# Public Disclosure and Private Information Acquisition: A Global Game Approach*

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April, 2023

**Abstract**

This paper studies public information disclosure in a model of dynamic financial markets with endogenous information acquisition. Due to an information complementarity, multiple equilibria may emerge, complicating comparative statics analysis. By adding noise to agents' information costs, we establish equilibrium uniqueness using global-game techniques. We show that while public information always crowds out private information in all underlying equilibria, it can crowd in private information acquisition in the unique global-game equilibrium. This result is driven by the strategic uncertainty introduced through the global-game refinement. The crowding-in effect is more pronounced when there is a high level of fundamental uncertainty, which supports the case for greater information disclosure during times of increased market volatility.

**Keywords**: Information Disclosure; Information Acquisition; Dynamic Complementarity; Global Games; Strategic Uncertainty.

**JEL classification:** D83, D84, E44, G14, G20.

*We thank the editor, Guillermo Ordoñez, an associate editor, and three anonymous referees for providing valuable suggestions. We also thank Jess Benhabib, Kim-Sau Chung, Liang Dai, Tony Xuezhong He, Shiyang Huang, Zhen Huo, Xuewen Liu, Stephen Morris, Michal Szkup (discussant), Laura Veldkamp, Pengfei Wang, Yi Wen, Yao Zeng, Lichen Zhang, along with seminar and conference participants at various places for useful discussions and comments. Feng Dong acknowledges the financial support from the National Natural Science Foundation of China (#72250064, #72122011) and Tsinghua University Spring Breeze Fund (#2020Z99CFW046).

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# 1 Introduction

Public information disclosure plays a critical role in contemporary financial market policy. Since the 2008 global financial crisis, financial transparency has become increasingly important, leading to the implementation of regulatory measures such as the Dodd-Frank Act of 2010, intended to enhance disclosure in financial markets.

Although information disclosure in financial markets has been demonstrated to offer benefits such as improved market liquidity and reduced firm financing costs, its overall desirability remains a topic of debate.[1] In particular, there are concerns that public information disclosure could suppress private information production (Colombo et al., 2014), leading to less informative asset prices overall or even negative welfare consequences. (Morris and Shin, 2002; Amador and Weill, 2010; Goldstein and Yang, 2019).[2] However, these results are often derived from settings where information held by various parties acts as substitutes, meaning that information provided by one party (e.g., a public authority) diminishes the incentive for others to acquire information. In contrast, recent research has indicated that information is not always a substitute but can also become complementary through various channels. These findings challenge the assumption that public disclosure always has a crowding-out effect on private information production and opens up new perspectives on the dynamics of information disclosure in financial markets.

This paper fills this gap by analyzing the implications of public information disclosure where private information acquisition exhibits strategic complementarity. A challenge arises as information complementarity leads to multiple equilibria, complicating comparative statics analysis. This multiplicity, however, hinges on the strong assumption that all agents possess perfect knowledge about others' expected returns to information acquisition and their corresponding strategic actions in equilibrium. We depart from this paradigm by incorporating strategic uncertainty into agents' information choices, in line with the global-game literature (Morris and Shin, 2003).

Our analysis reveals that this refinement not only ensures the existence of a unique equilibrium, as expected, but also surprisingly alters the model's implications concerning public information disclosure. We find that while the disclosure of information always crowds *out* private information in all equilibria under common knowledge, it can instead encourage the acquisition of private information under the unique global-game equilibrium. This is because in certain common-knowledge equilibria, agents have perfect knowledge of how much infor-

[1] Goldstein and Yang (2017) provides an excellent survey on the impact of information disclosure in financial markets.

[2] For recent works that demonstrate the crowding-in effect of public disclosure, see Bond and Goldstein (2015), Goldstein and Yang (2015, 2019), and Xiong and Yang (2021).

mation other agents have acquired. With information disclosure, the value of information function shifts to the left, resulting in fewer agents acquiring information, thereby leading to the crowding out of private information in all common-knowledge equilibria.

However, under the global-game equilibrium, agents are not able to directly observe the state of the economy and are only provided with a single piece of private information about it. This creates strategic uncertainty, meaning that agents are unsure about how many informed investors there are and must form an expectation about it. In this case, agents care about the changes in the *expected* value of information rather than the value of information at a particular point, as in the common-knowledge equilibria. In the presence of information complementarity, the public disclosure of information can increase the *overall* value of information, leading to the crowding in of private information acquisition.

In Section 2, we formalize this idea in a general strategic game framework, without specifying the source of strategic substitutability or complementarity. We demonstrate how the global-game refinement can be applied by introducing noise into agents' individual states (i.e., information costs). We then highlight the key difference between common-knowledge equilibria and the global-game equilibrium: under common-knowledge equilibria, agents are fully aware of where the equilibrium is located and focus on the *local* impact of information disclosure, whereas under the global-game equilibrium, agents consider the *global* shift of the payoff function due to strategic uncertainty.

Section 3 presents a model with dynamic information complementarity as in Avdis (2016), extending the Grossman and Stiglitz (1980) model by adding an additional round of trade conducted by new investors. Information complementarity arises through a discount-rate channel, where more informed trading results in less stock price loading on noisy supply variations. This translates to reduced unpredictable noise in future resale stock prices, leading to less discounting in asset prices and increased information value. The value of information becomes hump-shaped and initially increases then decreases with the share of informed investors, causing equilibrium multiplicity. We show that public information disclosure always shifts the hump-shaped information value to the left, causing all interior equilibria to shift accordingly. We conclude that public information always crowds out private information acquisition, even with dynamic information complementarity.

In Section 4, we apply global-game refinement to the model based on Section 2's insights. Our first main result demonstrates that in the unique global-game equilibrium, public information disclosure can crowd in private information acquisition. This crowding-in effect relies on a public-private information complementarity: more public information increases the value of information for private investors since it is observable to future investors, who then trade more aggressively, reducing noise in future *resale* prices. This reduces current investors' unlearnable risk, who trade more aggressively, raising the value of information. When this effect is sufficiently strong, the expected value of information increases, crowding in more information. The public-private complementarity is akin to the noise-reduction effect in Bond and Goldstein (2015) and Goldstein and Yang (2019) but with an important distinction: the complementarity effect's strength is endogenous and potentially varies with the model's primitives, as it depends on the overall shape of the information value determined in equilibrium.

Our second main result reveals the "state-dependent" nature of the crowding-in effect. We demonstrate that public-private complementarity is more pronounced when fundamental uncertainty is high. This occurs because under high uncertainty, risk-averse investors nearly cease trading entirely, rendering private information valueless. In this situation, releasing even a small amount of public information has a particularly potent marginal impact on private information acquisition, as it incentivizes investors to resume trading, thereby increasing the value of information. Due to this state-dependence, public information disclosure has a nonmonotonic effect on private information acquisition: as the public signal becomes more precise, it initially crowds in and later crowds out private information acquisition. The analysis thus highlights an advantage of disclosing more information during periods of high market volatility. Essentially, high market volatility/uncertainty diminishes agents' incentive to trade and acquire information. Public disclosure effectively stimulates private trading and boosts the value of information during periods of high uncertainty. This channel surpasses the conventional crowding-out effect, resulting in an improved information environment overall.

