(Z, z)$, increases as they become more productive. If a worker and firm separate, the worker can search at the start of the next period. This assumption that workers can search immediately after employment, without having to pay random cost c_w in the preceding period, assumes that employment gives workers a notion of momentum or connections in the labor market that allows them the ability to search immediately after an employment spell.

$$W_a(\tau, Z, z, \mu) = u(\mu f(Z, z)) + (1 - \delta_a(\tau))\beta\mathbb{E}\left[W_{a+1}(\tau, Z', z', \mu)\right] + \delta_a(\tau)\beta\mathbb{E}\left[\widehat{U}_{a+1}(\tau, Z', z')\right] \tag{7}$$

We assume that the exogenous separation probability depends on age and education because there is substantial heterogeneity in the job separation rate across different demographics (Menzio, Telyukova, and Visschers, 2016; Guo, 2018). In the quantification section we will discipline these separations rates using data from the CPS. We also assume that the job separation rate does not vary with aggregate productivity, consistent with a recent literature discovering that the main source of unemployment volatility comes from cyclical fluctuations in the job finding rates instead of separation rates (Shimer 2005, Hagedorn and Manovskii 2008, Elsby et al. 2009, Shimer 2012, and Kehoe et al. 2023).

The free-entry condition given by (8) summarizes the vacancy posting decision of firms in the model. Firms post vacancies in every sub-market until the cost of posting a vacancy (c_f) equals the expected benefit from posting a vacancy. Here the expected benefit is the value of employing a worker in the sub-market $J_a(\tau, Z, z, \mu)$ multiplied by the probability of matching with a worker $q\left(\theta_a(\tau, Z, z, \mu)\right)$. In sub-markets where vacancies are posted, free entry implies that firms will take advantage of any arbitrage opportunities until (8) holds with equality. Firms post no vacancies in sub-markets where the cost of posting a vacancy exceeds the expected benefit.

$$c_f \geq q\left(\theta_a(\tau, Z, z, \mu)\right) J_a(\tau, Z, z, \mu) \qquad \forall\ a, \tau, Z, z, \mu \tag{8}$$

$J_a(\tau, Z, z, \mu)$ denotes the value to a firm of employing an age a worker of education type τ with skill z earning piece-rate μ when aggregate productivity is Z.

$$J_a(\tau, Z, z, \mu) = (1-\mu) f(Z, z) + (1-\delta_a(\tau))\beta \mathbb{E}\left[J_{a+1}(\tau, Z', z', \mu)\right] \tag{9}$$

The worker and firm together produce $f(Z, z)$, and the firm pays fraction μ of this production to the worker. The job is destroyed with probability $\delta_a(\tau)$, leaving the firm with nothing. With probability $(1-\delta_a(\tau))$, the match remains intact into the next period, where the worker is one quarter older, and there may be changes to the aggregate state and the worker's skill.

3.2.2 Definition of Equilibrium

Definition 1: *A Recursive Equilibrium (RE) is given by:*

1. *Value functions* $\left\{N_a^S(\tau, Z, z), N_a^C(\tau, Z, z), N_a^N(\tau, Z, z), N_a(\tau, Z, z), \widehat{U}_a(\tau, Z, z), W_a(\tau, Z, z, \mu), J_a(\tau, Z, z, \mu)\right\}$
2. *Equilibrium market tightness function* $\{\theta_a^U(\tau, Z, z)\}$ *solves the workers' search problem (given by equation 6)*
3. *Optimal search and college attendance rules* $G_a(\tau, Z, z)$ *and* $C_a(\tau, Z, z)$
4. *Aggregate transition probabilities consistent with policy functions and stochastic* Z *process.*

We can also define a Block Recursive Equilibrium in this environment.

Definition 2: *A Block Recursive Equilibrium (BRE) is a RE where value and policy functions are independent of the aggregate distributions of agents across states.*

Menzio, Telyukova, and Visschers (2016) prove that a unique BRE exists, and that there is no other RE for this type of model.[18] This provides great convenience for obtaining the solution of the model when quantifying it. We now turn to our quantification strategy.

4 Quantification

4.1 Calibration

In solving the model outlined in Section 4, we assume $u(\cdot) = \ln(\cdot)$, $f(Z_t, z_t) = Z_t + z_t$, and $b(z_t) = b_c + z_t$. We approximate the AR(1) process followed by aggregate productivity (Z_t) as an N-state Markov chain following Tauchen (1986).[19] In addition to the calibrated parameter values recorded in *Table 3*, *Table 2* lists other assigned parameter values. The quarterly discount factor (β=0.99) corresponds to a 4% risk-free annual interest rate, while the autocorrelation of aggregate shocks (ρ) coincides with a common value in the literature chosen in den Haan et al. (2000), Hansen and Wright (1992), among others.

Table 2: Assigned Parameters

Parameter	Description	Value
β	Discount factor	0.99
ρ	Autocorrelation of aggregate shocks	0.95
$\delta_a(\tau)$	Separation rates by age and education	Figure 8

We calibrate the remaining eleven parameters recorded in *Table 3* to match eleven relevant moments. The first[20], third[21], fourth [22], fifth[23], and ninth[24] moments in the table are

[18]See Theorem 1 in Menzio, Telyukova, and Visschers (2016). This theorem extends this result in Menzio and Shi (2011) to an environment where workers are heterogeneous in terms of age and productivity characteristics.

[19]N=5 unless otherwise specified.

[20]U.S. Bureau of Labor Statistics, Unemployment Rate - 20 Yrs. & over [LNS14000024], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/LNS14000024

[21]Calculated using U.S. Bureau of Labor Statistics, Employed full time: Median usual weekly nominal earnings (second quartile): Wage and salary workers: 25 to 54 years [LEU0252887900A] and U.S. Bureau of Labor Statistics, Employed full time: Median usual weekly nominal earnings (second quartile): Wage and salary workers: 20 to 24 years [LEU0252887100A]

[22]Calculated using: U.S. Bureau of Labor Statistics, Employed full time: Median usual weekly nominal earnings (second quartile): Wage and salary workers: Bachelor's degree and higher: 25 years and over [LEU0252918500Q] and U.S. Bureau of Labor Statistics, Employed full time: Median usual weekly nominal earnings (second quartile): Wage and salary workers: High School graduates, no college: 25 years and over [LEU0252917300Q]

[23]Calculated using: U.S. Bureau of Labor Statistics, Employed full time: Median usual weekly nominal earnings (second quartile): Wage and salary workers: 55 to 64 years [LEU0252890900A] and U.S. Bureau of Labor Statistics, Employed full time: Median usual weekly nominal earnings (second quartile): Wage and salary workers: 25 to 54 years [LEU0252887900A]

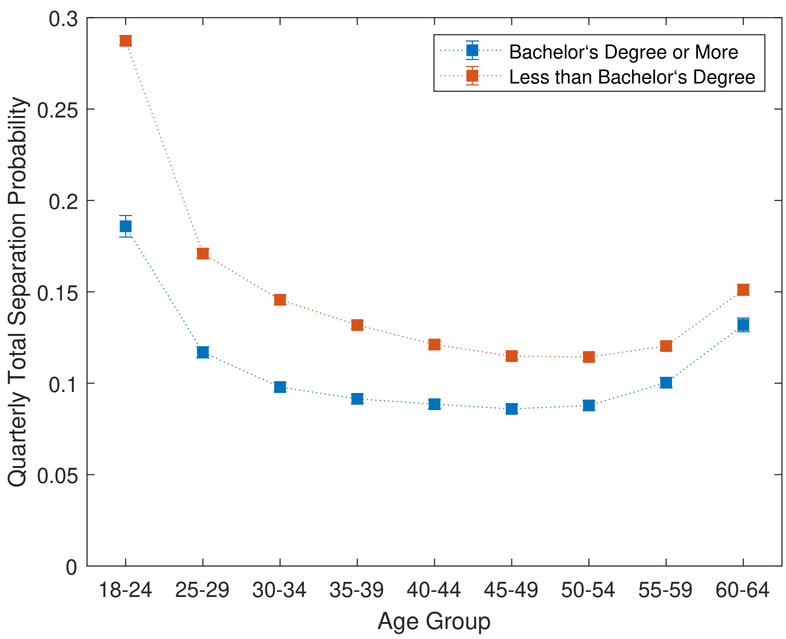
[24]Calculated using U.S. Bureau of Labor Statistics, Employment-Population Ratio - Bachelor's Degree and Higher, 25 Yrs. & over [LNS12327662] and U.S. Bureau of Labor Statistics, Employment-Population Ratio - High School Graduates, No College, 25 Yrs. & over [LNS12327660]

calculated as averages over 2015 through 2019 using data from the Federal Reserve Bank of St. Louis FRED site.

Table 3: Calibrated Parameter Values and Empirical Targets

Parameter	Estimate	Targeted Moment	Data	Model
ℓ	2.415631	Unemployment rate (%) for age 20+	4.035	4.066
κ	0.610879	Pop. age 25+ with Bachelor's degree (%)	31.920	30.962
Δ_g	0.240492	Wage ratio: prime age to 20-24	1.646	1.541
π_2	0.330988	College wage premium (age 25+)	1.814	1.775
π_1	0.112303	Wage ratio: 55-64 to prime age	1.086	1.334
c_f	0.811151	Vacancy posting rate	4.153	4.179
μ_{cw}	-0.144348	Age 20-64 participation rate (%)	87.140	88.364
σ_{cw}	0.828930	Prime age/60-64 participation rate	1.500	1.523
b_c	0.806085	College to non-college employment rate ratio	1.316	1.103
g	0.035700	College graduation rate over 4 yrs	0.441	0.441
σ_ϵ	0.015200	Participation % change w/ 1% shock: ages 18-24	0.522	0.520

Figure 8: Quarterly Total Separation Rates by Age and Education



We obtain the percentage of the population aged 25 and older with a Bachelor's degree or more as an average over 2015 through 2019 using data from the American Community Survey (ACS) 1-year estimates. The Job Openings and Labor Turnover Survey (JOLTS) data reports the vacancy posting rate. Specifically, the sixth moment is the total nonfarm, seasonally adjusted average job openings rate reported from 2015 through 2019. The quarterly separation rates for each age and education group is estimated from the CPS and includes all separations from the previous employer (including job-to-job separations). This separation rate most closely matches the separations occurring in the model because the agent can immediately search and become employed at the start of the next period following a separation, mimicking quits and relocations to other jobs.

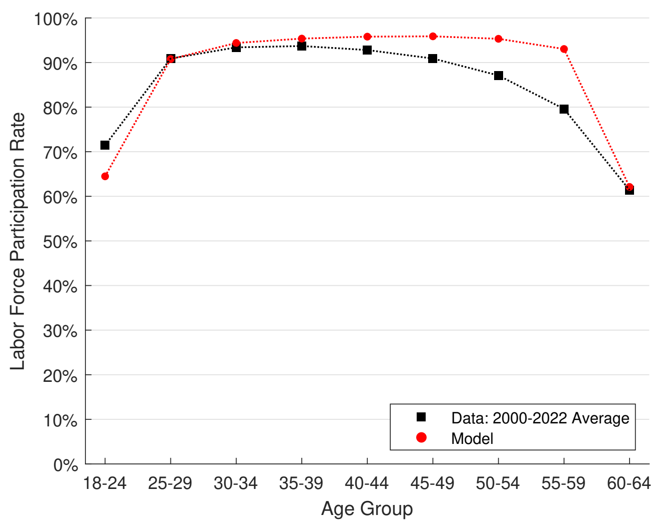
The National Center for Education Statistics reports the college graduation rate over four years, most recently for 2015-2016.[25]. Finally, we calculate the age 18-64 labor force participation rate and prime-age to age 60-64 participation rate ratio using data from the Current Population Survey (CPS). For these moments, individuals who reported non-participation due to illness, disability, or because they were taking care of their home or family were excluded, as in *Figure 2*. The final moment in the table corresponds with the same-quarter change in the participation rate of 18-24 year-olds associated with 1% higher GDP growth reported in *Figure 4*.

4.2 Model Predictions

To assess whether ours is a reasonable model of labor force participation over the life cycle, we examine what it implies for some key non-targeted moments regarding labor market dynamics and college attainment statistics.

A key characteristic of the data that the model captures is the profile of labor force participation by age. *Figure 9* plots the participation rate for each age group in the data and from the model. The participation rate forms an inverted U-shaped pattern over the

Figure 9: Participation Rate by Age Group: Model vs. Data (Untargeted)



life-cycle, with participation at its highest from around age 25-54. At the beginning of the life cycle, the benefit of education is at its greatest. The model only captures the

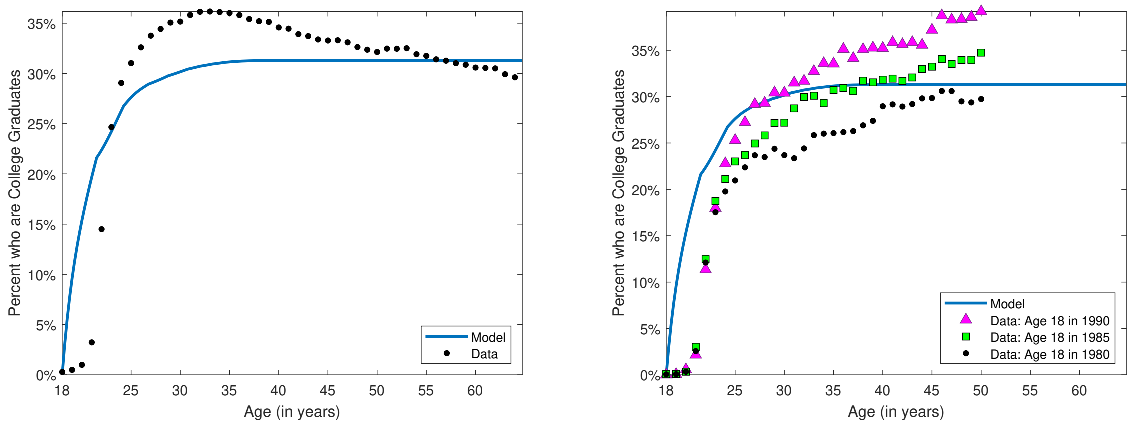
[25] See https://nces.ed.gov/fastfacts/display.asp?id=569

relatively lower participation rate among 18-24 year-olds by including the endogenous college attendance decision. As individuals approach age 65 in the model, the expected benefit of paying a search cost to re-enter employment declines. If a worker can successfully search and find a job, the time they can remain at that job is limited. Therefore, individuals are less likely to re-enter the labor force after a job loss as they approach age 65 in the model. The same effect seems to be present in the data, but it appears to set in sooner, perhaps due to differences in the age that individuals choose to retire or early retirements due to health or family circumstances.[26]

Individuals in the model can attend college at any age, but the benefit is greatest when they are young. *Figure 10* displays the fraction of the population who are college graduates by age. Recall that while the total fraction of the 25+ population who are college graduates is a targeted moment, the age profile of college graduates that the figure displays is untargeted. The data used in this figure is the average for each age from 2000 to 2022 from the monthly CPS. College attendance and graduation is most

Figure 10: Percent of Population who are College Graduates by Age (Untargeted)

2000-2022 Average over Cohorts | *Following 1980, 1985, and 1990 Cohorts*



common among those aged 18-30, and after around age 30, the percent of the population who are college graduates levels off. The data also captures changes in college attendance over time. Individuals became more likely to attend and graduate from college as time progressed. This leads to the decline in the percentage of the population who are college graduates after around age 30 in the data.

In the model all agents are born identical. Hence there is no ability dimension in driving college enrollment patterns. This raises a concern: does our model predict the

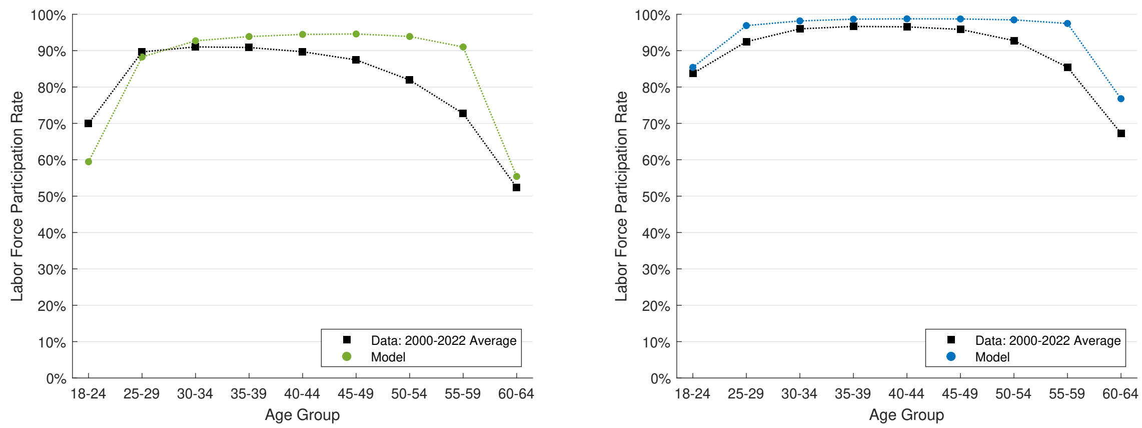
[26]The model performs well in matching labor participation for the relatively young and the old, but less well for those aged 50-59. One restriction we impose is that the value of leisure $b(z)$ does not change with age. In *Appendix F*, we explore an alternative calibration where the value of leisure can vary with age, and use the participation-age profile to infer the value of leisure for each age group. We find that the implied value of leisure increases with age and our main results in the quantitative policy experiment do not change under this alternative calibration.

right elasticity of college enrollment with respect to changes in economic fundamentals? To alleviate this concern, we compute how the enrollment of agents aged 18-24 responds to a positive aggregate shock lasting for one period, resulting in a 1% increase in model GDP. We find that in the model simulation, the 18-24 enrollment rate decreases by 0.28 percentage points in the period of the positive aggregate shock before moving back toward its steady-state value. This model prediction is close to the impulse response prediction in *Figure 6*, which showed a 0.30 percentage point decrease in the enrollment rate of this age group following a positive shock of the same size. The main difference in the model response compared to the data is the response timing. In the impulse response estimate from the data, the enrollment rate drops about four quarters after the shock, likely due to the time gap between applying and becoming a student.