**Related Literature.** The paper is related to three different strands of literature. First, it draws on financial market models with endogenous information acquisition, initiated by Grossman and Stiglitz (1980). Subsequent works illustrate that complementarity in information acquisition can arise for various reasons.[3] Information complementarity can arise because of increasing returns in the information sector (Veldkamp, 2006), private information on endowments (Ganguli and Yang, 2009), relative wealth concerns (García and Strobl, 2011), differential investment opportunities (Goldstein et al., 2014), nonnormal distributions (Breon-Drish, 2015), multiple sources of information (Goldstein and Yang, 2015), and Knightian uncertainty (Mele and Sangiorgi, 2015). This paper is built on dynamic trading models in which information complementarity arises (Froot et al., 1992; Chamley, 2007; Avdis, 2016; Cai, 2019; Glasserman et al., 2021). Benhabib and Wang (2015) extends Grossman and Stiglitz (1980) such that noise traders are replaced with sunspots. Benhabib et al. (2016) considers the idea of complementarity be-

[3]Due to space constraints, we focus on the works on information acquisition most related to ours. For the classical and recent progress made in the burgeoning literature on endogenous information acquisition, see also Verrecchia (1982), Hauswald and Marquez (2006), Van Nieuwerburgh and Veldkamp (2010), Han and Yang (2013), Huang (2015), Pei (2015), Yang (2015), Benhabib et al. (2016), Dai (2018), Yang and Zeng (2019), Zou (2019), Huang et al. (2020), and Yang (2020), among others.

tween public information and private information acquisition similar to ours but in a setting where the firm discloses its signal to traders. The contribution of the present paper is to propose a tractable global-game technique that refines multiplicity in information equilibria and thus enhances unique predictions. Another closely related paper is by Chamley (2007), who also applies global-game techniques to study complementarity in information markets.[4] Different from this paper, Chamley (2007) does not explore regime switches in response to fundamental changes (e.g., public disclosure), which is the focus of this paper.

Second, this work is related to the literature on disclosure in financial markets. Bond and Goldstein (2015) and Goldstein and Yang (2019) study multiple dimensions of information in essentially static financial markets. This paper instead studies single dimensions of information in a dynamic financial market. The dynamic channel of public information studied in this paper is similar to the mechanisms studied in Hirshleifer (1978), Dye (1990), Gao (2010), and Dutta and Nezlobin (2017). The key difference is that we study the implications for private information production. In a more recent work, Banerjee et al. (2018) illustrates that the crowding-out effect could be strong enough that disclosure makes asset prices less informative. Han et al. (2016) studies public disclosure in the presence of endogenous noise trading and finds that more precise information attracts more noise trading and that this crowds in private information production. Kurlat and Veldkamp (2015) investigates the social value of public information and shows that information can improve investors' welfare only when issuers strategically manipulate the supply of assets to obfuscate information or the information encourages firms to take on riskier investments. Gaballo and Ordoñez (2021) investigates the role of public information in market insurance and finds that more public information can be socially undesirable.

Finally, this paper relates to the literature on global games. The key insight that departing from common knowledge may restore uniqueness in coordination games stems from the seminal works by Carlsson and van Damme (1993) and Morris and Shin (1998). We consider a novel application to the information acquisition game in financial markets. This application also helps to address a critique of the global-game literature that comparative statics of global-game selections can be the same as the comparative statics of the equilibria of the unperturbed underlying game. This paper provides an example where the two yield strikingly different predictions.[5] Szkup and Trevino (2015) introduces endogenous information acquisition into a generic global game of regime switching. Ordoñez (2013) uses global-game techniques to study equilibrium fragility in a credit market with reputation concerns.

[4]See also Schaal and Taschereau-Dumouchel (2018) and Liu (2016) for recent applications of global games to macroeconomic fluctuations and bank runs, respectively.

[5]We thank Stephen Morris for pointing out this issue.

## 2 A General Framework

This section describes a generic model in which a continuum of agents take binary actions (e.g., information acquisition) with strategic motives. The purpose is to develop a general discussion on the different implications that can be derived from common-knowledge vs. global-game equilibrium.

Consider an economy with a continuum of agents, indexed by $i \in [0,1]$. Each agent decides on a binary action of whether to acquire information about an economic fundamental. The payoff from acquiring such information is represented by the following function $\pi(\cdot)$:

$$\pi(\lambda, \tau, \chi_i),$$

where $\lambda$ is the share of informed agents in this economy; $\tau$ is an aggregate state and in this case denotes the precision of the public disclosure; and $\chi_i$ are the individual costs of information acquisition, which are assumed to always reduce the payoff from information acquisition.[6]

**Assumption 1.** *The payoff function $\pi(\cdot)$ is differentiable with respect to its arguments and is always decreasing in the idiosyncratic cost $\chi_i$.*

This formulation encompasses different models of information acquisition used in the literature. The assumption that the payoff function $\pi$ depends not only on the state of the economy but also other agents' actions $\lambda$ reflects an important strategic aspect of the model.

We start by characterizing a common-knowledge information equilibrium in which all agents face the same information cost $\chi_i = \bar{\chi}$, $\forall i$. Since everyone is initially homogeneous, there is no issue of private information regarding the information costs. The (interior) equilibrium share of informed investors $\hat{\lambda}$ is then determined by equating the payoff to zero:

$$\pi\left(\hat{\lambda}, \tau, \bar{\chi}\right) = 0. \tag{1}$$

Given the equilibrium, we can derive how public disclosure affects the production of private information. This can be determined through total differentiation of the payoff function $\pi(\cdot)$ to obtain the derivative $\frac{d\hat{\lambda}}{d\tau}$ as below.

**Proposition 1.** *The effect of a public information release at an (interior) common-knowledge equilibrium is:*

[6] In the language of game theory, $\lambda$ can be interpreted as the average action of all agents. Let $a_i$ denote the information acquisition decision rule for agent $i$: $a_i = 1$ if she acquires information and is equal to 0 otherwise. Then, $\lambda = \int a_i di$.

$$\frac{d\hat{\lambda}}{d\tau} = -\frac{\left.\frac{\partial \pi}{\partial \tau}(\lambda, \tau, \bar{\chi})\right|_{\lambda=\hat{\lambda}}}{\left.\frac{\partial \pi}{\partial \lambda}(\lambda, \tau, \bar{\chi})\right|_{\lambda=\hat{\lambda}}}, \tag{2}$$

*where the value of* $\hat{\lambda}$ *is given by Equation 1.*

The numerator measures how public information $\tau$ directly affects individual payoffs $\pi$, while the denominator captures the strategic aspect of the model, namely how the others' actions affect one's own payoff. Both forces are evaluated at $\lambda = \hat{\lambda}$, implying that the prediction of the model is sensitive to the value of equilibrium $\lambda$ and the associated slope of the value of information at that particular value. This typically leads to ambiguity in predictions across different equilibria, making it difficult to evaluate robust model predictions.