Previously, the data displayed in *Figure 3* showed that the participation rate among college graduates is higher than that of non-graduates for each age group. *Figure 11* reveals this same feature is also present in the model. Although the total participation

Figure 11: Participation Rate by Age Group and Education (Untargeted)

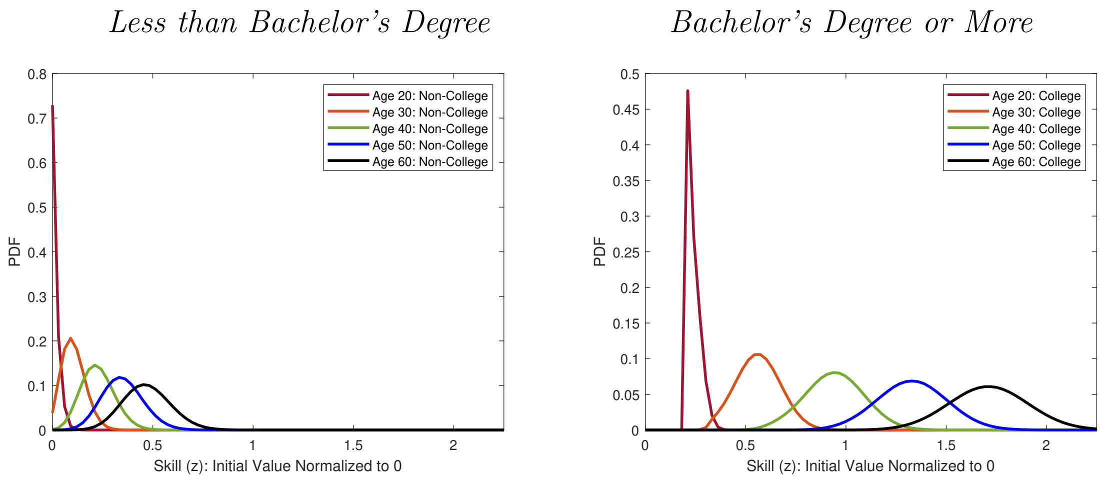
Less than Bachelor's Degree | *Bachelor's Degree or More*



rate and prime-age to age 60-64 participation rate ratio were targeted moments, the overall participation by age profile and the differences among college graduates and non-graduates are untargeted. While the model still overestimates participation among 55-59 year-olds when controlling for education, the overall shape of the participation by age profile and the difference in the level of participation between college graduates and non-graduates match the data closely. In particular, it successfully matches the empirical feature that labor participation declines faster for non-college graduates compared to college graduates. This is due to the endogenous human capital channel. In the model, non-college workers experience higher non-employment rates because of lower human capital. Due to a lack of employment opportunities and on-the-job learning, their human capital grows at a slower rate compared to the college graduates (see *Figure 13*, the college wage premium increases with age). Hence their participation rate declines at a faster rate as they age.

Agents in the model all start with a skill level (z) equal to zero. They can acquire skill through learning on the job and/or through college graduation. Recall that college graduation leads to an immediate increase in skill by the amount Δ_g and allows agents to acquire skills more quickly on the job. *Figure 12* displays the skill distributions across different ages for college graduates and non-graduates. The initial increase in skill gained from college graduation allows even young graduates to have a higher average z than non-graduates. As both groups age, college graduates acquire skills more quickly when employed leading to larger differences in average skill among agents of the same age.

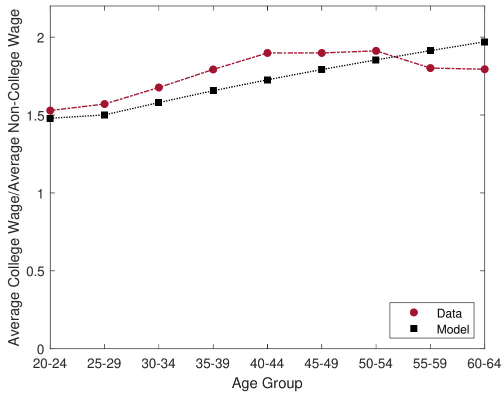
Figure 12: Model Skill Distributions



The assumption that college graduation has both an immediate and longer-term impact on worker productivity allows us to more accurately match the wage-age profile for college graduates and non-graduates. The data shows that there is an immediate college wage premium, even among those aged 20-24 who would have just recently graduated. As previously indicated in *Table 3*, the initial increase in skill of Δ_g upon college graduation allows the model to capture the immediate college wage premium enjoyed by younger workers (see *Figure 13*). Additionally, the figure shows that, in both the data and model, the college wage premium is increasing with age. In other words, on average, those with a bachelor's degree experience faster wage growth throughout their lives. The model can capture this feature of the data because the probability of skill increase on the job is allowed to differ by education (leading to different estimates of π_1 and π_2), and these different probabilities target the average wage growth among each education type.

Given that the model can capture relevant steady-state features of the data, we turn to look at how well it captures the responsiveness of labor force participation to aggregate shocks. *Figure 14* displays how the labor force participation rates of different age groups in the model respond to a one-period shock to aggregate productivity (Z), resulting in

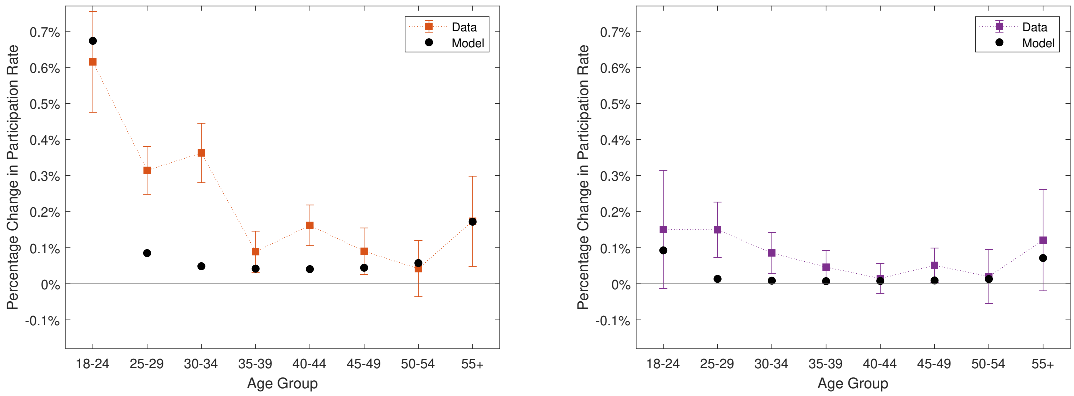
Figure 13: College Wage Premium by Age (Untargeted)



a 1% increase in output produced from employment (GDP). While the overall responsiveness of those aged 18-24 (not conditioning on educational attainment) is targeted in the calibration, all points displayed in *Figure 14* are untargeted. Just as in the data, we see that the participation of those without a Bachelor's degree is more responsive to aggregate productivity changes.

Figure 14: Same-Quarter Change in Participation Rate Associated with 1% Higher GDP Growth: Model vs. Data by Education Group (Untargeted)

Less than Bachelor's Degree *Bachelor's Degree or More*



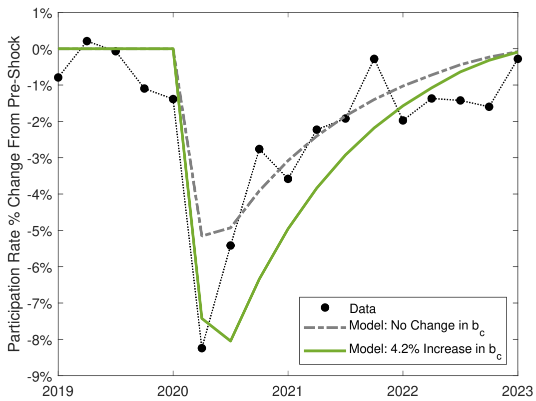
In both the data and model, the participation of younger and older workers is most responsive to changes in aggregate productivity. Workers beyond age 30 are very unlikely to attend college and effectively choose between two options when non-employed: unemployment and non-participation. However, younger workers get a greater lifetime return from college attendance and choose from three options when non-employed. When the value of searching for a job increases, as in cases where there is a positive aggregate shock, young agents substitute away from the college attendance option into labor force participation. Older workers benefit less from search due to the horizon effect discussed previously, but this effect is lessened when there is a boost to aggregate productivity and thereby their expected return from employment.

4.3 Participation Response to Large Aggregate Shocks: A Comparison with the 2020 Recession

We now examine the ability of the model to replicate salient features of the 2020 pandemic recession, particularly the dynamics of labor participation across different ages. The 2020 recession saw a sharp 8.48% drop in real GDP, which *Figure 1* illustrates had lasting effects on labor force participation. We investigate how well the model mimics the relative differences in participation observed across age groups following the shock.[27] First, we consider a negative one-period shock to aggregate productivity large enough to result in approximately an 8.48% decline in model GDP. We plot the participation response of each age group to this shock to aggregate productivity (Z) alone in *Figures 15* and *16*. The figures show that when there is only the shock to aggregate productivity, the decline in participation is not as dramatic as reported following the 2020 recession. To match the approximate magnitude of the change in participation observed in the data, we introduce a simultaneous one-period increase in the leisure value b_c (recall the total leisure value an agent receives when non-employed is $b(z) = b_c + z$).[28] We assume that during the period of the shock, agents act as if they believe the change in b_c to be permanent. We find that a one-period 4.2% increase in b_c along with an aggregate shock resulting in an overall 8.48% reduction in model GDP results in a participation response among those age 20-24 that matches the data reasonably well.

Figure 15 plots the percent deviation of age 20-24 participation from February 2020 in the data. (It reports quarterly averages of the monthly data displayed in *Figure 1* to

Figure 15: Age 20-24 Participation Response



[27]For these model experiments, we use a 7-state Markov chain approximation of the AR(1) process that aggregate productivity follows, increasing the accuracy of the approximation relative to the steady-state results previously presented.

[28]The change in the value of b_c can be motivated in different ways. A direct interpretation of the increase in b_c is the increased generosity of unemployment insurance. However, because b_c does not enter into the government budget constraint, we are more inclined to consider it as capturing workers' changing taste for work-life balance or the benefit obtained from avoiding illness at work.

compare with the model where each period represents a quarter.) *Figure 15* also displays the model response of age 20-24 participation when there is an 8.48% drop in GDP due only to an aggregate shock and no change in b_c, and the same response when there is an immediate one-period 4.2% increase in b_c. Both the initial decline in prime-age participation and the persistence of its response match the data well.

Figure 16 displays the participation response of agents aged 25-54 and age 55 and older to the same shocks. From the figure we see that when there is no change in b_c and when b_c increases 4.2%, the model under-predicts the response of prime age participation to the 2020 shock. The initial response of those age 55 and older matches the data, but the data response is more persistent. We find that an increase in b_c of 11.7% allows the model to match the prime age participation shock well. Additionally, an assumption that the increase in b_c is more persistent among those 55 and older allows the model to match that series more accurately as well. We could consider that agents of different ages saw different changes in b_c following the 2020 recession and could determine the series of shocks needed to match the exact responses of each age group. For example, one could assume that b_c remained lower for older workers for much longer due to health risks imposed by working during and following the pandemic. Although these considerations could be reasonable, we instead show that even when all agents face exactly the same shocks, individuals of different age groups have different participation responses in the model that match what was observed in the data reasonably well.

Figure 16: Prime Age and Older Participation Responses

Age 25-54 Response | *Age 55+ Response*

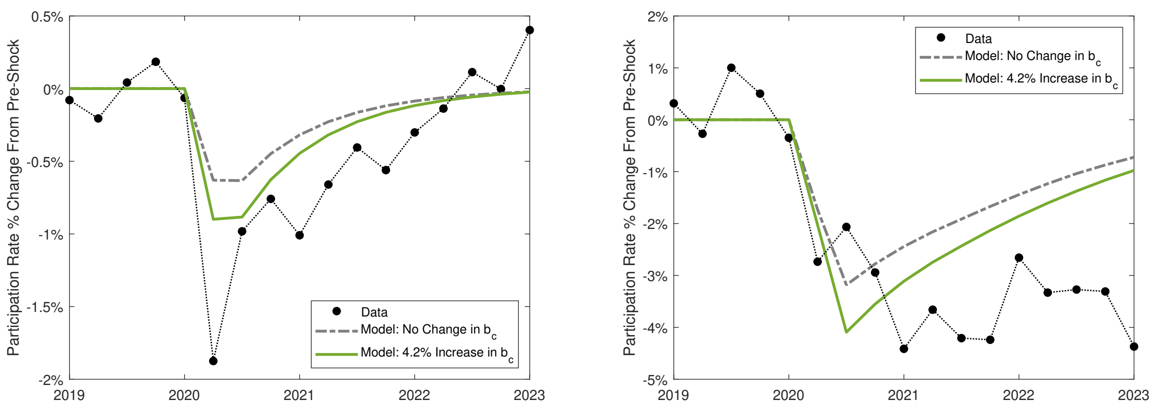
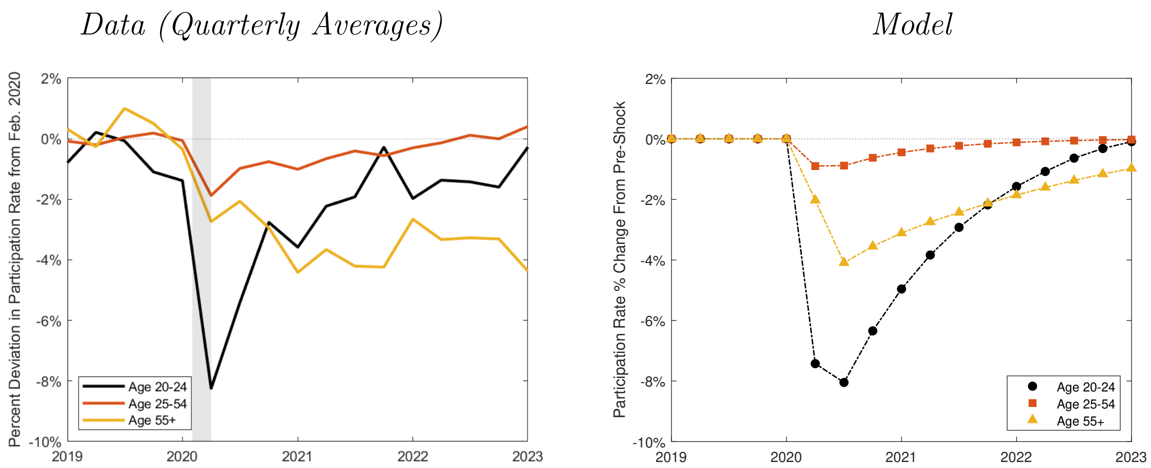


Figure 17 presents the labor participation response for different age groups, comparing the model to the data. The right panel of the figure shows the model response when all age groups experience a 4.2% increase in b_c and a shock to Z resulting an an overall decline in GDP of 8.48% in the period of these shocks (consistent to the decline in real GDP observed in the data). The model aligns with several key aspects of the data:

the prime-aged population experienced the least severe decline, while younger and older age groups faced more substantial drops, with the youngest group being hit the hardest on impact. Regarding the pace of recovery, the older age group exhibits the slowest recuperation among all three groups.

Figure 17: Participation Dynamics Around Pandemic



5 Optimal Search Subsidies over Life and Business Cycle

We now use our model to examine the efficiency of labor force participation choices by introducing a simple subsidy that incentivizes workers to search for jobs. This subsidy can be interpreted as capturing various components of workfare programs implemented in different countries, such as the "New Deal" introduced in the UK in 1998. These programs typically provide unemployment insurance, assisted job search, and/or subsidized jobs to encourage workers to join the labor force. Our subsidy represents these elements in a parsimonious manner, allowing us to focus on how such policies should be designed over the life and business cycle.

5.1 Search Subsidies: Benefits and Costs

In the steady-state of our model, a search subsidy can enhance welfare by insuring risk-averse agents against two sources of risk. First, the subsidy smooths consumption as agents move between employment and unemployment. Second, a search subsidy can help ensure that investments in human capital, specifically college enrollment, will pay off with a higher likelihood in future periods.

This first risk channel is fairly standard, while the second is not widely considered.

When agents in the model consider enrolling in college, they face some chance of realizing high participation costs, which negatively affect the expected payoff of their education. This is consistent with the observation that some individuals with a college degree do not participate in the labor force, even after excluding non-participation due to disability, illness, or caring for family members. By introducing a search subsidy, the likelihood that an agent will invest in their education and then remain out of the labor force declines.

While a search subsidy could enhance welfare by acting as an insurance mechanism, these subsidies and the taxes necessary to finance them create fiscal externalities and interact with agents' directed search and college enrollment incentives. If policies are to improve welfare, they must act to provide insurance against the two sources of risk discussed, while also being carefully implemented with their incentive-distorting effects in mind.

First, a subsidy specifically for job seekers, rather than the non-employed in general, provides insurance against unemployment risks while also addressing a fiscal externality associated with job-seeking activities. When individuals find work, the government collects more in taxes, allowing it to lower tax rates and benefit all other workers. Hence, a worker's job-seeking behavior positively impacts other workers through the government's budget constraint. By subsidizing job search efforts rather than non-employment, the government can mitigate this fiscal externality.

Next, the search subsidy can distort directed search and college enrollment incentives. When agents in the model receive a subsidy in unemployment, they are willing to spend more time engaged in search. This leads them to direct their search to higher-wage jobs with lower matching probabilities. Moreover, subsidizing search attracts agents away from college attendance and into the labor market by raising the opportunity cost of college enrollment. Education serves as an investment in higher future earnings, but with social insurance being funded through income taxes, part of the returns to college accrue to the government in the form of tax revenue instead of to college graduates. These effects can lead to fewer college enrollments than what would be socially optimal.

Our counterfactual experiment shows that the cost of search subsidies in terms of their distorting effects on search and college enrollment can outweigh their benefits. In the case where a uniform search subsidy is provided to all ages, we find that the steady-state welfare is decreasing in the level of search subsidies.[29]

Since college enrollment decisions are typically made before the age of 25, as in the data and our model, we conjecture that the efficiency costs associated with a search subsidy are more significant for the young. This suggests room for enhancing welfare through age-dependent subsidies. We find that offering subsidies to older individuals

[29]We consider a utilitarian social planner who maximizes average utility. In our model with zero population growth, this is equivalent to maximizing total utility. See *Appendix B* for further results with other welfare metrics.

(aged 55 and above) yields the greatest overall long-term welfare. The intuition is as follows: subsidizing the young may crowd out college education while subsidizing those in their prime ages would not significantly impact labor participation, as most of them are already in the labor force. As a result, targeting subsidies toward older individuals is the most cost-effective way to increase labor participation while preserving incentives for the young to pursue higher education.

We then show that allowing the subsidy to respond to business cycles yields negligible welfare gains in comparison to its steady-state counterparts. In the model, income is subject to two types of risk: an aggregate productivity shock Z and an idiosyncratic separation shock $\delta_a(\tau)$. Given the relatively modest volatility of the productivity shock compared to the separation shock, it is more important for the government to offer insurance against the latter rather than the former.[30] Our steady-state policy already addresses the idiosyncratic risk, implying a considerably smaller window for welfare gain when adjusting the policy based on the business cycle. Moreover, an inefficiency arises when a more generous search subsidy is provided during a recession, as agents are incentivized to search at times when search is least productive. Firms post fewer vacancies during recessions and the job-finding rate is lower, so agents are encouraged to pay a search cost when the benefit of doing so is lower (and during booms, they are not as encouraged to search when the benefit is higher).

5.2 Steady-State Effects of Subsidizing Search: Role of Age Dependence

In this section, we study policy implications at the steady-state, focusing on the effects of age-based subsidies for search. Specifically, suppose that the government pays a lump-sum subsidy s to unemployed individuals in the economy if they search for a job. A proportional income tax on wages offsets the cost of this subsidy and balances the government's budget. With tax rate τ, an employed individual earning wage w receives after-tax income $w(1-\tau)$. We consider the effects of providing this subsidy for search when the subsidy is paid to all individuals, only to those 18-24, only prime age (25-54), and only to age 55+. In each scenario, taxing employed individuals of all ages funds the subsidy. We ensure that the corresponding tax rate balances the government's budget in the risky steady-state.[31]

Let $\underline{a_s}$ be the lowest age for which the government subsidizes search, and let $\overline{a_s}$ be the highest age that receives the subsidy. Let $u(a)$ be the total mass of agents unemployed

[30] As table 3 illustrates, the idiosyncratic separation risk is substantial in the model: there is around a 10 percent or higher probability ($\delta_a(\tau)$) the workers may lose their job, resulting in a potential drop in consumption by as much as 20 percent (b_c). The standard deviation of aggregate productivity shock (σ_ϵ), on the other hand, is 0.0152, resulting in relatively modest changes in income.

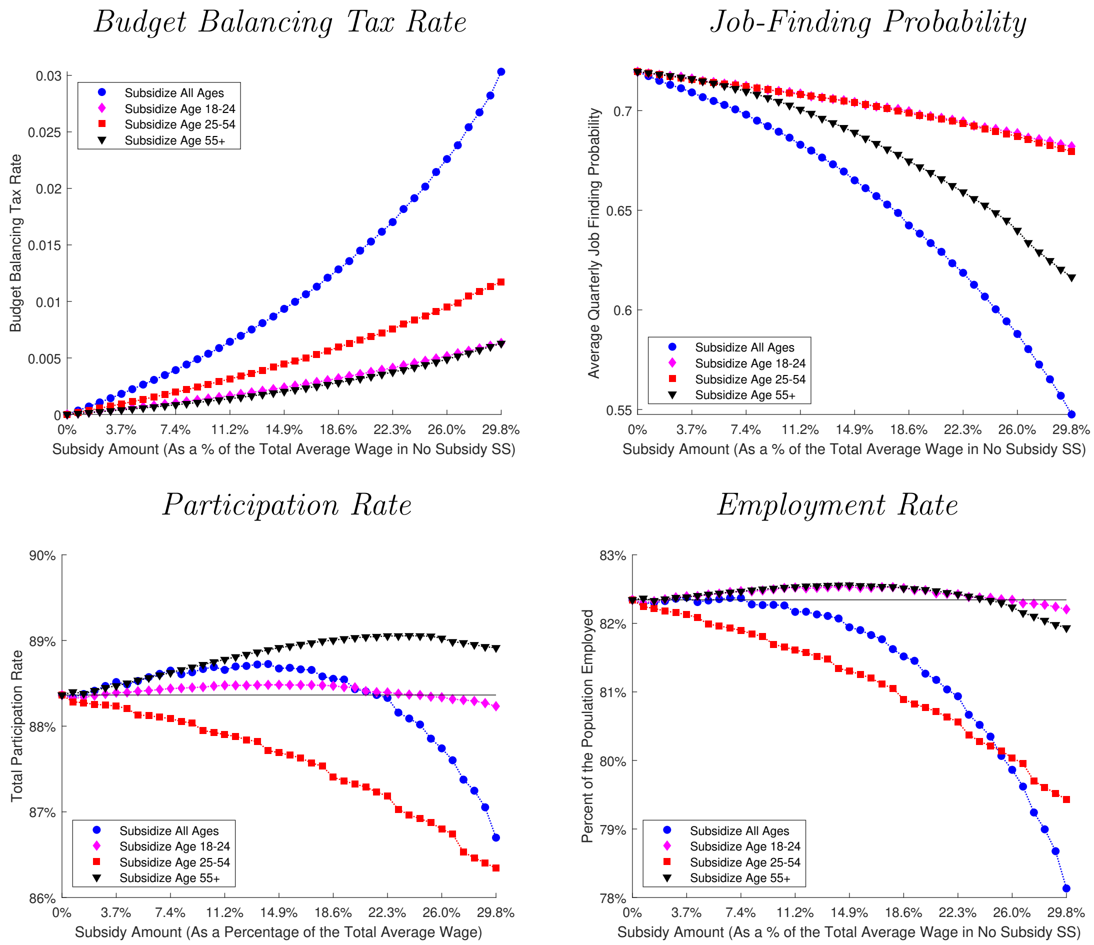
[31] This is the same risky steady-state concept described by Coeurdacier et al. (2011).

of age a in the risky steady-state, and let $e(a,w)$ be the mass of agents employed at age a earning wage w. Then, the total cost of the subsidy is equal to the subsidy amount multiplied by the mass of agents receiving the subsidy $\left(s \sum_{a=\underline{a_s}}^{\overline{a_s}} u(a)\right)$. The tax revenue from the proportional tax on wages equals the sum of the tax rate multiplied by the wage over the total mass of agents employed at each wage. Therefore, for any subsidy, the corresponding tax rate τ must satisfy the following government budget constraint.

$$s \sum_{a=\underline{a_s}}^{\overline{a_s}} u(a) = \sum_{a} \sum_{w} \tau w e(a,w) \tag{10}$$

The top left panel of *Figure 18* displays the tax rate on wages (τ) needed to offset the cost of the subsidy. Increasing the subsidy on search requires a higher tax rate τ to balance

Figure 18: Impacts of Search Subsidy on Labor Market

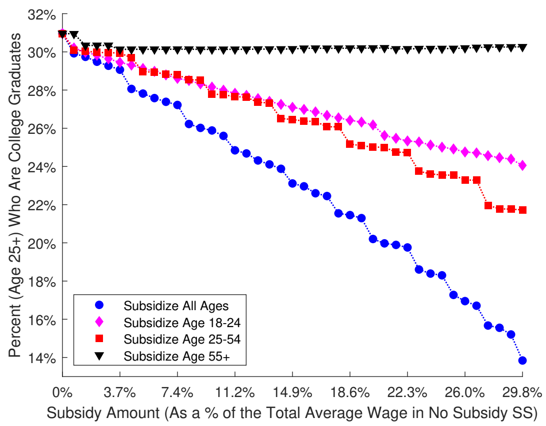


the government's budget and can result in two opposing effects on incentives. First, paying a subsidy to those who search increases the incentive to search. However, because higher subsidies result in higher taxes when employed, this reduces the benefit of finding employment. The top right panel of *Figure 18* plots the average job finding probabilities, which decrease with higher subsidies. The bottom panels of *Figure 18* display the effects that subsidizing different age groups has on the labor force participation rate and the total percentage of the population who are employed. In both cases, small subsidy amounts can increase participation and employment. However, because higher subsidies must be offset by higher taxes on the employed, eventually very high subsidies can reduce participation and employment.

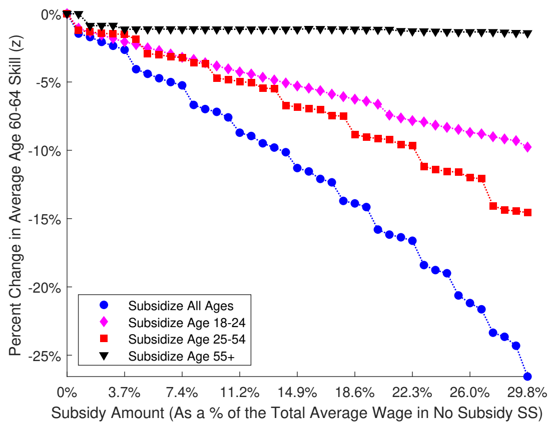
The effect of subsidizing search on total output is determined not only by the mass of agents that enter employment, but also by the productivity of those agents. In the model, non-employed agents choose between search, attending college, and engaging in no activity. Encouraging search by providing a subsidy makes search relatively more attractive than pursuing education. The top panels of *Figure 19* illustrate how subsidizing search affects both the percentage of agents who choose to attend college and the total skill accumulated by the end of the life cycle. The top left panel of *Figure 19* shows that generally, subsidizing search makes the option of college attendance less attractive and results in a reduction in educational attainment. *Figure 10* showed that in both the data

Figure 19: Impacts of Search Subsidy on Education, Productivity, and Production

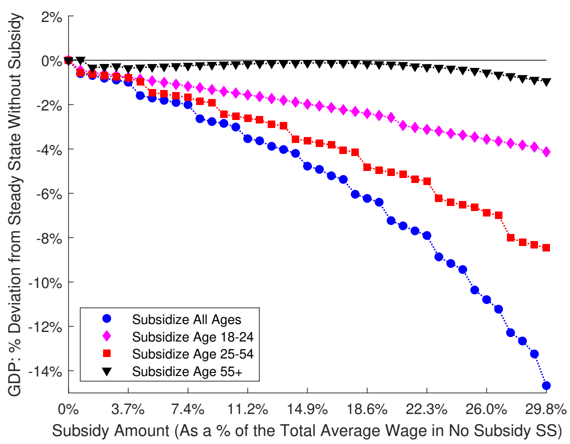
Percent College Graduates Among Age 25+



Percent Change Avg. Age 60-64 Skill



GDP (Total Production from Employment)

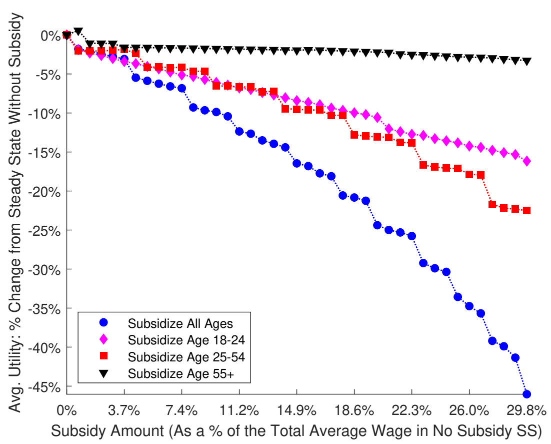


and model, individuals largely make their college attendance choice when young, and most individuals who graduate college do so by age 30. Subsidizing the search only of those 55 and older does not make search relatively more attractive compared to college attendance during the time when individuals benefit most from college attendance. Each subsidy's impact on college attendance has an almost symmetric influence on average skill near the end of the life-cycle, computed as the average skill (z) among those aged 60-64.

Subsidizing search can generally increase participation and employment at low subsidy levels. However, these subsidies disincentivize college attendance unless provided only to those closer to the end of the life-cycle, and otherwise lead to a sizeable decline in individual productivity. These two effects, generally increasing employment but decreasing individual productivity, simultaneously determine how the subsidy influences total GDP in the economy, measured as output produced from employment. The bottom panel of *Figure 19* shows that generally subsidizing search reduces production from employment because although smaller subsidy amounts increase employment, they also immediately start to disincentivize college attendance. However, because subsidizing the search only of agents 55 and older encourages employment while not notably discouraging college attendance, the economy experiences no significant change in GDP when subsidizing the search of this oldest age group for modest subsidy amounts.

Finally, we investigate how subsidizing the search of different age groups influences average utility.[32] *Figure 20* shows that only small age-targeted subsidies to older individuals lead to steady-states with higher average utility. *Figure 20* shows that subsidizing search

Figure 20: Average Utility

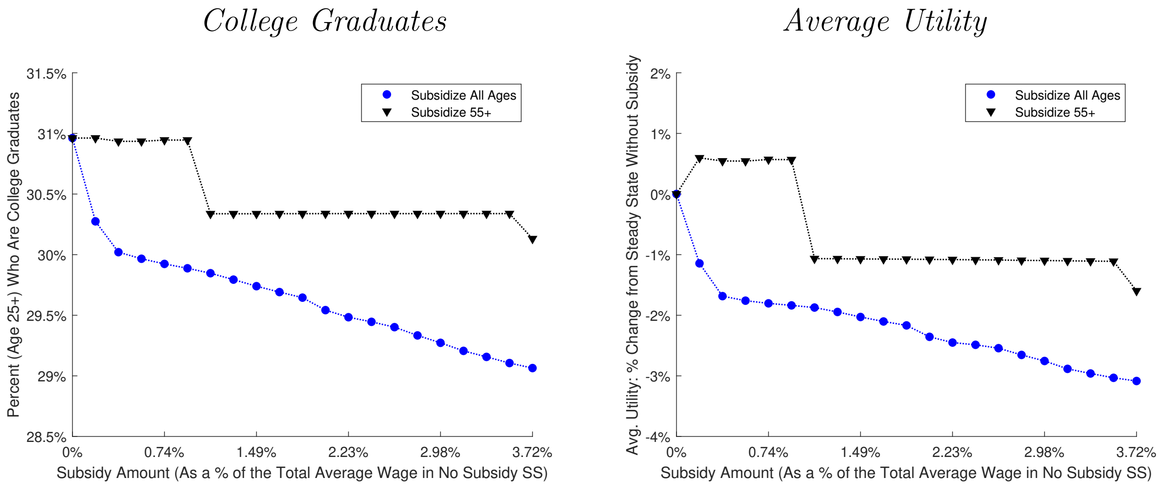


among individuals of all ages, those aged 18-24, and those aged 25-54 reduces average utility for all nonzero subsidy amounts. This reduction is due to the crowding-out effect of subsidizing search on college enrollment. *Figure 19* showed that subsidizing search for those apart from the older 55+ age group lead to immediate and sharp declines in the

[32]Because the population size in our model economy is constant, policies that increase average utility also must increase total utility.

percentage of the population who are college graduates and average skill at the end of working life. *Figure 20* depicts a slight welfare gain when subsidizing the search of age 55 plus individuals for relatively small subsidy amounts. Increasing subsidies necessitates higher taxes on the employed, and eventually average utility starts to decline at higher subsidy amounts.