To overcome this difficulty, we introduce the global-game refinement into this setting by assuming that agents have private information regarding their heterogeneous information costs. Specifically, assume that agents' information costs are distributed according to

$$\chi_i = \bar{\chi} + \sigma \varepsilon_i,$$

where $\sigma$ is positive and $\varepsilon_i$ is some random variable with zero mean. As in the global-game literature, private agents in this case cannot directly observe the entire cost distribution but can use their own signals $\chi_i$ to imperfectly infer it. This creates strategic uncertainty for all agents, whose expected payoff is given by:

$$E(\pi(\lambda, \tau, \chi_i)|\chi_i),$$

where the expectation is taken over $\lambda$ given $\chi_i$ as a signal of the unknown cost distribution. This payoff function is decreasing in $\chi_i$ due to a real channel and a signaling channel. On the real side, by assumption 1 the payoff function $\pi$ is decreasing in $\chi$. On the signaling side, a higher $\chi_i$ signals a higher mean for the entire distribution of $\chi$, which implies a lower share of informed agents $\lambda$, which reduces the expected value of information when complementarity prevails.[7]

Thus there exists a *unique* monotone equilibrium: an agent chooses to acquire information if and only if his or her information cost $\chi_i$ is below some cutoff $\chi^*$. As is standard in the global-game literature, it can be derived that the expected payoff of an investor of type $\chi_i$ is equal to a

[7]We will primarily consider environments where both substitutability and complementarity are present, but the latter prevails for most ranges of $\lambda$.

simple integration of the payoff function over all possible values of $\lambda$

$$E(\pi(\lambda,\tau,\chi_i)|\chi_i) = \int_0^1 \pi\left(\lambda,\tau,\chi_i\right)d\lambda. \tag{3}$$

Thus, the expected payoff of the marginal investor is $\int_0^1 \pi\left(\lambda,\tau,\chi^*\right)d\lambda$. Therefore, the threshold $\chi^*$ is determined by equating the marginal investor's *expected* payoff (Equation 3) to zero.

We now study how a public information release affects private information acquisition. Since the equilibrium $\lambda^*$ is increasing in the cutoff $\chi^*$, it suffices to conduct comparative statics with respect to $\chi^*$. This can be achieved through total differentiation of Equation 3. We summarize the results in the following proposition:

**Proposition 2.** *Under certain conditions specified in Morris and Shin (2003), a unique cutoff equilibrium exists. The unique cutoff threshold $\chi^*$ is given by*

$$\int_0^1 \pi\left(\lambda,\tau,\chi^*\right)d\lambda = 0, \tag{4}$$

*and the equilibrium share of informed agents $\lambda^*$ is monotonically increasing in $\chi^*$. Given a global-game equilibrium, the effect of a public information release is:*

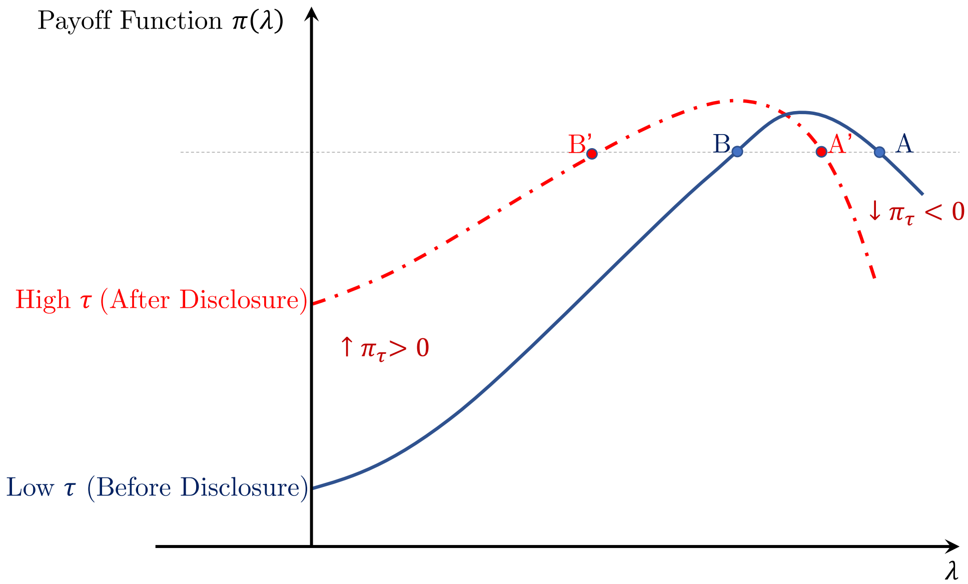
$$\frac{d\chi^*}{d\tau} = -\frac{\int \frac{\partial\pi}{\partial\tau}\left(\lambda,\tau,\chi^*\right)d\lambda}{\int \frac{\partial\pi}{\partial\chi}\left(\lambda,\tau,\chi^*\right)d\lambda}, \tag{5}$$

The most striking difference between a common-knowledge equilibrium and the global-game equilibrium is that under the former, agents only care about the *local* impact of disclosure at the exact equilibrium value of $\lambda$ (Equation 2), while under the latter, agents care about the *global* impact of public disclosure over the full range of $\lambda$ (Equation 5). This is due to the presence of strategic uncertainty whereby agents are unable to perfectly infer others' action. Hence, their optimal strategy depends on their expectation over all possible scenarios:

**Theorem 1.** *The global-game equilibrium and the common-knowledge equilibrium deliver comparative statics of different signs if and only if the local impact of an information disclosure is different from its global impact on the payoff function:*

$$sign\left(\frac{\frac{\partial\pi}{\partial\tau}\left(\lambda,\tau,\bar{\chi}\right)\Big|_{\lambda=\hat{\lambda}}}{\frac{\partial\pi}{\partial\lambda}\left(\lambda,\tau,\bar{\chi}\right)\Big|_{\lambda=\hat{\lambda}}}\right) = -sign\left(\frac{\int \frac{\partial\pi}{\partial\tau}\left(\lambda,\tau,\chi^*\right)d\lambda}{\int \frac{\partial\pi}{\partial\chi}\left(\lambda,\tau,\chi^*\right)\ d\lambda}\right). \tag{6}$$

Figure 1: A Graphic Illustration




*where $\hat{\lambda}$ is the equilibrium share of informed investors in the common-knowledge equilibrium defined in Equation 1 and $\chi^*$ is the global-game cutoff threshold defined in Proposition 2.*

The key observation is that the prediction from the common-knowledge equilibrium depends on how the public signal $\tau$ affects the payoff function at a particular point $\hat{\lambda}$, while under the global game, the prediction depends on how it affects the entire payoff function, as captured by the integration operator.

To illustrate why this leads to differences in prediction, consider Figure 1, where we plot an ad hoc payoff function $\pi$ as a function of $\lambda$. Focus first on the solid line. This is the case with low precision of the public signal $\tau$, so one can think of it as before a public information disclosure. The shape of the payoff function is nonmonotonic: first increasing then decreasing with $\lambda$, reflecting the competing forces of strategic complementarity and substitutability. There are two interior common-knowledge equilibria: A and B.

Suppose now that a public disclosure takes place, raising the value of $\tau$ and shifting the value of information function to the red line. Specifically, it reduces the value of information at the very top values of $\lambda$ and increases it for the rest of the regions because it shifts the value of information function shifts to the left. The disclosure shifts the common knowledge equilibrium to the left from point A (B) to point A' (B'), indicating that the disclosure crowds out private information production. To see this more formally, take point A as an example. Disclosure at point A depresses the value of information:

$$\left.\frac{\partial \pi}{\partial \tau}\right|_{\lambda=\lambda_A} < 0. \tag{7}$$

Thus, the numerator of Equation 2 is negative. Note also that due to the negative local slope of the value of information function around point A ($\frac{\partial \pi}{\partial \lambda}|_{\lambda_A} < 0$), the denominator of Equation 2 is negative. Thus, by Equation 2, crowding-out arises at this particular equilibrium: $\frac{d\lambda_A}{d\tau} < 0$.[8]

What about the global-game equilibrium? By Assumption 1, the payoff $\pi$ is always decreasing in the information cost $\chi$. Hence, the denominator of Equation 5 is always negative. The numerator, on the other hand, measures the impact of $\tau$ on the value of information over all ranges of $\lambda$. Comparing the dashed line (post disclosure) to the solid line (pre disclosure), one can see that the value of information generally increases, except at the very top. Hence:

$$\int \frac{\partial \pi}{\partial \tau} d\lambda > 0. \tag{8}$$

Thus, by Equation 5, crowding in arises at the global-game equilibrium.