Figure 21: Subsidy Effects on College Incentives and Average Utility



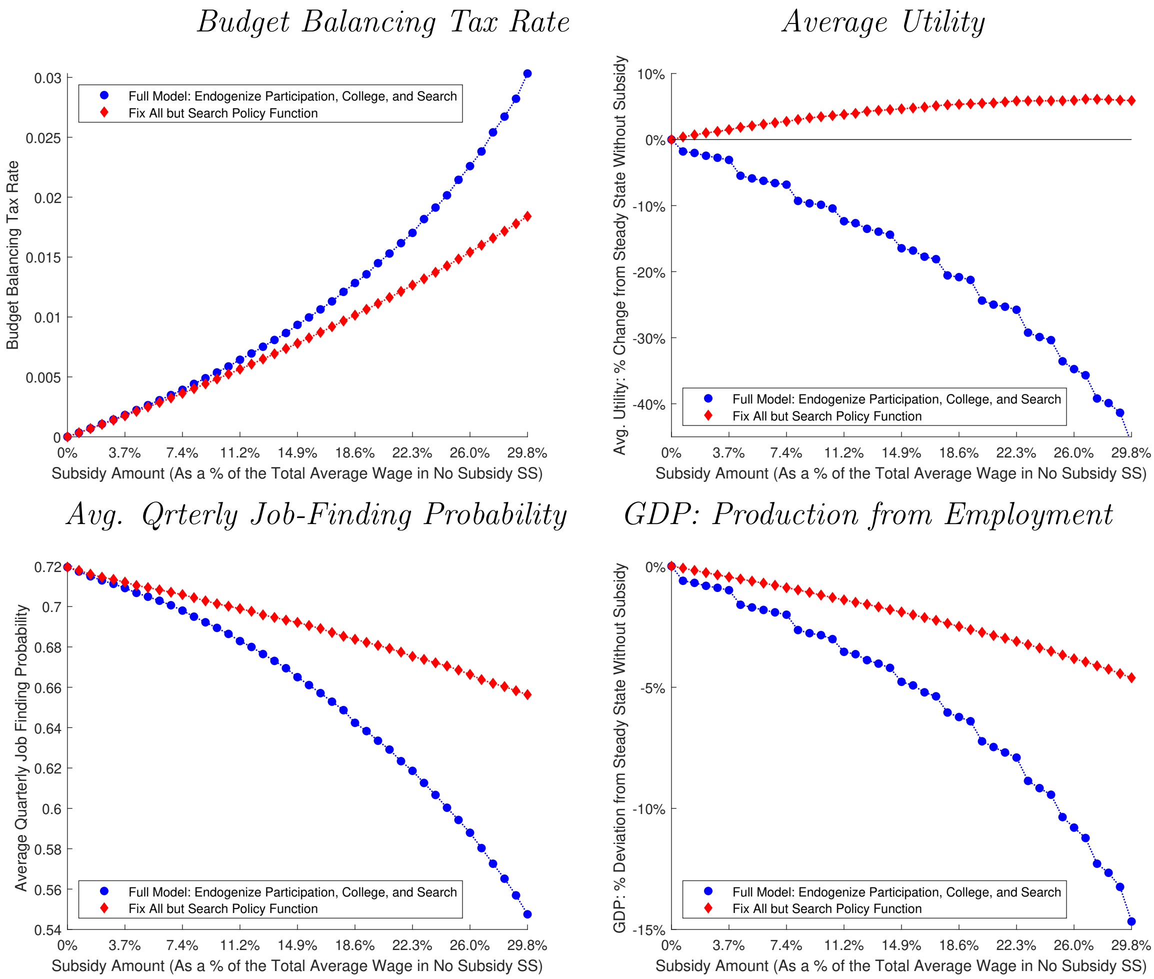
In *Figure 21* we examine college attendance and average utility, around the subsidy amount (given to 55+) that delivers the highest utility. We find that they are highly correlated. The reason is that college attendance provides human capital accumulation and boosts of worker's lifetime income and it has a dominating impact on worker's long term welfare.[33] From the plot one can also see that the optimal amount of subsidy is relatively small, amounting to around 0.74% of the average wage in this economy.

The result that subsidizing the search of all individuals reduces average utility may be initially surprising, especially given that many models where the participation and college decisions are not endogenized find a welfare-increasing role for search subsidies. In these models with directed search, risk-averse agents, and incomplete insurance markets, a search subsidy makes unemployed workers willing to spend more time searching, so they direct their search to decrease their probability of moving into employment. Despite this negative side effect, a reasonable search subsidy can still increase welfare by smoothing agents' consumption as they move between employment and unemployment. However, these models generally do not consider the effect of subsidizing search on participation and schooling decisions. To further investigate this result, we compare the predictions of our full model with a version of our model where participation and college attendance policy functions are fixed to the case where there is no subsidy. This version of the model

[33]The discrete jumps in college attendance and utility in the case of subsidizing 55+ is due to limited heterogeneity in the initial stages of life-cycle. We could, for example, introduce heterogeneous tastes for college attendance to smooth out the jump.

considers the effects of subsidizing the search of all ages when only the (directed) search policy functions react to the subsidy. *Figure 22* displays the resulting predictions.

Figure 22: Effects of Search Subsidy given to all Ages: Comparison of Full Model and Model Where Only Directed Search Decision is Endogenized



When our model only considers how agents' directed search choice is impacted by the subsidy but not their participation and college attendance choices, we underestimate the budget-balancing tax rate, especially for higher subsidy amounts. Additionally, when we do not allow the participation and college attendance policy functions to react to the subsidy, the model predicts that subsidizing the search of all ages can substantially raise average utility. These results demonstrate that considering the subsidy's effects on participation and college attendance not only significantly impacts the model's quantitative predictions, it also results in qualitatively different results and policy recommendations.

Although the version of our model where only the directed search decisions of agents react to the subsidy predicts the same qualitative effects on the average job-finding rate and production on-the-job, it significantly underestimates the decline in each of these in response to larger subsidy amounts. As discussed in Section 4, with directed search, agents face a trade-off between job quality and the probability of finding a job. Better jobs take longer to find on average. When unemployed agents are subsidized, they are willing to spend more time searching, so they search for higher wages and their quarterly job-finding probability declines. This is true in both versions of the model, but the effect is significantly stronger in the full model, where the percentage of agents who graduate from college declines with larger subsidy amounts. The same is true regarding the predicted response of GDP. Although GDP declines when only the search decisions of agents are affected by the subsidy, as agents spend more time in unemployment, the decline is much larger in the full model due to the drop in productivity resulting from lower college attendance.

5.3 Subsidizing Search Over the Business Cycle

We now compare how the economy that offers search subsidies behaves over a long series of simulated shocks with an economy that does not. Recall that aggregate productivity (Z) follows an AR(1) process such that $\ln(Z_t) = \rho \ln(Z_{t-1}) + \epsilon_t$ where $\epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2)$.[34] In 5.3.1, we track the economy as it experiences this simulated series of shocks when there is no search subsidy and when there is the subsidy to agents age 55 and older that maximizes average utility in the risky steady-state. Then, in 5.3.2, we compare these results with the case where the subsidy amount is dependent on aggregate productivity.

5.3.1 Fixed Subsidy

In this section, we study the case where the subsidy does not vary with the business cycle. While the tax rate ensures that the government's budget is balanced in the risky steady-state, the budget is closely balanced in expectation over the series of shocks. *Figure 23* shows the cumulative government net income as a percentage of cumulative steady-state GDP. Over the 250 simulated years, the cumulative government net income is slightly negative, but is just around -0.0004% of the economy's GDP. Over the simulation, total consumption is consistently higher when the search subsidy is given to those 55 and older. Average utility is also consistently slightly higher in the economy where the subsidy is given. *Figure 24* shows that while there may be differences in the volatility of consumption and average utility when a search subsidy is offered, the differences are slight. Next, we explore how allowing the subsidy amount to vary with aggregate productivity affects both the volatility and average levels of these same variables of interest.

[34] Computationally, this AR(1) is approximated as an N-state Markov chain following Tauchen (1986). In this section, we simulate a series of shocks over 1,000 quarters (250 years) where the AR(1) process aggregate productivity follows is approximated with a 17-state Markov chain. *Figure 40* in *Appendix D* displays the simulated AR(1) and its Markov approximation.

Figure 23: Fixed Subsidy Amount: Government Budget

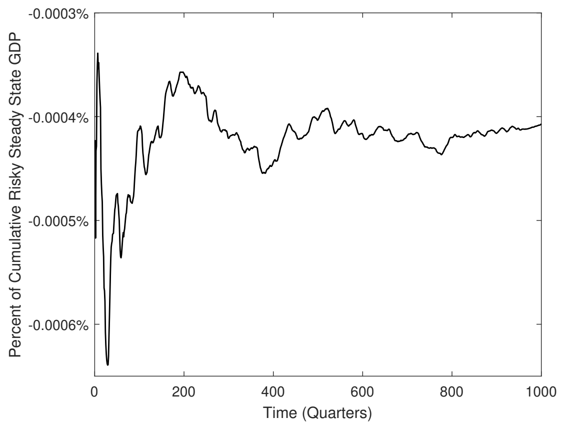
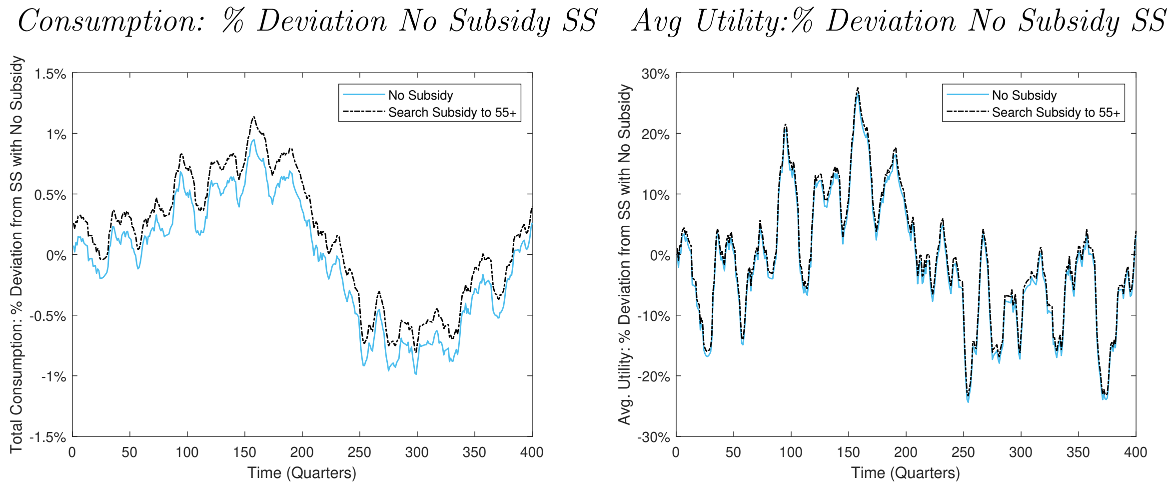


Figure 24: Comparing No Subsidy with a Fixed Subsidy

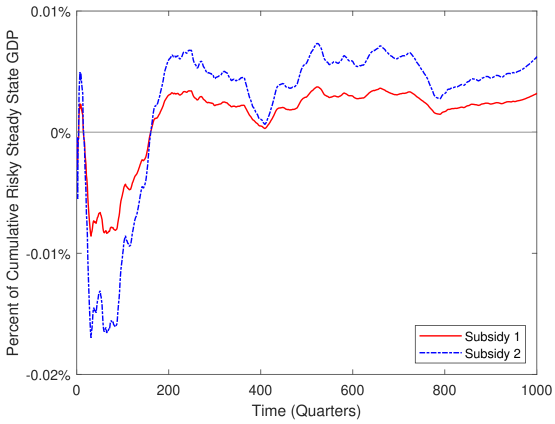


5.3.2 Aggregate State-Dependent Subsidy

Consider the same steady-state subsidy amount given only to agents 55 and older that maximizes steady-state average utility. Rather than keeping the subsidy amount fixed over the business cycle, we examine how allowing the subsidy amount to vary with aggregate productivity affects how the economy responds over the simulated series of shocks. In this section, we consider two variations of this subsidy. Additional variations, including subsidizing search for individuals of all ages, which we found lowered average steady-state utility, are discussed in *Appendix E*. Let s^* be the subsidy amount given only to agents 55 and older that maximizes steady-state average utility, and consider two variations of this policy. "Subsidy 1" pays amount $s(Z) = s^* - 0.5\left(\frac{(Z-Z_{SS})}{Z_{SS}}\right)$ to agents 55 and older who search for a job, where Z_{ss} indicates the steady-state value of Z. "Subsidy 2" varies even more over the business cycle and pays amount $s(Z) = s^* - \left(\frac{(Z-Z_{SS})}{Z_{SS}}\right)$.

While the subsidy cost is higher during recessions compared to the case where the subsidy did not vary with the aggregate state of the economy, the subsidy cost is also relatively lower following positive shocks to aggregate productivity. *Figure 25* shows that while the per-period government deficit or surplus may at times be larger than in the fixed subsidy case, the government budget is still closely balanced in expectation when the subsidy amount varies over the business cycle. *Figure 25* shows that after the

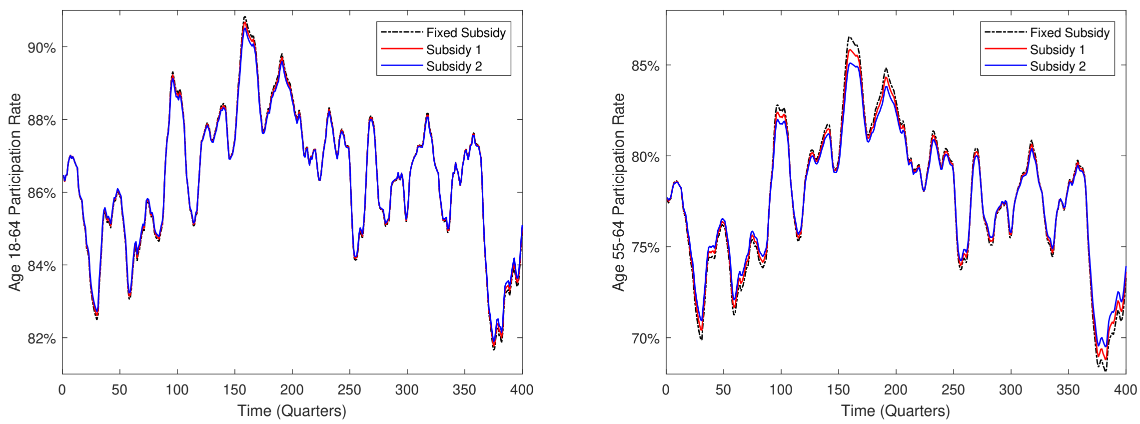
Figure 25: Aggregate State-Dependent Subsidy: Cumulative Net Government Income



simulated 250 years, the government's cumulative net income is close to zero (surplus less than 0.01%). While the government's budget is balanced in the long-run, it experiences larger deficits and surpluses in certain periods with more volatile subsidies. While having no notable effect on mean participation, allowing the subsidy amount to vary with the business cycle has evident dampening effects on the volatility of participation. Since the search subsidy is only offered to those aged 55 and older, the effects are concentrated in this age group (*Figure 26*).

Figure 26: Aggregate State-Dependent Subsidy: Labor Force Participation

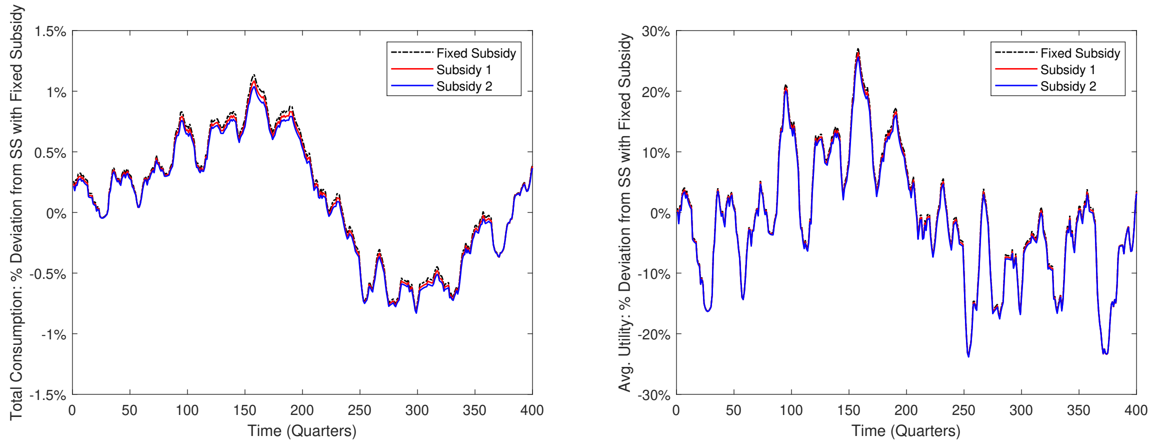
Age 18+ Participation Rate *Age 55+ Participation Rate*



Although having a noticeable impact on the volatility of labor force participation, *Figure 27* shows that allowing the subsidy amount to vary with the aggregate state results in only a slight reduction in the volatility of consumption and average utility.

Figure 27: Aggregate State-Dependent Subsidy: Consumption and Average Utility

Consumption: % Deviation Fixed Subsidy SS *Avg Utility: % Deviation Fixed Subsidy SS*



The results of these policy experiments are summarized in the following table. *Table 4* displays how various measures of concern change under different policies. Consistent with *Figure 20* we find that offering a search subsidy to agents age 55 and older increases average utility, consumption, GDP, and participation over the series of shocks. A noteworthy feature of this experiment is that the average consumption gain declines slightly from 0.184% to 0.137% with an increasingly generous subsidy during economic downturns (refer to *Table* 4, second row), leading the average utility gain to decline from around 1.013% to 0.317% as well (first row). This unveils an inefficiency associated with offering search subsidies during recessions: firms post fewer vacancies during such times resulting in a lower job-finding rate, so agents are encouraged to pay a search cost when the benefit of doing so is lower (and during booms, they are not as encouraged to search when the benefit is higher). Therefore, such a subsidy results in lower consumption and correspondingly, reduced welfare.[35] Therefore, while business cycle policies can mitigate the volatility of several aggregate variables, they predict marginal or even negative welfare gains when compared to their steady-state policy counterparts.[36] *Appendix D* shows that the results reported in *Table 4* are robust to different series of simulated shocks.

[35] To understand this argument more formally, consider the following Cobb-Douglas matching function, where the total number of matches m is given by:

$$m = Av^{0.5}u^{0.5}.$$

A represents the match efficiency, v denotes the total number of vacancies, and u is the share of unemployed workers. Note that the marginal efficiency of boosting u to increase the total number of matches is $Av^{0.5}$, which decreases in recessions because vacancy posting rates are typically low in those periods.

[36] Table 4 in Appendix E reports variance statistics over the simulated periods.

Table 4: Comparing Different Subsidy Policies: Mean over Simulated Series of Shocks

	Fixed Subsidy to 55+	Subsidy 1	Subsidy 2
Avg. Utility: % Change	1.013%	0.653%	0.317%
Consumption: % Change	0.184%	0.159%	0.137%
GDP: % Change	0.030%	0.023%	0.010%
18-64 Participation: Rate Change	0.032%	0.021%	0.009%
55+ Participation: Rate Change	0.163%	0.115%	0.057%

Note: All changes are relative to the case of no subsidy. Subsidy 1 and 2 refer to policies with different sensitivity to business cycle shocks.

6 Conclusion

The economic downturn caused by the pandemic led to a considerable decline in labor force participation, particularly among the younger and older demographics. This situation prompts us to explore policy measures that encourage workers to return to the job market and how a government should design such measures. This paper employs a heterogeneous-agent search model to shed light on this issue. The model's key elements are endogenous college participation and human capital accumulation over workers' life cycle. The model successfully matches disaggregate evidence on labor participation rates across different age cohorts and their responses to GDP growth shocks.

The policy recommendation from our model is as follows: subsidies should be aimed at older workers rather than younger ones, as subsidizing the youth could unintentionally discourage college enrollment, adversely impacting long-term productivity. Furthermore, these subsidies should be provided consistently, not only during periods of economic downturn. The rationale is that subsidizing job search in recessions is less productive, as good jobs are harder to find during recessions than booms.

Our model provides a foundation that can be expanded in various ways. For instance, this model does not consider asset accumulation, and broadening this aspect would enable the study of the interplay between labor market dynamics and wealth inequality. Such a framework can be further extended with an equilibrium model of colleges, akin to Cai and Heathcote (2022), to examine the role of college education in this interaction. Furthermore, the model does not consider active on-the-job training offered by a wide range of firms in practice. Considering human capital's significant role in driving labor participation, as shown in our model, such training could have crucial implications for labor market dynamics and workers' participation choices. We leave these to future research.

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Appendix A: Further Empirical Results and Robustness

Participation Response to Alternative Measures of Productivity

Table 5 displays the results of regressing the participation rate of different age groups on real GDP per capita growth and the past four lags of both real GDP per capita growth and percentage change in the participation rate of that age group. Compared to the results of *Table 1*, which used real GDP rather than real GDP per capita, we see that this alternative approach does not affect our results in any meaningful way.

Table 5: Responsiveness of Participation Rate to Changes in Real GDP Per Capita Growth

% Change in Participation	Age 20-24	Age 25-54	Age 55+
Real GDP Per Capita Growth	**0.502*****	**0.145*****	**0.160*****
	(0.062)	(0.014)	(0.052)
Real GDP Per Capita Growth (t-1)	0.267***	0.047**	0.085
	(0.078)	(0.020)	(0.055)
Real GDP Per Capita Growth (t-2)	0.027	-0.005	0.005
	(0.081)	(0.019)	(0.054)
Real GDP Per Capita Growth (t-3)	0.079	-0.007	0.108**
	(0.081)	(0.019)	(0.053)
Real GDP Per Capital Growth (t-4)	0.102	0.005	0.010
	(0.080)	(0.019)	(0.052)
% Change Participation (t-1)	-0.347***	-0.139	-0.233**
	(0.097)	(0.097)	(0.094)
% Change Participation (t-2)	-0.179*	0.025	0.204**
	(0.103)	(0.096)	(0.089)
% Change Participation (t-3)	-0.054	0.265***	0.384***
	(0.104)	(0.099)	(0.090)
% Change Participation (t-4)	-0.100	0.092	0.143
	(0.098)	(0.096)	(0.092)
Constant	-0.525***	-0.079***	-0.036
	(0.115)	(0.024)	(0.090)
Observations	112	112	112
R^2	0.495	0.531	0.268

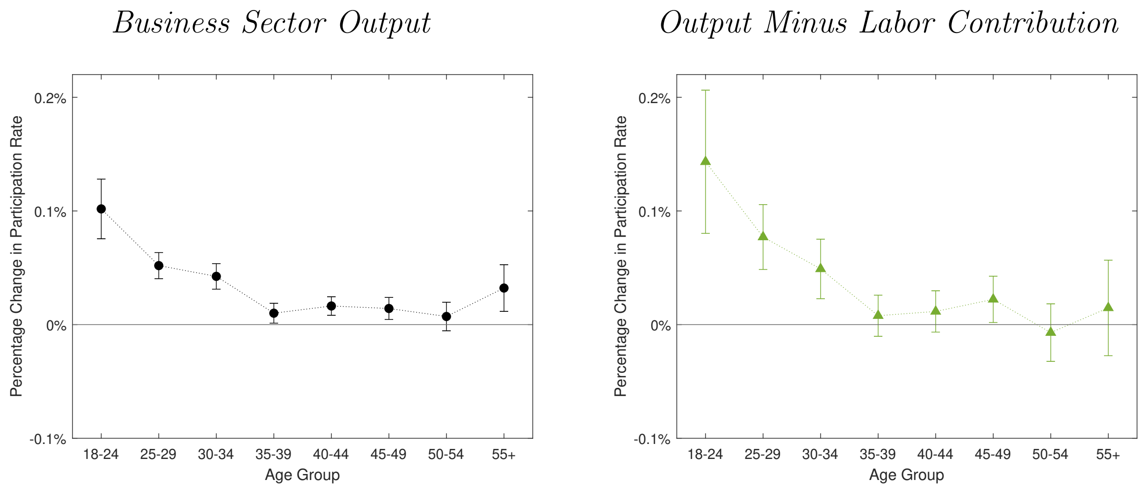
Notes: The participation data used here are quarterly averages of monthly, seasonally adjusted, data from the CPS. Individuals who reported not participating due to illness, disability, or because they were caring for their home/family were excluded from the sample. The data spans from 1994 Q1 to 2023 Q1. Stars denote statistical significance: $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$.

We now present a set of empirical findings comparing the response of participation to changes in business sector output and total factor productivity (TFP) estimated in

Fernald (2014).[37] Business sector output in that paper is estimated as an equally weighted average of business output measured from the expenditure side and business output measured from the income side. The percentage change in business sector output is then approximated as the difference in the log of business sector output from one quarter to the next. This approximation of the quarterly percentage change in business sector output differs from the percentage change in real GDP, especially in instances of larger deviations.

The left panel of *Figure 28* displays the response of the percentage change in participation, also approximated as the difference in logs, to changes in the approximated percentage change in business sector output. While the pattern of the relative responses is similar, with the participation of younger individuals being the most responsive, the size of the responses to this measure of the change in output are smaller. The key question of interest, however, is how the estimated response of participation to TFP changes compares to the estimated response to output changes. Fernald (2014) estimates the percentage change in TFP (again approximated as the difference in logs) as the percentage change in output minus changes in the contributions of labor and capital. The right panel of *Figure 28* shows the response of participation to changes in output minus changes in the contribution of labor.[38]

Figure 28: Same-Quarter Percentage Change in Participation Rate Associated with 1% Higher Business Sector Output from Fernald (2014)



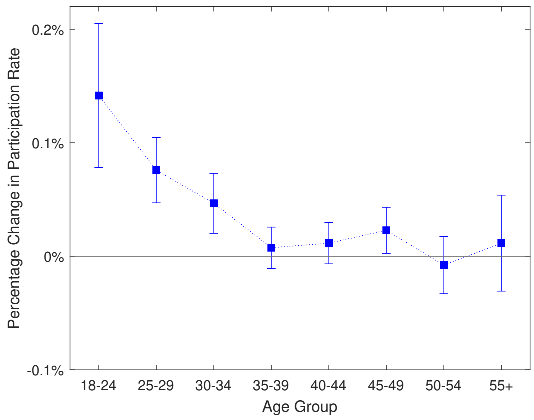
The estimated response of participation to changes in output after subtracting out

[37]This data was obtained from https://www.johnfernald.net/TFP, which provided an updated data set including estimates through the fourth quarter of 2022.

[38]In the data set provided by https://www.johnfernald.net/TFP, the percentage change in business sector output is given by the variable "dY". We compute the percentage change in output minus changes in the contribution of labor as "dY - (1-alpha)*(dhours + dLQ)". Fernald computes the the percentage change in business sector TFP as "dY - alpha*dk - (1-alpha)*(dhours + dLQ)". The measure of changes in the labor input is adjusted for changes in hours worked and changes in worker quality.

changes due to the contribution of labor appears similar to the estimated response to changes in business sector output, and is slightly larger. We also find a similar result when estimating the percentage change in participation related to a 1% higher percentage change in business sector TFP, shown in *Figure 29*.

Figure 29: Same-Quarter Percentage Change in Participation Associated with 1% Higher Business Sector Total Factor Productivity from Fernald (2014)



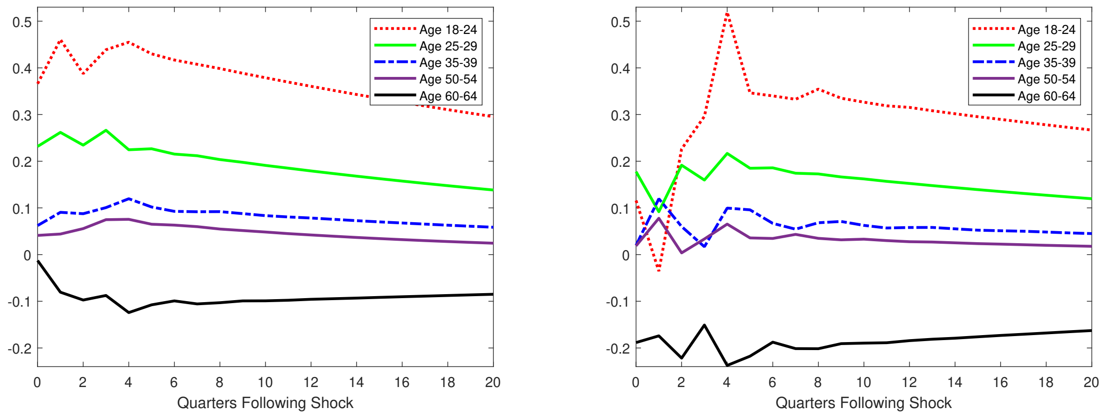
Because the estimated response of the percentage change in participation related to changes in business sector output and business sector TFP estimated via Fernald (2014) are similar, we believe our empirical results are robust and are not driven by an external factor affecting both output and participation.

Participation Response with and without Pandemic Data

This section investigates how estimated participation rate responses to aggregate shocks would differ if only pre-pandemic data were used. *Figure 30* shows the impulse response functions (IRFs) estimating the response of the participation rate of each age group to a positive shock resulting in 1% greater GDP growth. The left panel of the figure shows the IRFs estimated using the full data spanning 2000-2022. (The 95% confidence bands are excluded so as to make the figure readable.) The right panel of *Figure 30* shows the same IRFs estimated using only pre-pandemic data from 2000-2019. We see that including data from the pandemic recession changes the estimated timing of the response of individuals aged 18-24, but not the general size of the response after around four quarters. The estimated responses of age groups 25-29, 35-39, and 50-54 appear similar when only using pre-pandemic data. However, each group's initial same-quarter response to the shock appears slightly lower. Agents aged 60-64 have a surprisingly negative estimated participation response to positive shocks when we use only pre-pandemic data. Once the response of 60-64 year-olds during the pandemic is included in the data set, the estimated IRF shifts upward.

Figure 30: Participation Response to a 1% GDP Growth Shock

IRFs Using 2000-2022 Data *IRFs Using 2000-2019 Data*



Participation Level and Response Differences by Demographics

This paper uses participation rate data from the Current Population Survey, where those not participating due to disability, illness, or because they were caring for their home or family are discluded from the sample. *Figure 31* displays the average participation rate from 2000-2022 by age, race, and gender when non-participation due to those listed reasons are discluded. The left panel of the figure shows a persistent gap in participation over the life-cycle between Black and White or Hispanic respondents. The right panel of the figure shows that while there is no gap in participation among men and women early in the life-cycle (when non-participants due to disability, illness, or caring for home or family are discluded), a small gap starts to emerge later in the life-cycle.

Figure 31: Participation over the Life Cycle by Race and Gender

Participation by Race *Participation by Gender*

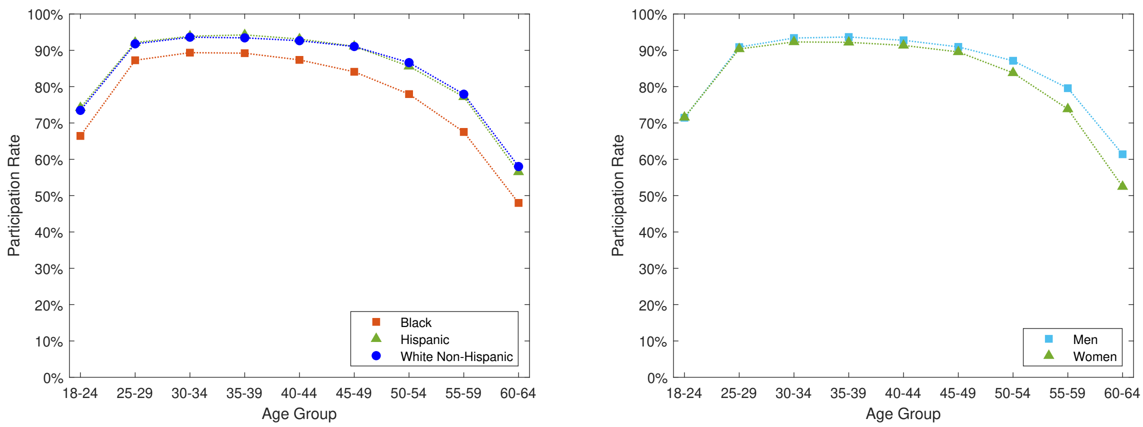


Figure 32 mimics *Figure 4* in that it displays the percentage change in the participation rate of each defined group associated with 1% higher GDP growth in the same quarter.

These responses are estimated using a VAR including four lags of both the percentage change in the seasonally-adjusted participation rate from the preceding quarter and the percentage change in real seasonally-adjusted GDP from the preceding quarter. The left panel of the figure shows that across the age groups considered, the participation of Black respondents was most responsive to changes in GDP growth, followed by Hispanic respondents. The right panel of *32* shows that the estimated responsiveness for men and women is relatively similar throughout the life-cycle.