What is driving the differences? Under common-knowledge equilibrium, there is no strategic uncertainty, and agents know perfectly that they are coordinating at point A. Thus, they only care about the *local* impact of $\tau$, which is negative around A, and ignore the fact that for most of the other values of $\lambda$, the payoff actually increases with $\tau$ (see Equation 7). This effect is captured by the global-game equilibrium due to the presence of strategic uncertainty. In this case, agents care about the *global* impact of $\tau$ because they are never certain about which $\lambda$ they are coordinating on. As a result, they integrate over all possible values of $\lambda$ in evaluating the payoff from information acquisition (see Equation 8). Because $\tau$ substantially increases $\pi$ over low values of $\lambda$, the value of the integration is positive, implying that disclosure crowds in more private information production.

We will now provide a microfounded model with trading and endogenous information acquisition that can generate the type of value of information functions similar to Figure 1.

## 3 A Microfounded Model of Information Acquisition

The model is a multiperiod extension of Grossman and Stiglitz (1980), similar to Avdis (2016). Time is discrete and divided into three periods: $t = 0, 1, 2$. There is a long-lived stock of fixed

[8] At point B, both the slope of the payoff function ($\frac{\partial \pi}{\partial \lambda}|_{\lambda_B} > 0$) and the impact of public information on the payoff function ($\frac{\partial \pi}{\partial \tau}|_{\lambda_B} > 0$) are positive; hence, by Equation 2, crowding out also yields $\frac{d\lambda_B}{d\tau} < 0$.

supply (normalized to zero), which pays out a dividend $D_1$ and $D_2$ at the end of days $t = 1$ and $t = 2$. The dividend stream consists of a persistent component $F$, which is assumed to be time-invariant, and a noise component $\varepsilon_t^D$, which is i.i.d. over time.

$$D_t = F + \varepsilon_t^D,$$

where the persistent component $F$ is the asset fundamental and the precision of the dividend noise is $\tau_D$. There is also a bond of perfectly elastic supply, which delivers return $R$ across consecutive periods. All investors have exponential utility:

$$u(c) = -\exp(-\alpha c),$$

where the parameter $\alpha > 0$ measures the degree of risk aversion.

In the beginning of period 0, first generation (G1) investors are born with a certain amount of wealth, in the form of bonds and stocks. They do not directly observe the value of $F$ but are endowed with a noisy public signal of $F$:

$$S = F + \varepsilon^F,$$

where the noise $\varepsilon^F$ is unbiased and has precision $\tau_F$, which captures the strength of public disclosure.[9] These G1 investors are then offered an opportunity to acquire information about the true value of $F$ at some cost $\bar{\chi}$. Investors who choose to purchase this information are labeled "informed", and the others are labeled "uninformed". This concludes period 0.

At the beginning of period 1, the financial market opens, and G1 investors, both informed and uninformed, engage in trading. There is also a group of noise traders whose demand is denoted by $x_1$, which is a normally distributed random variable with mean 0 and precision $\tau_{x1}$. Without noise trading, stock prices would become fully revealing, and no equilibrium could exist with a positive information cost. At the trading stage, the information set for the uninformed investors is $\Omega_1^U = \{S, P_1\}$, and that for the informed investors is $\Omega_1^I = \{S, F, P_1\}$, where $P_1$ denotes the equilibrium stock price in the first round of trading. After the trading stage, the dividend $D_1$ is delivered, and this concludes the period. If the world ended here, this would be the standard Grossman and Stiglitz (1980) model. Adding a period 2 creates the resale value of the stock and hence information complementarity.

At the beginning of period 2, the second generation of investors is born. It is assumed that there is no information acquisition choice available to them.[10] The financial market then opens,

[9] For exposition, we assume that $F$ has an improper uniform prior following Morris and Shin (2002). One could obtain similar results if instead assuming that the prior of $F$ is a normal distribution with finite variance.

[10] The assumption is made so that the model remains as close to the existing literature as possible (e.g., Avdis

and all investors (G1 and G2) engage in trading. In particular, G1 investors sell their holdings because they exit the market after trading. In addition, there is a group of new noise traders with demand $x_2$ of precision $\tau_{x2}$. In this benchmark model, we assume that noise trading is serially uncorrelated.[11] This is a special case of a mean-reverting stock supply process used in the literature (e.g., Campbell and Kyle, 1993; Wang, 1993; Wang, 1994; and Avdis, 2016). In Section 3.4, (with detailed derivations in Appendix D), we discuss the model implications for a more general, mean-reverting process for noise trading. The main conclusion is that our result is robust and holds for stock supply processes with relatively strong mean reversion.

Period-2 investors observe the price history and the public signal with the information set $\Omega_2 = \{S, D_1, P_1, P_2\}$, where $P_2$ denotes the equilibrium stock price in period 2. After trading, G1 investors exit the market and consume. Then, the dividend $D_2$ is distributed to the stock holders. This concludes period 2. The timeline is displayed in Figure 2, in which the extension to the Grossman and Stiglitz (1980) model is marked in red.

### 3.1 Characterization of Equilibrium and Payoff from Information Acquisition

We solve the model backward. At time 2 the G2 investors observe, among other things, the first-period trading price $P_1$ as a signal of the stock fundamental $F$.[12] As we focus on linear equilibria, observing the trading price is equivalent to observing the following price signal, denoted by $S_{P1}$:

$$S_{P1} = F - \frac{1}{\theta_1} x_1.$$

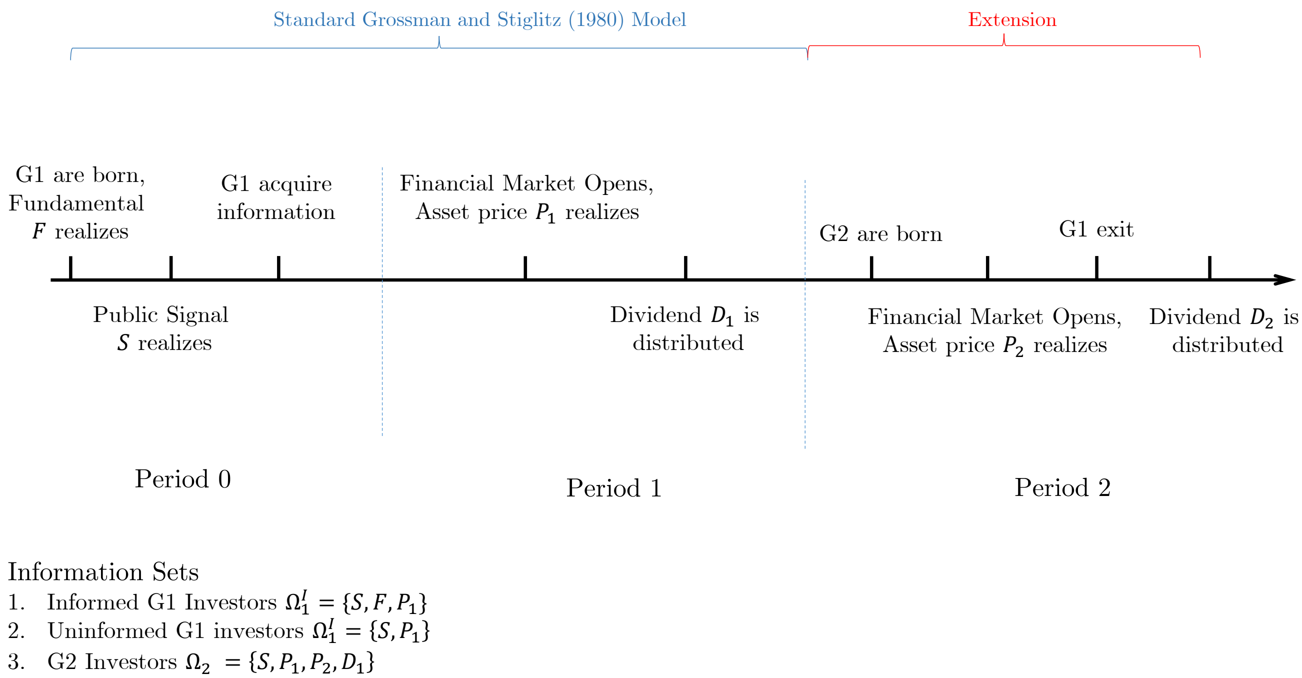
Where $\theta_1$ is an equilibrium coefficient that measures price informativeness. A higher value of $\theta_1$ means that the stock price is more sensitive to economic fundamentals $F$ relative to the noise. $\theta_1$ is an endogenous variable determined in equilibrium by both public and private information acquisition choices. Given $\theta_1$, we can derive the second-period stock price function $P_2$ from the market clearing condition, where the G2 investors' demand is equal to the noisy stock supply

(2016)). It also enables a straightforward application of the global game, as information choice in this setting is essentially a static choice (although the following financial market trading stage is dynamic). For a model with repeated information acquisition and the associated information complementarity, see Cai (2019). For issues related to applying global games in settings of dynamic strategic actions, see Angeletos and Werning (2006).

[11] A noisy stock supply is an important driver of dynamic information compelemntarity. The more persistent stock supply becomes, the less likely information complementarity is to arise (Avdis, 2016). The i.i.d. supply assumption has been used in the literature by, e.g., Allen et al. (2006) and Peress (2014) . Peress and Schmidt (2021) empirically document that this persistence parameter can be large under high-frequency settings but tends to be small and close to i.i.d. at a monthly or lower frequency. Our mechanism mainly works in settings featuring low frequencies, as we intend to capture issues related to public information disclosure by governments, regulators, and central banks. In this spirit, in our numerical simulation, we set the per-period interest rate to 1%, intending to capture relatively lower (i.e., quarterly) frequency movements in asset prices and investor behavior.

[12] Note that G2 investors also observe the second-period trading price $P_2$. However, because all G2 investors are homogeneous (there is no endogenous information choice), there is no new information to be learned from $P_2$ beyond what has been incorporated into $P_1$.

Figure 2: Timeline




$x_2$. It can be shown that the stock price function $P_2$ is a linear combination of the price signal, the public signal, the noisy stock supply, and first-period dividend payment:

$$P_2 = aS_{P1} + bS - cx_2 + dD_1,$$

where the price coefficients are all positive and functions of price informativeness $\theta_1$ (see Lemma A.1 in the Appendix for derivations). This expression is intuitive: any good news such as more favorable disclosure, market information, and higher dividend payouts tend to raise the stock price, while a larger stock supply serves as a negative shock that tends to reduce the stock price.

Given the expression for $P_2$, we can now turn to period 1. The first-period investors' problem is identical to Grossman and Stiglitz (1980) except that the total stock payoff (denote by $Q_1$) consists of both the dividend payment and the resale value of the stock:

$$Q_1 = D_1 + P_2. \tag{9}$$

We can then derive the stock demand functions for informed and uninformed investors, *holding fixed* the share of informed investors $\lambda$. Equating demand to supply, we obtain a financial market equilibrium with an equilibrium value of $\theta_1$. The following proposition illustrates that such an equilibrium exists and is unique.

**Proposition 3** (Financial Market Equilibrium)**.** *Under the condition*

$$\frac{\alpha}{\tau_D} + 1 - R > 0, \tag{10}$$

*a unique financial market equilibrium exists for any $\lambda \in [0,1]$ at which the price informativeness $\theta_1$ is monotonically increasing in the share of informed investors $\lambda$.*

We take condition 10 to hold for all the subsequent analysis and numerical simulations. Although we cannot solve for a closed-form solution of $\theta_1$ as a function of $\lambda$, the advantage of this approach is that $\theta_1$ is a monotonic transformation of $\lambda$. Thus, to explore the effect of changing $\lambda$, we simply need to examine the impact of varying $\theta_1$. Note that in a similar environment, Avdis (2016) shows that financial market equilibrium is unique under appropriate conditions (p. 572, Proposition 3.3).

We move to time 0, in particular the information-acquisition stage that determines the share of informed $\lambda$. We denote the ex ante expected utility of the informed and uninformed investors as

$$J^i\left(\lambda, \tau_F, \chi\right),$$

where $i \in \{I, U\}$ represents informed and uninformed investors, respectively. This value function depends on $\lambda$ and $\tau_F$ because they affect the precision of the price signal $S_{p1}$ and the public signal $S_1$ and, hence, the information content in the stock price $\theta_1$. It also depends on the information cost $\chi$, which affects the payoff from information acquisition.[13]

We can now define the payoff from information acquisition in this environment as the difference in the value of being informed and uninformed:

$$\pi\left(\lambda, \tau_F, \chi\right) = J^I\left(\lambda, \tau_F, \chi\right) - J^U\left(\lambda, \tau_F, \chi\right). \tag{11}$$

Thus, this payoff function summarizes the expected financial returns that investors would obtain by becoming informed, and they choose to acquire information if and only if this payoff is positive. With this payoff function at hand, we can define the notion of equilibrium and study comparative statics as in Section 2. In a common-knowledge equilibrium where all investors share the same information cost $\chi = \bar{\chi}$, an information equilibrium $\hat{\lambda}$ is given by equating the

[13]Note that the value function does not depend on agents' wealth because agents have exponential utility, which displays no wealth effect.

payoff $\pi\left(\hat{\lambda},\tau_F,\bar{\chi}\right)$ to zero, unless at boundary:

$$\pi\left(\hat{\lambda},\tau_F,\bar{\chi}\right)\begin{cases}\leq 0 & \text{if } \hat{\lambda}=0\\ =0 & \text{if } \hat{\lambda}\in(0,1)\\ \geq 0 & \text{if } \hat{\lambda}=1\end{cases}.$$

Figure 3 plots an example of the equilibrium in this model. Panel A plots equilibrium price informativeness $\theta_1$ as a function of the share of informed investors. The stock price becomes more informative with more informed agents, which in turn provides more information to the uninformed investors. This is illustrated in Panel B where we plot the precision of the stock fundamental conditional on public information:

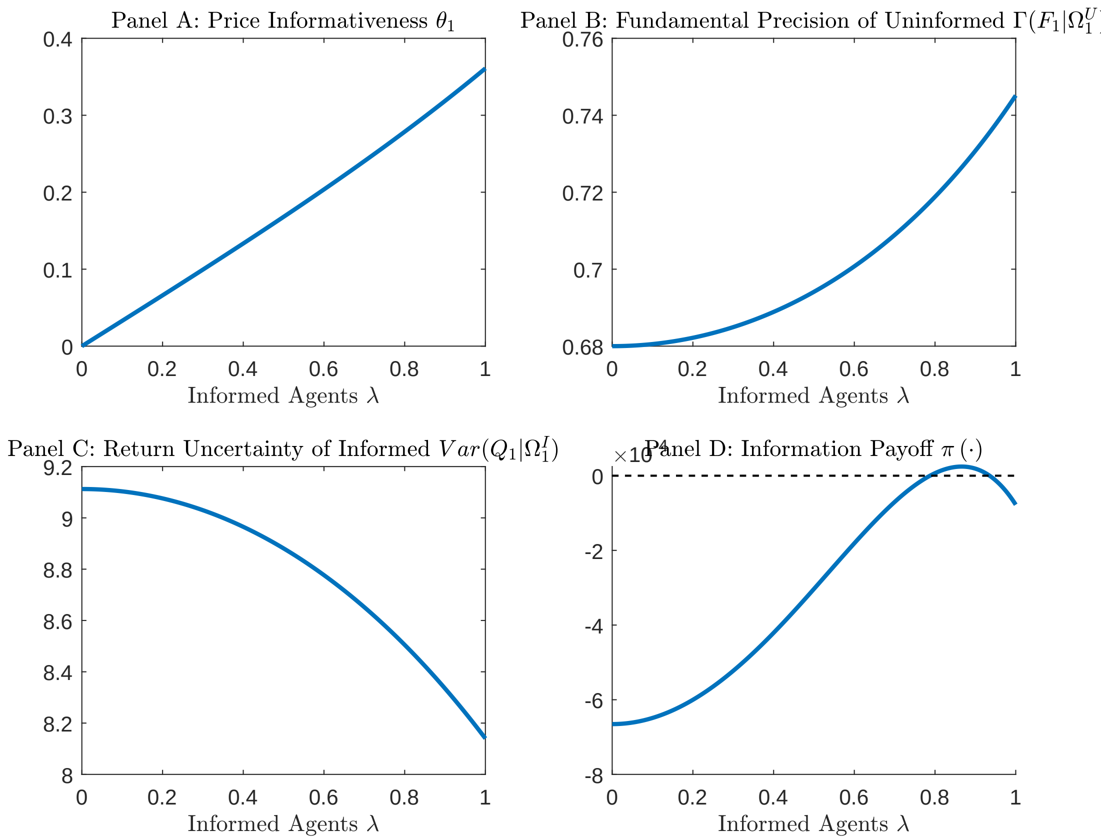
$$\Gamma(F|S_{P1},S)=\tau_F+\theta_1^2\tau_{x1}, \tag{12}$$

One can see that a more precise price signal $\theta_1$ increases the amount of information available to uninformed investors. Panel C plots the *residual* uncertainty in the stock payoff $Q_1$ faced by informed investors. These investors perfectly know the stock fundamental $F$, but because future resale stock prices are less sensitive to supply noises, the uncertainty they face decreases with $\lambda$. Finally, we plot the information payoff function $\pi$ in Panel D. In this example, the payoff function takes a nonmonotonic shape, similar to what we see in Figure 1. The nonmonotonicity results from the interplay between strategic substitutability and complementarity in information acquisition in this model, to which we now turn.

## 3.2 Value of Information: Substitutability vs. Complementarity

This section investigates how the value of information changes with $\lambda$. From Proposition 3, we know that $\theta_1$ is monotonically increasing in $\lambda$; hence, it suffices to examine how $\theta_1$ affects the value of information. We follow the literature and consider the ratio of expected utilities between informed and uninformed investors as a measure of the value of information. Since agents live for one period, they effectively solve a static problem, and the value of information

Figure 3: Comparative Statics with respect to $\lambda$




**Notes**: This figure plots comparative statics with respect to the share of informed $\lambda$. Panel A plots price informativeness. Panel B plots information available to uninformed investors upon observing the price, the public signal, and the first-period dividend. Panel C plots the residual uncertainty faced by informed investors in the resale stock returns. Panel D plots the value of information, defined as the ratio of the ex ante utility of the uninformed and informed. The crossings of the value of information and the zero dashed-black line are common-knowledge equilibria. $\tau_D = 2, \tau_{x1} = \tau_{x2} = 0.5, R = 1.01, \alpha = 1, \bar{\chi} = 0.195, \tau_F = 0.68$.

is proportional to the ratio of stock return volatility perceived by each group of investors:[14]

$$V = \frac{Var(Q_1|\Omega_1^U)}{Var(Q_1|\Omega_1^I)} = 1 + \frac{\overbrace{\frac{1}{\Gamma(F|S_{P1},S)}}^{\textbf{static substitutability}} \downarrow}{\frac{1}{\tau_D} + \underbrace{\frac{1}{\tau_{x2}}\left(\frac{c}{1+d}\downarrow\right)^2}_{\textbf{dynamic complementarity}}}. \quad (13)$$

The numerator captures the static substitutability effect as in Grossman and Stiglitz (1980). As $\theta_1$ increases, there is less fundamental uncertainty as the price signal becomes more informative ($\Gamma(F|S_{P1},S) \uparrow$). As there is now more information available to uninformed investors, their incentive to acquire information decreases. On the other hand, there is the dynamic complementarity effect captured by the $\frac{c}{1+d}$ in the denominator: as $\theta_1$ increases, the ratio $\frac{c}{1+d}$ decreases, raising the value of information. This dynamic complementarity effect works through the following discount rate channel. Recall that the stock payoff $Q_1$ is given by

$$Q_1 = D_1 + P_2 = (1+d)\,D_1 + aS_{P1} + bS - cx_2.$$

Consider the risk faced by informed investors, to whom the fundamental $F$ is perfectly observable. An important source of residual risk for them comes from variations in future stock supply $x_2$, and the coefficient $c$ measures the size of this risk. When there are more informed investors, price informativeness $\theta_1$ increases, which allows future rational investors to trade more aggressively. This means that $P_2$ tends to be more sensitive to fundamental information but less sensitive to noise trader risks. As a result, the equilibrium coefficient $c$ decreases. This implies less risk in the future stock payoff, which in turn means that there is less discounting on the future returns to investment. Hence, investors are induced to invest more, raising the value of information.

Given the presence of both substitutability and complementarity, it is natural to ask what force dominates and under what circumstances.