Figure 32: Participation Responsiveness by Race and Gender: Percentage Change in Participation Associated with 1% Higher GDP Growth in the Same Quarter

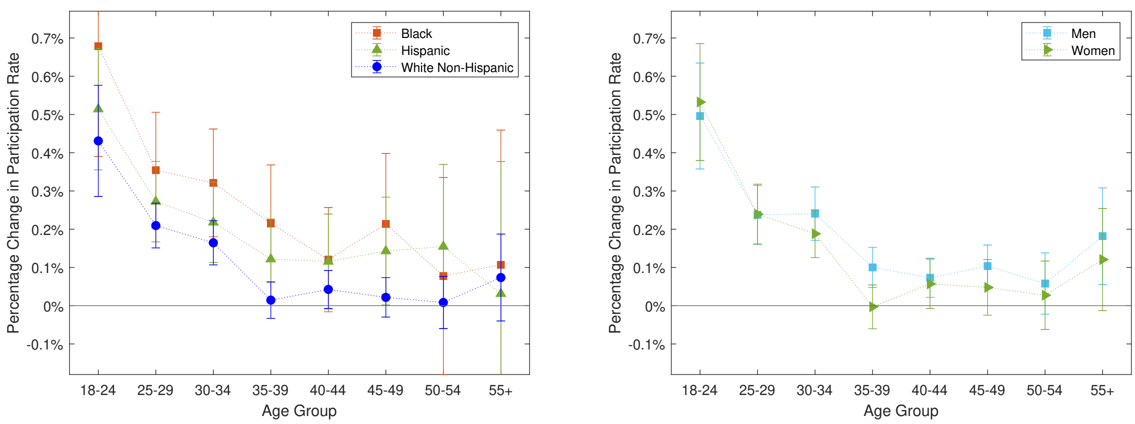
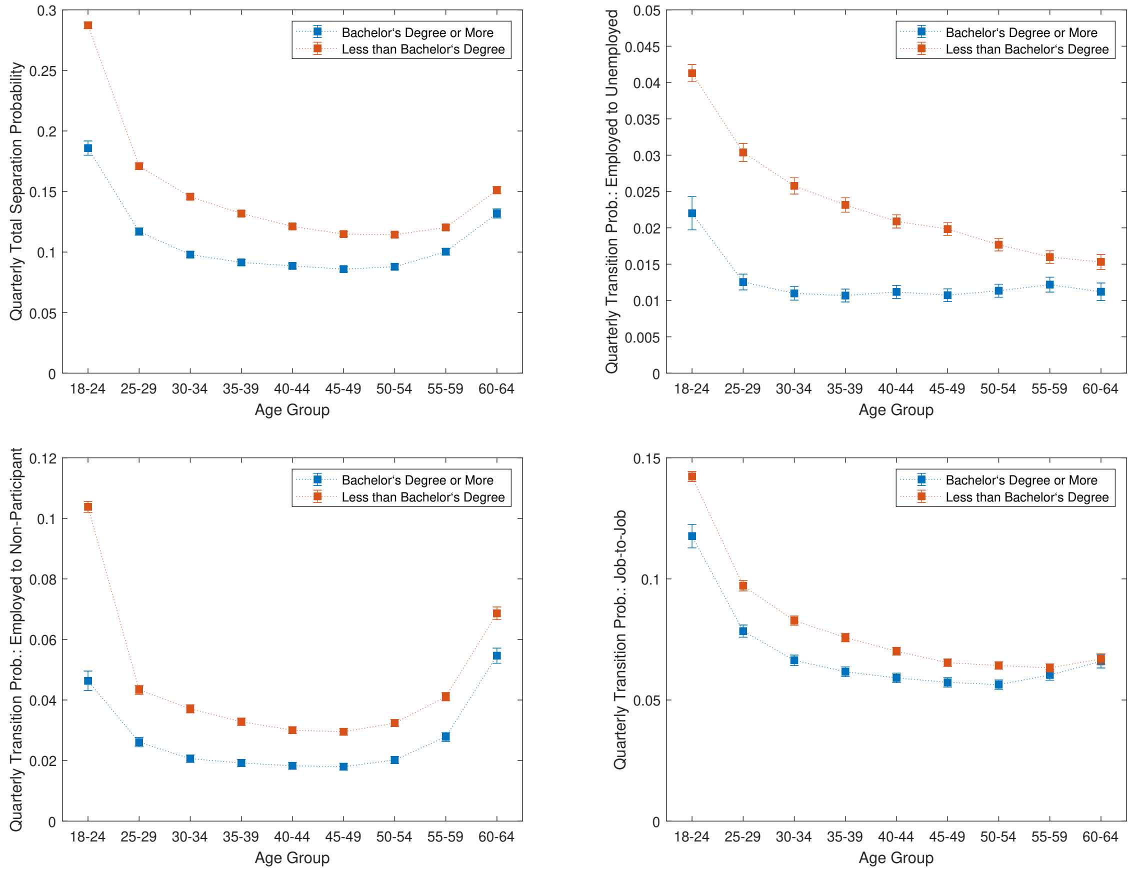


Figure 33 plots the estimated overall separation probabilities (top left panel, the same as figure 8), along with the decompositions into transition rates to unemployment (top right panel), non-participation (bottom left panel), and to another employer (bottom right panel). The top right panel plots the transition rate to unemployment by age and education. One notable feature is that the separation probability differential across different education levels is driven by separations to non-employment, not to another employer. In fact, the job-to-job transition rates are virtually the same for college graduates and non-college graduates. These patterns are consistent with what the existing literature has found. See Guo (2018) for overall separation rates estimated from the CPS and Menzio et al. (2016) for EU rates estimated from the SIPP for men only.

Figure 33: Estimated Separation Probability and Decomposition



Appendix B: Alternative Welfare and Equivalent Variation Calculations

In this appendix, we discuss alternative welfare measures that could be considered to examine the effects of offering different search subsidies. First, we consider focusing on how different subsidies affect new agents entering the economy. We report how search subsidies offered to all agents and to different age groups affect the discounted expected lifetime utility value of new entrants as well as the consumption equivalent variation of new entrants. To compute the latter, we estimate the share of remaining lifetime consumption each agent would be willing to forgo (or must receive) to enter the economy with the policy change.

Let $\left(\{c_{jt}\}_{t=1}^{T}\right)$ be agent j's consumption over future periods and contingencies in the baseline economy, and let $\left(\{\tilde{c}_{jt}\}_{t=1}^{T}\right)$ denote the same series after the change in policy. We first estimate the scaling parameter η_j such that individual j is indifferent between living the remainder of their life under both policies. For each individual j, η_j is such that

$$\sum_{t=1}^{T} \beta^{t-1} \ln(\eta_j c_{jt}) = \sum_{t=1}^{T} \beta^{t-1} \ln(\tilde{c}_{jt}).$$

Let $\tilde{V}_j \equiv \sum_{t=1}^{T} \beta^{t-1} \ln(\tilde{c}_{jt})$. With the chosen utility function, we can solve for η_j as follows.

$$e^{\sum_{t=1}^{T} \beta^{t-1} \ln(\eta_j c_{jt})} = e^{\tilde{V}_j}$$

$$e^{\ln(\eta_j c_{j1})} e^{\beta \ln(\eta_j c_{j2})} e^{\beta^2 \ln(\eta_j c_{j3})} ... = e^{\tilde{V}_j}$$

$$\eta_j c_{j1} (\eta_j c_{j2})^{\beta} (\eta_j c_{j3})^{\beta^2} ... = \left(\eta_j^{\left(1+\beta+\beta^2+...\right)} \right) \left(e^{\ln(c_{j1})} e^{\beta \ln(c_{j2})} e^{\beta^2 \ln(c_{j3})} ... \right) = e^{\tilde{V}_j}$$

$$\left(\eta_j^{\sum_{t=1}^{T} \beta^{t-1}} \right) \left(e^{\sum_{t=1}^{T} \beta^{t-1} \ln(c_{jt})} \right) = e^{\tilde{V}_j}.$$

Let $V_j \equiv \sum_{t=1}^{T} \beta^{t-1} \ln(c_{jt})$. Then

$$\left(\eta_j^{\sum_{t=1}^{T} \beta^{t-1}} \right) e^{V_j} = e^{\tilde{V}_j}$$

$$\eta_j = \left(e^{\tilde{V}_j - V_j} \right)^{\left(\frac{1}{\sum_{t=1}^{T} \beta^{t-1}} \right)}.$$

So that η_j can be used to determine the share of consumption that agent j is willing to give up (or must receive) to enter the economy with the new policy, we use the following transformation

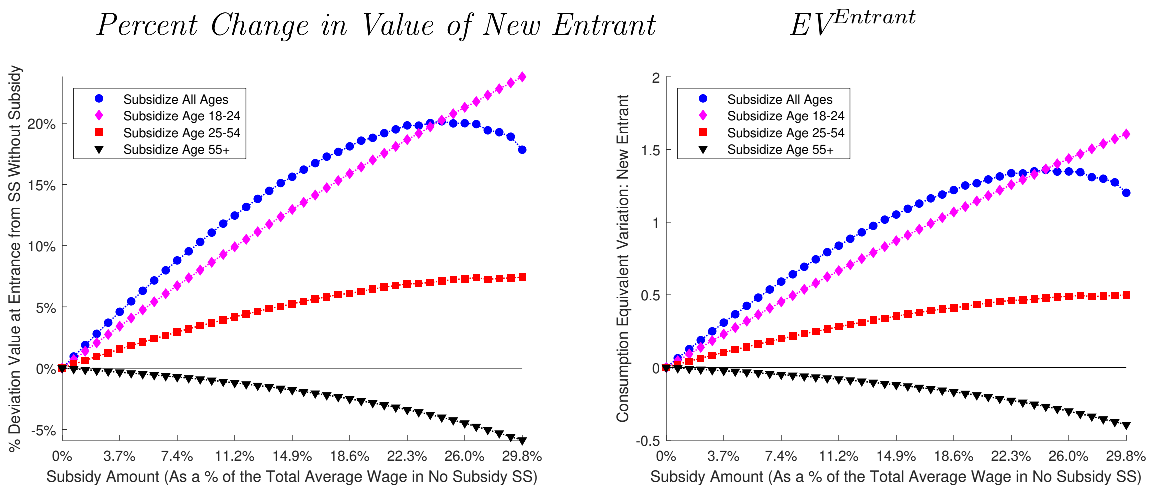
$$\widehat{\eta}_j = 100(\eta_j - 1).$$

We use the equivalent variation solved for any individual j first to compute the share of consumption a new agent would be willing to forgo (or receive) in all future periods and contingencies to enter into the economy with the policy change as opposed to the baseline economy. The expected discounted series of consumption that the agent would receive upon entering the baseline economy is specified by equation (5) where $a = 1$, $\tau = 1$, $Z = Z_{ss}$, and $z = 0$. Let $\tilde{N}_1(1, Z_{ss}, 0)$ denote this entrance value after the policy change considered. The consumption equivalent variation of a new entrant is

$$EV^{Entrant} = 100\left(\left(e^{\tilde{N}_1(1,Z_{ss},0)-N_1(1,Z_{ss},0)}\right)^{\left(\frac{1}{\sum_{t=1}^{T}\beta^{t-1}}\right)} - 1\right).$$

Figure 34 plots the percentage change in the value of expected discounted lifetime utility and the consumption equivalent variation of a new entrant under different policy changes. In both panels of the figure, we see that all subsidies except subsidies to those 55 and older increase the entrance value of new agents. When subsidies are given only to those 55 and older, new entrants who discount the future see a far-away benefit and an immediate cost of having to pay higher taxes when employed. Each panel of the figure shows qualitatively the same results but offers a different way to quantify these results.

Figure 34: Alternative Welfare Measures: Focus on New Entrant



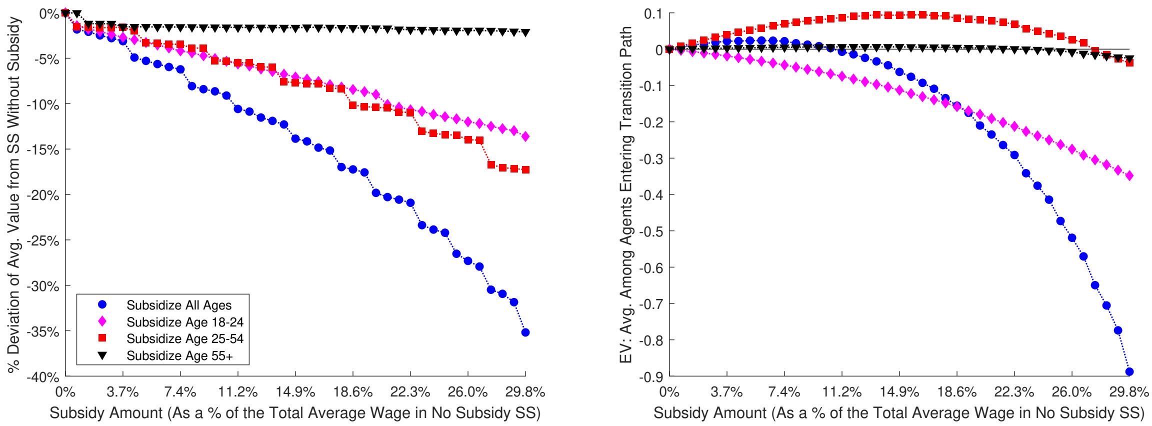
Now we consider how policy changes affect all agents who are in the economy at the time of the policy change. These agents vary in age, skill, education, and employment status. We report the percentage change in the average *remaining* expected discounted lifetime utility of all agents in the economy at the time of the policy change. We also report the average consumption equivalent variation of agents at the time of the policy change. This second metric that we consider is the average share of consumption that existing agents in the steady-state of the baseline economy would be willing to give up (or must receive) for the remainder of their lives before retirement to undergo the policy change. Specifically, we consider that in the steady-state of the baseline economy, there are agents of all ages and employment statuses and consider values of $\widehat{\eta}_j$ for their remaining consumption. Considering a large representative sample of N agents from the baseline steady-state, we compute

$$EV^{Transition} = \frac{1}{N}\sum_{j=1}^{N}\widehat{\eta}_j.$$

The left-hand side of *Figure 35* displays the percentage change in the average remaining expected discounted lifetime utility of all agents in the economy at the time of the policy change. The results look similar in nature to *Figure 20*, as only the subsidy paid to agents age 55 or older increases the average remaining expected discounted lifetime utility. The right-hand side of *Figure 35* plots the average consumption equivalent variation of agents at the time of the policy change ($EV^{Transition}$).

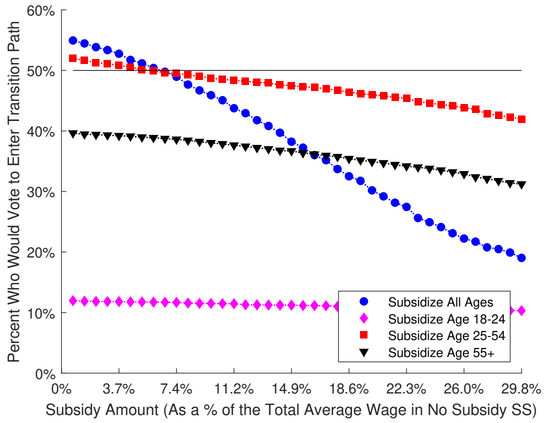
Figure 35: Alternative Welfare Measures: Weighting All Agents Present at Time of Policy Change

% Change in Avg. Value At Policy Change $EV^{Transition}$



Finally, we consider the percentage of agents in the risky steady-state of the baseline economy with no search subsidy who would vote for each policy considered. We assume that agents are only self-interested, and they vote for a policy only if it increases their remaining expected discounted lifetime utility. Unsurprisingly, we find that policies that offer a search subsidy to a larger group of agents, rather than just the very young or old, are more likely to gain the approval of a majority of agents. Policymakers might consider expanding the ages to which the policy is offered to ensure that the policy would meet the approval of a majority of voters. (For example, they might consider the effects of offering the search subsidy to agents 50 and older rather than 55 and older to find a utility-increasing policy that would also likely be approved by voters.)

Figure 36: Percent Who Would Vote For Each Policy Change

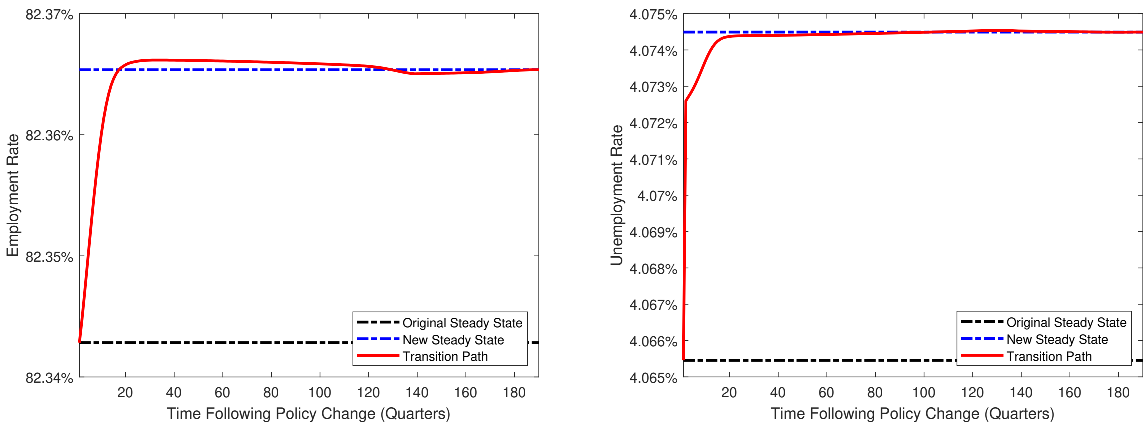


Appendix C: Transition Paths To New Steady-State for Utility Maximizing Subsidy to Ages 55+

This appendix discusses the transition path of the economy from the risky steady-state with no search subsidy to the risky steady-state with the utility-maximizing subsidy paid to unemployed agents age 55 and older. *Figure 37* shows that the employment and unemployment rates transition to their new steady-state values relatively quickly after the policy change. The search subsidy encourages those who receive it to pay the cost to search for employment, so it is unsurprising that the policy change would have a sudden and positive impact on the percentage of the total population that is employed. Because the subsidy is only given to older workers, the overall impact on the total employment rate is modest, resulting in an increase from around 82.34% to 82.37% in the new steady-state.