**Proposition 4.** *Strategic complementarity dominates: $V'(\lambda) > 0$ when fundamental uncertainty is sufficiently high.*

To see why complementarity dominates when there is high uncertainty, consider the extreme case in which there is no publicly available information $\tau_F = 0$ and there are no informed

[14] For proof of the original theorem, see p. 398, theorem 2 (Grossman and Stiglitz, 1980). See Lemma A.2 in the Appendix for derivations in this environment.

investors $\lambda = 0$, so that stock price informativeness $\theta_1 = 0$. In this case, the fundamental uncertainty would be very high, with little useful information available to uninformed investors $\Gamma(F|S_{P1}, S) = 0$. As there is no useful information to be observed, period-2 rational investors are driven out of the market, and therefore $P_2$ is largely driven by noise trades and hence extremely noisy. Thus, the stock payoff entails very high uncertainty even for informed investors. As a result, risk-averse investors stay away from the stock market. Hence, the value of acquiring information about the stock market approaches zero. Now, consider an influx of informed investors: $\lambda > 0$. More fundamental information would be incorporated into the stock price $\theta_1 > 0$, which in turn would reduce the trading risks faced by informed investors. Thus, rational investors would invest more heavily in the stock market, raising their payoff from acquiring information. Due to this trading effect, complementarity tends to dominate when uncertainty is very high. Hence, in Panel D of figure 3, the information payoff function is first increasing in $\lambda$ when $\lambda$ is very low and then decreasing in $\lambda$ when $\lambda$ becomes higher.

### 3.3 The Impact of Information Disclosure

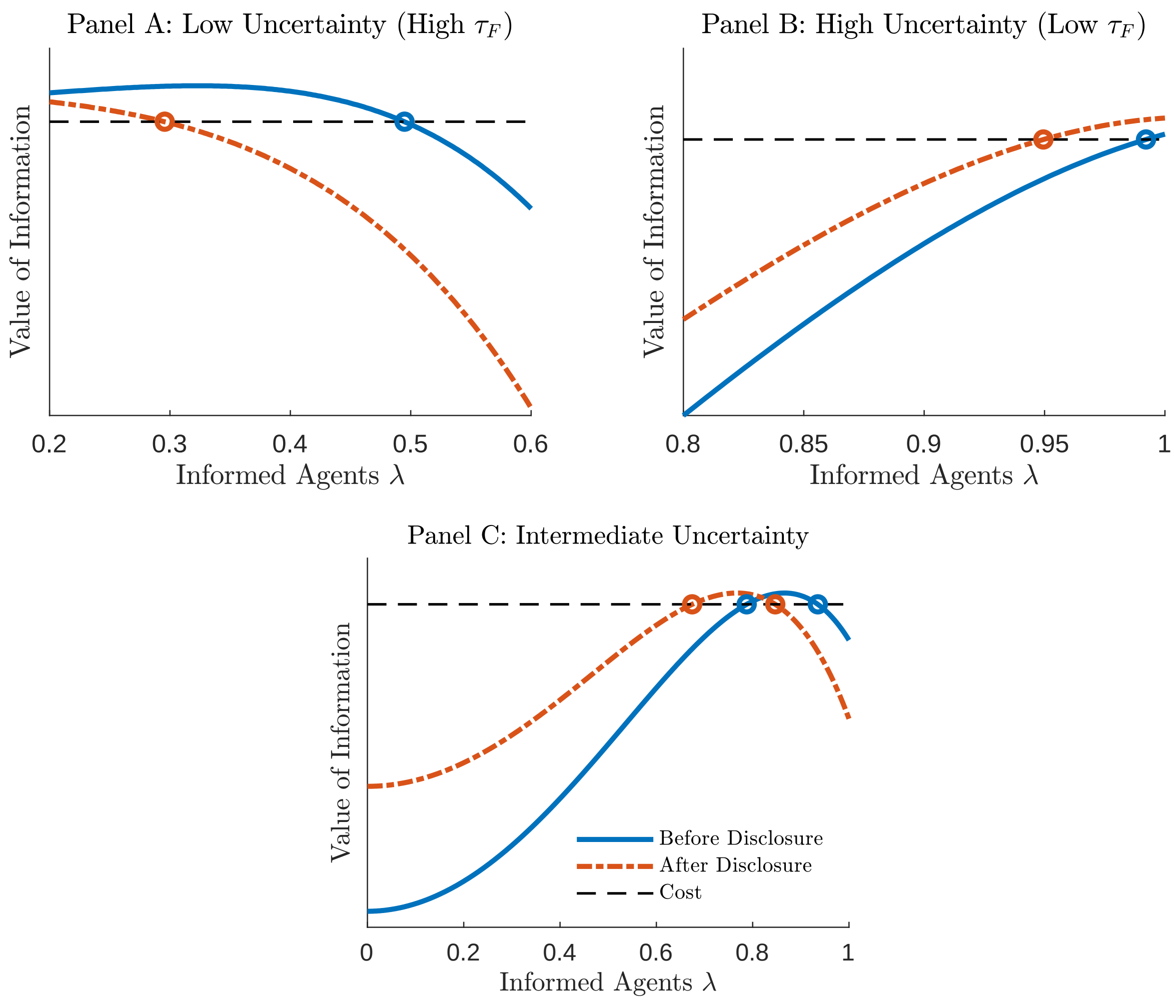
We now examine the impact of varying public information $\tau_F$. Figure 4 illustrates the impact of increasing $\tau_F$ under various circumstances with different levels of uncertainty. As illustrated in the last section, higher uncertainty is associated with information complementarity and hence an upward-sloping value of information, while lower uncertainty is associated with the opposite pattern. Note that, regardless of its shape, the value of information shifts to the left, and public disclosure (an increase in $\tau_F$) always reduces the value of an endogenous $\lambda$.

How should we understand this result? From Equation 2, we learn that the impact of $\tau_F$ on $\lambda$ in a common-knowledge equilibrium depends on how $\tau_F$ and $\lambda$ affect the value of information, i.e., the signs of $\frac{\partial \pi}{\partial \tau}$ and $\frac{\partial \pi}{\partial \lambda}$. In this example, both $\tau$ and $\lambda$ affect the value of information only through their impacts on the aggregate public information $\Gamma(F|S_{P1}, S) = \tau_F + \theta_1(\lambda)^2 \tau_{x1}$. Hence, both derivatives take the same sign regardless of the shape of the value of information. This is because they both introduce useful information into the trading price $P_1$, which is available for G2 investors. Hence, they enter into the value of information in a similar way. We conclude with the following proposition.

**Theorem 2.** *Public information releases always crowd out private information acquisition at any interior common-knowledge equilibrium:*

$$\frac{d\lambda}{d\tau_F} < 0.$$

Figure 4: Impact of Information Disclosure Under Common-knowledge Equilibria




**Notes**: This figure plots value of information and equilibrium for different levels of $\tau_F$. For low (high) uncertainty, the value of information is monotonically decreasing (increasing). For an intermediate level of uncertainty, the value of information is nonmonotonic. In all three cases, the information equilibria shift to the left with public disclosure. Parameters used: $\tau_D = 2, \tau_{x1} = \tau_{x2} = 0.5, R = 1.01, \alpha = 1, \bar{\chi} = 0.195$; Panel A solid line $\tau_F = 0.72$, Panel A dashed line $\tau_F = 0.73$; Panel B solid line $\tau_F = 0.658$, Panel A dashed line $\tau_F = 0.663$; Panel C solid line $\tau_F = 0.68$, Panel C dashed line $\tau_F = 0.69$.

### 3.4 Mean-Reverting Supply Shocks

In this section, we discuss the implications of the model when the supply shocks follow a more general, mean-reverting process as in Avdis (2016). Assume that:

$$x_2 = \rho_x x_1 + \varepsilon_{x2},$$

where $1 \geq \rho_x \geq 0$ denotes the degree of persistence in stock supply $x_t$ across the two periods and $\varepsilon_{x2}$ is a noise term with mean 0 and variance $\tau_{x2}$. Other aspects of the model are kept the same as the benchmark, and when $\rho_x = 0$, the model collapses to the benchmark with i.i.d. stock supply shocks.