Figure 37: Transition Paths of Employment and Unemployment

Employment Rate Transition Path *Unemployment Rate Transition Path*

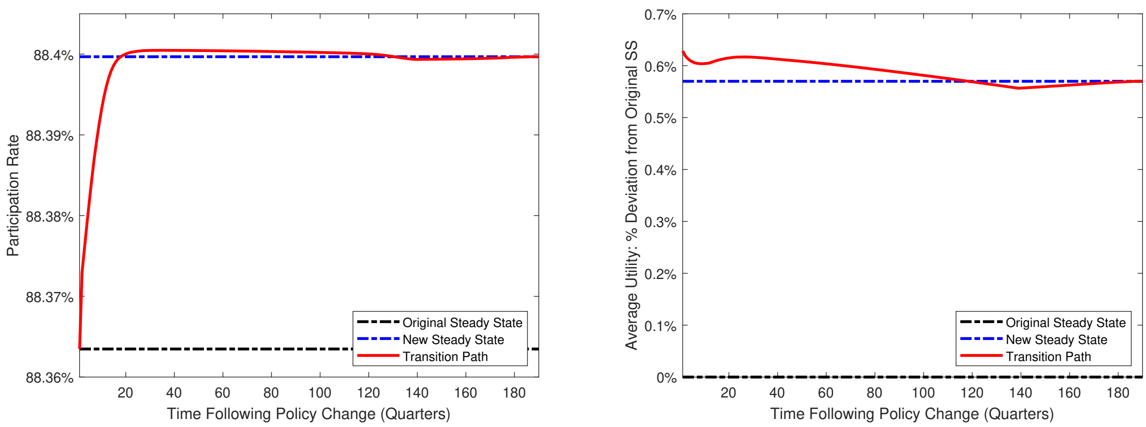


The total unemployment rate in the economy also transitions fairly quickly to its new steady-state value. The search subsidy lessens the benefit of moving from unemployment to employment, so when agents direct their search, they are willing to trade off a lower job-finding rate for a higher wage if matched. Therefore, agents receiving the subsidy spend more time in unemployment on average, and the total unemployment rate increases.

Figure 38 displays the transition paths of the total participation rate and average utility following the policy change. Like employment and unemployment, the participation rate responds relatively quickly to the policy change, while average utility moves more slowly to its new steady-state value. The search subsidy immediately promotes participation, and the participation rate is close to its new steady-state value around 30 quarters following the policy change. We see that the increase in average utility is also sharpest in approximately the first 30 quarters after the policy change.

Figure 38: Transition Paths of Average Utility and Total Participation

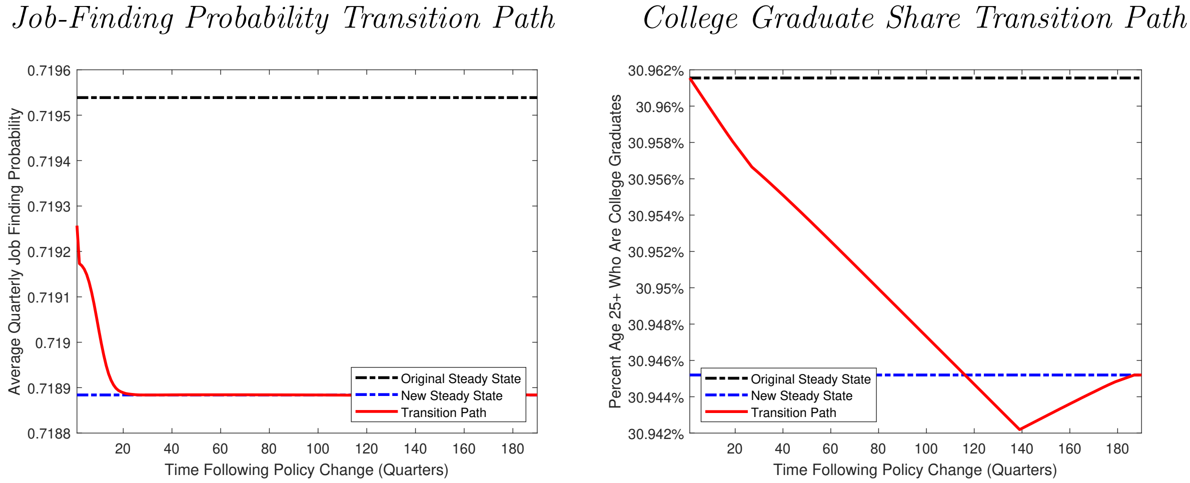
Participation Rate Transition Path *Average Utility Transition Path*



Although participation and employment respond quickly to the policy and influence the transition of average utility, the policy change also influences the percentage of the population who attend and graduate from college. *Figure 39* shows how the share of college graduates among agents age 25 and older responds to the policy change. Although young agents are not initially eligible for the subsidy, the policy change affects their enrollment decision because it impacts the length of time they expect to be employed and, therefore, the length of time they expect to benefit from obtaining a college degree. The policy change does not immediately have much impact on the share who have graduated because although it affects the enrollment decision of younger agents, these agents do not immediately graduate and are only included in the average for educational attainment after they turn 25 (as is common in many data series reporting educational attainment). The gradual transition of the share of agents who are college graduates towards its new steady-state value explains the more gradual increase in average utility after the initial

30 quarters following the policy change. *Figure 39* also shows how the average job-finding probability of those who are unemployed responds to the policy change. The immediate drop in this probability coincides with the immediate increase in the unemployment rate shown in *Figure 37*.

Figure 39: Transition Paths of Job-Finding Rate and Percentage of Agents 25+ Who are College Graduates



Appendix D: Additional Aggregate State-Dependent Subsidy to Agents 55 Plus Results

This section provides additional details and results regarding the aggregate state-dependent subsidies offered only to agents age 55 and older discussed in 5.3.2. For each of the subsidies considered, we simulated the model economy with the subsidy over 1,000 quarters (250 years). So that the results are comparable over different simulations, we generate a series of shocks to aggregate productivity and apply this same series of shocks to the model economy under each simulation. Recall that aggregate productivity is assumed to follow an AR(1) process that is approximated using an N-state Markov chain.[39] *Figure 40* displays the simulated AR(1) and its Markov approximation.

[39]In this case, the AR(1) process is approximated with a 17-state Markov chain.

Figure 40: Simulation of Aggregate Productivity AR(1) Process

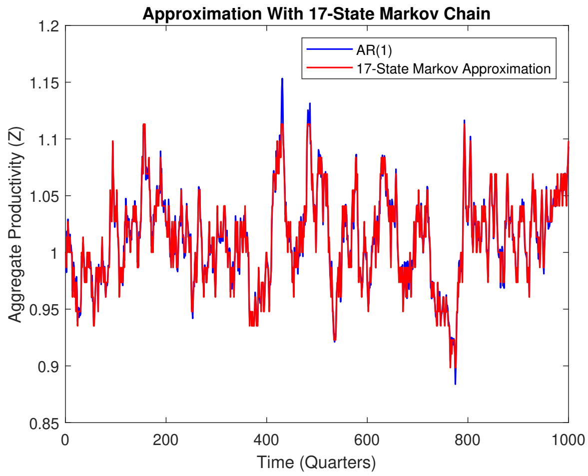


Table 6 reports the coefficient of variation (CV) for each series of interest over the simulated shocks. Consistent with *Figures 26* and *27*, allowing the subsidy amount to increase during recessions and decline during expansions reduces the volatility of utility, GDP, and participation. Consumption, which includes leisure consumption, is notably less volatile than GDP, and its volatility only slightly declines as we consider subsidies more responsive to the aggregate state.

Table 6: Coefficient of Variation over Simulated Shocks

	No Subsidy	Fixed Subsidy to 55+	Subsidy 1	Subsidy 2
Average Utility	0.1127	0.1122	0.1109	0.1097
Consumption	0.0050	0.0050	0.0050	0.0049
GDP	0.0262	0.0262	0.0255	0.0247
Participation Rate: 18-64	0.0261	0.0261	0.0253	0.0244
Participation Rate: 55+	0.0606	0.0603	0.0561	0.0517

Subsidy Simulation Robustness

In addition to considering how the model economy responds to the series of simulated shocks displayed in *Figure 40*, we check that our results are consistent when looking over ten total series each consisting of 1,000 simulated quarters in the model economy. *Table 7* replicates the results previously reported in *Table 4*, using instead the average over the ten different simulated series of shocks. We see that the results are quantitatively very similar to those previously reported, and do not change the policy recommendation of our model.

Table 7: Comparing Different Subsidy Policies: Robustness

	Fixed Subsidy to 55+	Subsidy 1	Subsidy 2
Avg. Utility: % Change	0.996%	0.653%	0.335%
Consumption: % Change	0.180%	0.156%	0.136%
GDP: % Change	0.030%	0.001%	-0.004%
18-64 Participation: Rate Change	0.032%	0.030%	0.026%
55+ Participation: Rate Change	0.163%	0.156%	0.140%

Note: All changes are relative to the case of no subsidy. Subsidy 1 and 2 refer to policies with different sensitivity to business cycle shocks.

Similarly, *Table 8* replicates the results reported in *Table 6*, using not only the first series of simulated shocks, but nine additional series of the same length. We see that including the results of different simulations does not notably change the results of our analysis.

Table 8: Coefficient of Variation over Simulated Shocks: Robustness

	No Subsidy	Fixed Subsidy to 55+	Subsidy 1	Subsidy 2
Average Utility	0.1274	0.1269	0.1258	0.1247
Consumption	0.0068	0.0068	0.0068	0.0068
GDP	0.0292	0.0292	0.0284	0.0276
Participation Rate: 18-64	0.0294	0.0294	0.0285	0.0276
Participation Rate: 55+	0.0693	0.0690	0.0642	0.0593

Appendix E: Effects of Aggregate State-Dependent Search Subsidies Offered to All Ages

In this appendix, we evaluate how offering a search subsidy that depends on aggregate productivity to agents of all ages affects the economy as it evolves over the same series of shocks displayed in *Figure 40*. This series simulates 250 years in the model economy. Recall that subsidizing search for all ages resulted in lower GDP and average utility. This is because subsidizing search makes job search relatively more attractive compared to college attendance. This results in younger workers having a lower college enrollment rate and reduces average worker productivity. Therefore, since subsidizing the search of all ages has negative steady-state implications, we consider a subsidy that equals zero when aggregate productivity is at its steady-state value.

Specifically, we consider "Subsidy 1" which pays amount $s(Z) = -0.5\left(\frac{(Z-Z_{SS})}{Z_{SS}}\right)$ to all agents who search for a job, where Z_{ss} indicates the steady-state value of Z. "Subsidy 2" varies even more over the business cycle and pays amount $s(Z) = -\left(\frac{(Z-Z_{SS})}{Z_{SS}}\right)$. *Figure 41* shows the per-period subsidy cost and tax revenue of each policy on the left-hand panel, while the right-hand panel shows the cumulative deficit or surplus over the entire simulation. Just as when the state-dependent subsidy was offered only to agents 55 and older, the government's budget is closely balanced in the long-run. The government does experience relatively larger deficits and surpluses in certain periods as the variability of the subsidy increases and as more individuals (age groups) are eligible to receive the subsidy.

Figure 41: Aggregate State-Dependent Subsidy offered to all Ages: Government Budget

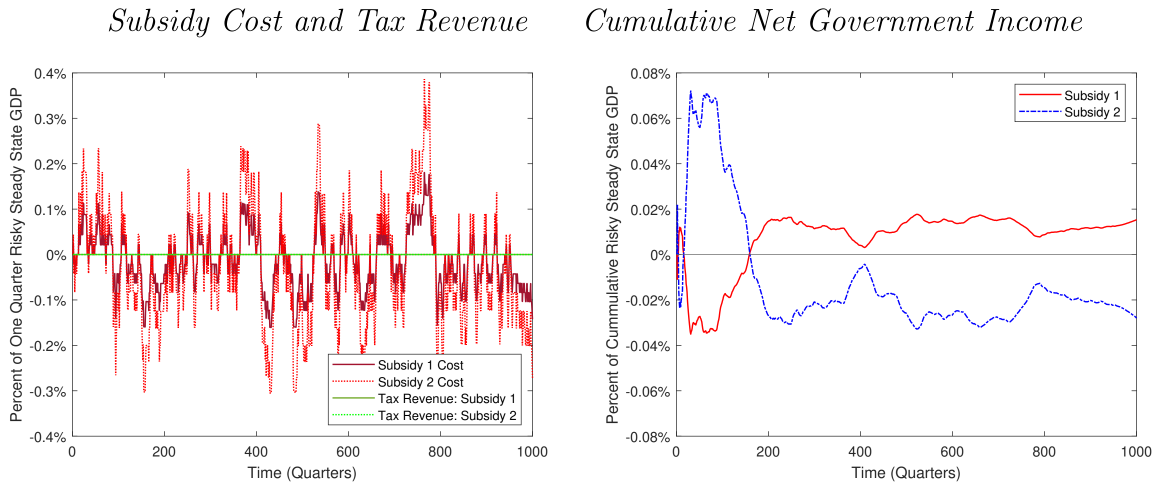
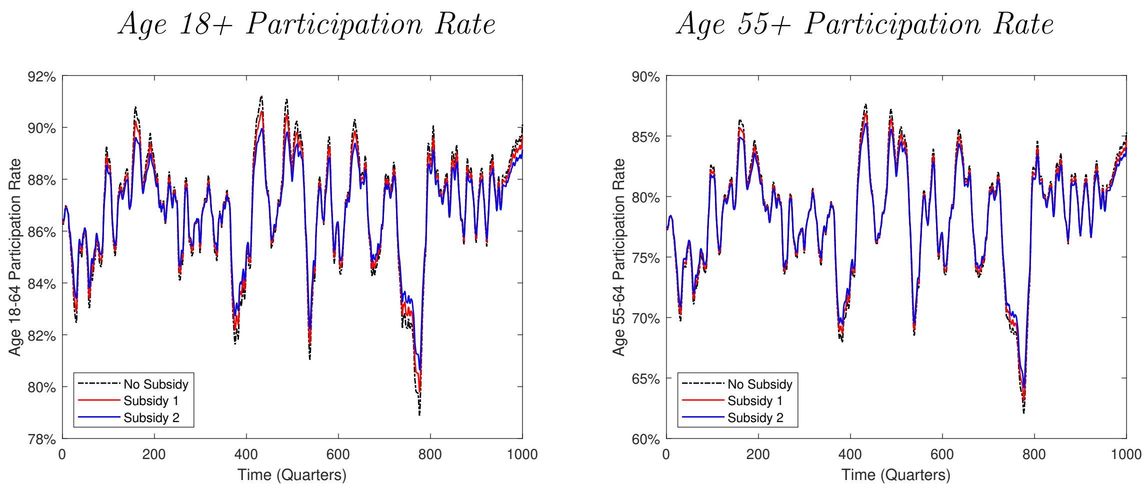


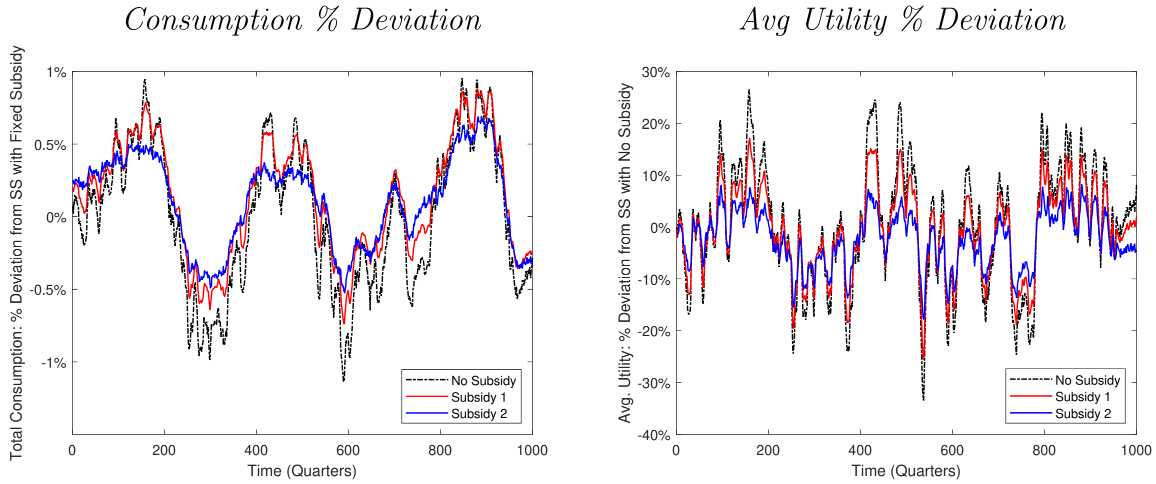
Figure 41 shows how each subsidy affects the total and age 55 plus participation rates. The means of the participation rates over the series of shocks are very similar.

Figure 42: Aggregate State-Dependent Subsidy offered to all Ages: Participation



Although the means are similar, we see that the volatility of the total and 55 plus participation rates are much lower in the case where the subsidy amount varies with the aggregate state. In the case of Subsidy 2, where the subsidy is the most responsive to changes in aggregate productivity, the drop in participation during recessions and the increase during booms is much less than in cases where no subsidy is applied.

Figure 43: Aggregate State-Dependent Subsidy offered to all Ages: Avg. Utility and Consumption % Deviation from No Subsidy Steady State



In addition to smoothing participation, the aggregate state-dependent subsidies reduce the volatility of both consumption and average utility.

Offering a search subsidy smooths consumption as agents move between employment and unemployment and also encourages participation. However, encouraging participation makes the option of attending college relatively less attractive. With directed search, the subsidy also makes agents willing to spend more time in unemployment, reducing the probability they will move out of unemployment. In the cases considered here, the subsidy equals zero when aggregate productivity is at its steady-state value. Allowing this baseline of zero to become slightly positive during recessions and slightly negative during booms very slightly reduces average utility over the simulated shocks.

Table 9: Mean over Simulated Shocks

	Subsidy 1	Subsidy 2
Avg. Utility: % Change from No Subsidy	-0.926%	-2.691%
Consumption: % Change from No Subsidy	0.136%	0.112%
GDP: % Change from No Subsidy	0.020%	0.055%
18-64 Participation: Rate Change 18-64	-0.028%	-0.060%
55+ Participation: Rate Change	-0.050%	-0.096%

Note: All changes are relative to the case of no subsidy. Subsidy 1 and 2 refer to policies with different sensitivity to business cycle shocks.

This reduction in average utility indicates that the benefit of offering a mechanism for consumption smoothing during times when unemployment is outweighed by the undesirable incentives in terms of search and college enrollment. *Table 10* reports the coefficient of variation (CV) for each series of interest over the simulated shocks. As with previous subsidies considered, subsidies that vary more in response to changes in aggregate productivity reduce the CV for utility, consumption, GDP, and participation.

Table 10: Coefficient of Variation over Simulated Shocks

	No Subsidy	Subsidy 1	Subsidy 2
Average Utility	0.1127	0.0828	0.0530
Consumption	0.050	0.0040	0.0031
GDP	0.0262	0.0242	0.0221
Participation Rate: 18-64	0.0261	0.0231	0.0200
Participation Rate: 55+	0.0606	0.0560	0.0514

Subsidy Simulation Robustness

We ensure the robustness of our findings by examining their consistency across ten distinct series, each comprising 1,000 simulated quarters within the model economy. *Table 11* presents a replication of the outcomes initially depicted in *Table 9*, but this time, we employ the average results derived from these ten different sets of simulated shocks. These results closely align with our previous findings.

Table 11: Mean over Simulated Shocks: Robustness

	Subsidy 1	Subsidy 2
Avg. Utility: % Change from No Subsidy	-0.768%	-3.307%
Consumption: % Change from No Subsidy	0.134%	0.113%
GDP: % Change from No Subsidy	0.035%	0.084%
18-64 Participation: Rate Change 18-64	0.002%	-0.002%
55+ Participation: Rate Change	-0.005%	-0.009%

Likewise, *Table 12* restates the findings presented in *Table 10*, incorporating not only the initial series of simulated shocks but also nine additional series of equivalent length. It is evident that the inclusion of results from various simulations has a negligible impact on the outcomes of our analysis.

Table 12: Coefficient of Variation over Simulated Shocks: Robustness

	No Subsidy	Subsidy 1	Subsidy 2
Average Utility	0.1274	0.0961	0.0652
Consumption	0.0068	0.0056	0.0044
GDP	0.0292	0.0270	0.0247
Participation Rate: 18-64	0.0294	0.0261	0.0226
Participation Rate: 55+	0.0693	0.0641	0.0588

Appendix F: Age-Dependent Subsidy Robustness

This appendix considers an alternative calibration strategy that targets additional data moments and allows the leisure benefit enjoyed in non-employment to be age-dependent. First, we discuss the advantages and disadvantages of this alternative calibration strategy compared to the strategy adopted in the main text. Then, we re-examine the results of introducing age-dependent search subsidies, initially discussed in Section 5.1. While the optimal search subsidy is larger under this alternative calibration, the key results remain the same. We find that it is optimal to subsidize only agents 55 and older under this alternative calibration, consistent with the results in the main text.

Alternative Calibration: Age-Dependent Leisure Values

In this appendix, we solve the model outlined in Section 4, with the additional assumption that the leisure value $b(z)$ is no longer skill-dependent but varies by age group. We calibrate the leisure value for each of the age groups displayed in *Figure 44* to target the participation rate of that specific age group. Additionally, to aid in matching the data, we now assume that the firm's vacancy posting cost c_f is a function of worker skill z so that $c_f = c_c + z$. In addition to the calibrated parameter values recorded in *Table 13*, the assigned model parameters remain the values listed in *Table 2*.

Figure 44: Participation Rate by Age Group: Model vs. Data (Targeted)

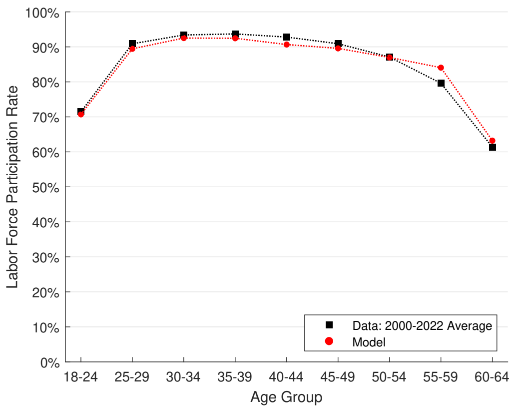


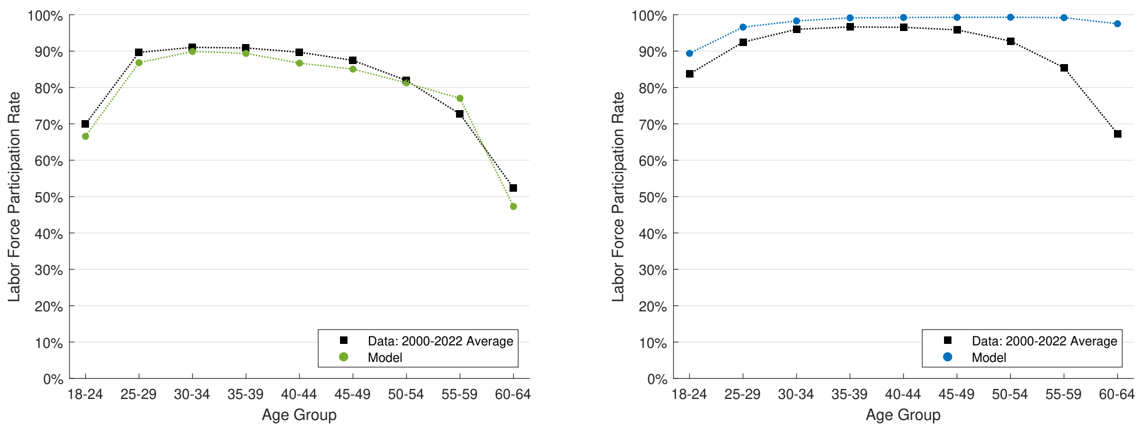
Table 13: Calibrated Parameter Values and Empirical Targets

Parameter	Estimate	Targeted Moment	Data	Model
ℓ	2.629671	Unemployment rate (%) for age 20+	4.04	4.05
κ	0.521257	Pop. age 25+ with Bachelor's degree (%)	31.9	31.1
Δ_g	0.241367	Wage ratio: prime age to 20-24	1.65	1.54
π_2	0.344238	College wage premium (age 25+)	1.81	1.60
π_1	0.129721	Wage ratio: 55-64 to prime age	1.09	1.34
c_c	0.679508	Vacancy posting rate	4.15	4.03
μ_{cw}	-0.518127	College to non-college employment rate ratio	1.32	1.25
σ_{cw}	0.692976	Prime age/60-64 participation rate	1.50	1.43
g	0.035700	College graduation rate over 4 yrs	0.44	0.44
σ_ϵ	0.018200	Participation % change w/ 1% shock: ages 18-24	0.52	0.56
$b_{18\text{-}24}$	0.781847	18-24 Participation Rate (%)	71.4	70.7
$b_{25\text{-}29}$	0.875283	25-29 Participation Rate (%)	90.9	89.5
$b_{30\text{-}34}$	0.937329	30-34 Participation Rate (%)	93.4	92.5
$b_{35\text{-}39}$	0.961273	35-39 Participation Rate (%)	93.7	92.5
$b_{40\text{-}44}$	1.052476	40-44 Participation Rate (%)	92.8	90.7
$b_{45\text{-}49}$	1.070725	45-49 Participation Rate (%)	90.9	89.6
$b_{50\text{-}54}$	1.164913	50-54 Participation Rate (%)	87.1	87.0
$b_{55\text{-}59}$	1.165635	55-59 Participation Rate (%)	79.6	84.1
$b_{60\text{-}64}$	1.294147	60-64 Participation Rate (%)	61.4	63.2

Although *Figure 44* showed that the alternative calibration presented in this section closely matches the total participation rate for each age group, the following figure shows significant discrepancies when examining participation conditional on education. The participation rate for those with a Bachelor's degree no longer aligns as closely with the data compared to the results in the main text. Additionally, when comparing the college

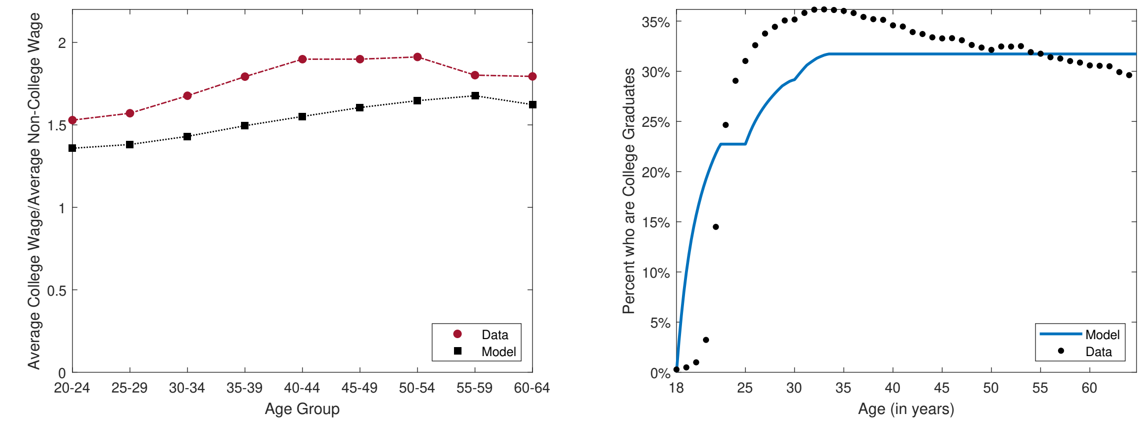
Figure 45: Participation Rate by Age Group and Education (Untargeted)

Less than Bachelor's Degree | *Bachelor's Degree or More*



premium for each age group with the data, we find that the model now produces a college premium that is too low for every age group. This provides a convenient robustness check of our subsidy results when the return to college attendance is lower than in the calibrated model presented in the main text. *Figure 47* shows that under this alternative calibration,

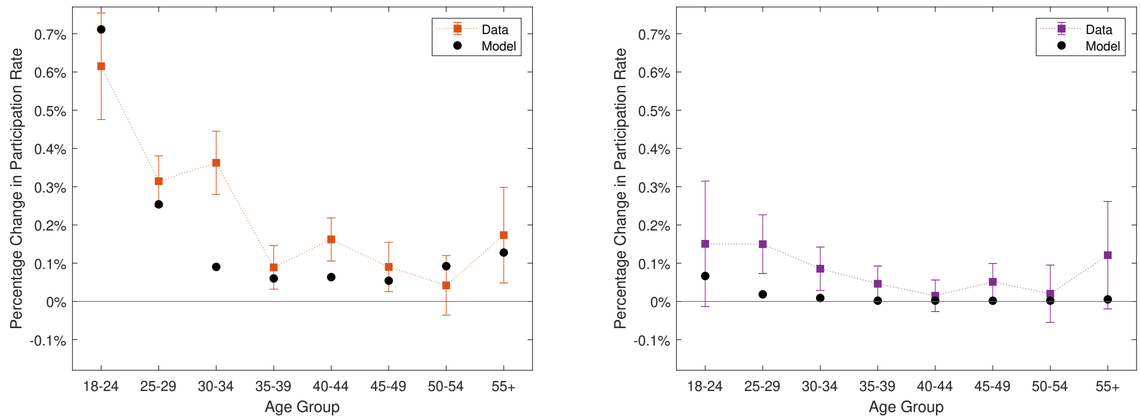
Figure 46: College Wage Premium and Percent with Bachelor's by Age (Untargeted)



the response of the participation rate of each age and education group to same-quarter GDP shocks aligns closely with the data. In summary, under this alternative calibration,

Figure 47: Same-Quarter Change in Participation Rate Associated with 1% Higher GDP Growth (Untargeted)

Less than Bachelor's Degree *Bachelor's Degree or More*

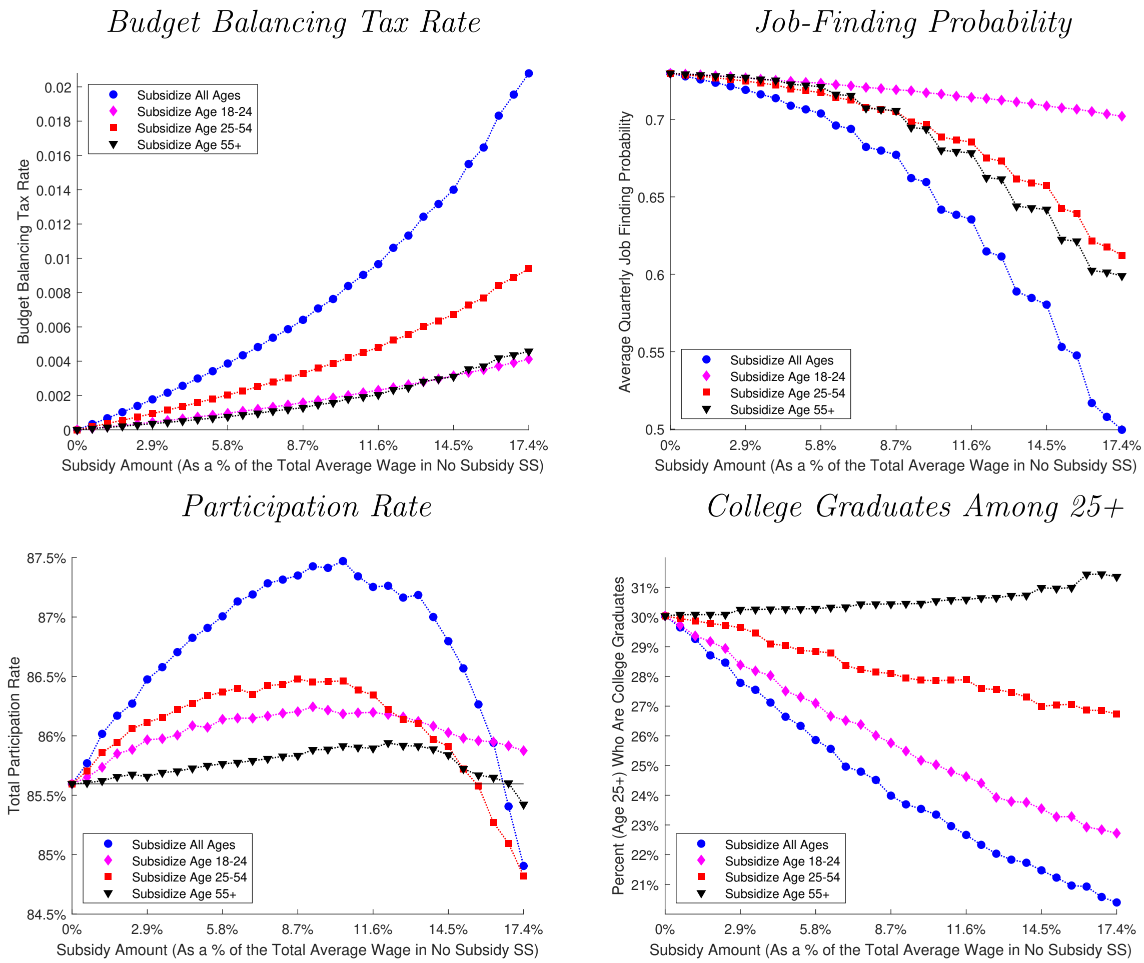


the model can more closely match the overall profile of participation by age and the participation rate responses to aggregate shocks. However, the model does worse at matching the participation by age profile of college graduates and at matching the college wage premium.

Subsidy Results Robustness Under Alternative Calibration

Now consider the effects of introducing age-dependent subsidies, as in Section 5.1 of the main text, under this alternative calibration. *Figure 48* displays the budget-balancing tax rate, quarterly job-finding probability, aggregate participation rate, and the percent of agents who are college graduates among those age 25+ under each subsidy considered. As in Section 5.1, we see that subsidies which are enjoyed by a larger share

Figure 48: Robustness: Impacts of Search Subsidy on Labor Market



of the population require a higher tax on the employed. Although introducing a subsidy for those who search for a job initially increases the participation rate, it reduces the job-finding probability as the incentive to leave unemployment declines and the unemployed become pickier in their directed search decision. Interestingly, the effect of subsidizing search among the older age groups increases college attendance more than in the main-text calibration. Given this result *Figure 49* shows that larger gains can be achieved by subsidizing the oldest age group. While the optimal subsidy to the 55+ age group was just around 0.74% of the average wage in the pre-subsidy economy, average utility under the alternative calibration peaks when a search subsidy equal to about 17.3% of the average wage in the pre-subsidy economy is given to those 55+. While the size of the optimal search subsidy differs, the overall result regarding which group to subsidize remains unchanged.

Figure 49: Robustness: Impacts of Search Subsidy on Average Utility