In Appendix D, we show that the value of information in this setting is given by:

$$V = 1 + \underbrace{\left(1 - \frac{c}{1+d}\rho_x\theta\right)^2}_{\text{Extra Term}} \frac{\frac{1}{\tau_F + \theta^2\tau_{x1}}}{\frac{1}{\tau_D} + \left(\frac{c}{1+d}\right)^2 \frac{1}{\tau_{x2}}}.$$

This expression differs from the benchmark value of information expression (Equation 2) by the additional term in brackets. When $\rho_x = 0$, the additional term is equal to 1, and hence the value of information expression reduces to Equation 2. Thus, to study the further implications in this extended model with $\rho_x > 0$, it suffices to study how this additional term varies with public information disclosure.

Using this expression, we can show that a more persistent stock supply process (higher values of $\rho_x$) weakens information complementarity, namely, the slope of the value of information with respect to the share of informed investors decreases with the value of $\rho_x$. This is an important point made in Avdis (2016): when the stock supply process is more persistent (or, equivalently, less mean reverting), the current level of stock supply is less predictive of its future changes, weakening information complementarity. For moderate levels of supply persistence ($\rho_x$ greater than 0 but small), therefore, the predictive discount-rate channel presents, and hence information complementarity carries over.

It can also be shown that public information disclosure can shift up the value of information function under a more persistent stock supply. To see this, note that $\rho_x$ is multiplicative to $\theta$ but not to the precision of the public information $\tau_F$. Hence, the impact of $\tau_F$ is similar to the benchmark model. We thus conclude that the result of our model is robust to a moderate level of persistence or relatively strong mean reversion in the noisy stock supply process.

# 4 Global-Game Equilibrium

This section introduces the global game into the information market. We will primarily work with the payoff function $\pi(\lambda, \tau_F, \chi)$ defined in Equation 11, as this payoff function summarizes all the information necessary to make the information choice.

As in Section 2, we perturb the model with a noise $\varepsilon_i$ affecting investors' cost of information acquisition:

$$\chi_i = \bar{\chi} + \sigma\varepsilon_i.$$

Observing $\chi_i$, investors then decide whether to acquire information about the stock fundamental $F$. Given public information release $\tau_F$ and the agent's individual information cost $\chi_i$, her payoff from information acquisition would be $\pi(\lambda, \tau_F, \chi_i)$ if she perfectly knew the value of $\lambda$. In equilibrium, however, agents cannot form perfect expectations of $\lambda$ due to strategic uncertainty. They therefore compare expected payoffs when making information choices.[15]

The information acquisition stage thus can be formulated as a symmetric binary-action global game with private valuation as in Section 2. The next proposition shows, by invoking a result in Morris and Shin (2003), that there exists a unique information equilibrium in which private investors follow a monotone strategy.

**Proposition 5** (Existence and uniqueness of global game equilibrium)**.** *Suppose that $\tau_F$ is sufficiently low. Then, the payoff function $\pi(\cdot)$ satisfies A.1 through A.5 in Morris and Shin (2003) (listed in the Appendix). Thus, by Proposition 2.1 of Morris and Shin (2003), the model admits a unique equilibrium in which investors acquire information if and only if their information cost is below a certain threshold $\chi^*$. The unique equilibrium cutoff $\chi^*$ and the share of informed investors $\lambda^*$ are both given as in Proposition 2.*

In the theoretic proof, we focus on the case with high uncertainty (low $\tau_F$). This is for two reasons. First, this is the situation where information complementarity prevails (as shown in

[15] The financial market equilibrium remains the same once investors have made their information choices. The only technical assumption we make is that the mean $\bar{\chi}$ becomes publicly observable at the beginning of period 1, prior to the financial market opening. The information sets for different types of investors are given by $\Omega_1^U = \{\bar{\chi}, S, P_1\}$, $\Omega_1^U = \{\bar{\chi}, S, F, P_1\}$, and $\Omega_2 = \{\bar{\chi}, S, P_1, P_2, D_1\}$. This assumption is required here with private information because otherwise agents will form posterior beliefs about the cost distribution from observing the equilibrium price signal, and this breaks the Gaussian-linear framework. One interpretation is of investors being fund managers and information acquisition representing the decision of whether to become (more) skilled at some (extra) learning cost relative to peers. Part of this cost may be incurred *before* one becomes an asset manager (for example, whether to pursue a master's degree at an expensive institution). Under this interpretation, this assumption states that it is only after one becomes an asset manager and enters the "club"that he or she observes the stock-picking ability of his or her peers. Without private information on information costs, this assumption is not needed because agents can rationally infer the equilibrium share $\lambda$ from the publicly observed cost distribution. In this case, the result of the paper still holds if there is sufficient heterogeneity in the cost distribution. We explore this case in Appendix B.1.

Proposition 4). This results in equilibrium multiplicity, and therefore it is meaningful to apply global game to refine equilibria. The second reason is that the payoff function $\pi$ is difficult to work with in most cases because it involves calculating the *difference* in expected utility, which in the class of noisy rational expectation models can be extremely complicated. When uncertainty is sufficiently high, the payoff function $\pi$ converges to some monotonic transformation of $V$, which is easier to work with.[16]

## 4.1 Impact of Information Disclosure Under the Global Game

This section analyzes the impact of public information disclosure under the unique global-game equilibrium, in particular how it affects the cost threshold $\chi^*$. Recall that this comparative static is given by Proposition 2:

$$\frac{d\chi^*}{d\tau_F} = -\frac{\int_0^1 \frac{\partial \pi}{\partial \tau_F}(\lambda, \tau, \chi^*)\, d\lambda}{\underbrace{\int_0^1 \frac{\partial \pi}{\partial \chi}(\lambda, \tau, \chi^*)\, d\lambda}_{<0}}. \tag{14}$$

The denominator of this equation is always negative because costly information always reduces payoff. Thus, the sign of the comparative statics depends on the numerator, which measures how the entire payoff function shifts with $\tau$:

$$\int_0^1 \frac{\partial \pi}{\partial \tau_F}(\lambda, \tau, \chi^*)\, d\lambda.$$

When fundamental uncertainty is sufficiently high, public and private information are complements: injecting more public information tends to increase the payoff from information acquisition: $\frac{\partial \pi}{\partial \tau_F} > 0$. Thus, we obtain the following crowding-in effect under the global-game refinement:

**Theorem 3** (Crowding in under the global game equilibrium)**.** *When $\tau_F$ is sufficiently low, public information disclosure crowds in more private information acquisition at the unique global-game equilibrium:*

$$\frac{d\lambda}{d\tau_F} > 0.$$

[16]We can show that when $\tau_F$ is sufficiently low, the payoff gain function $\pi(\cdot)$ (defined in Equation 11 ) and the value of information expression $V(\cdot)$ are linked by:

$$\pi(\lambda, \tau_F, \chi) \to 1 - \frac{\exp \alpha R \chi}{V}.$$

See the Online Appendix (Lemma A.3) for details.

What drives the differences in prediction between the global-game equilibrium (Theorem 3) and the underlying common-knowledge equilibria (Theorem 2)? Figure 5 plots how the value of information shifts under different values of $\tau_F$. The figure is analogous to Figure 1, with the crucial difference that the payoff functions here are not ad hoc but numerically generated using the microfounded model.

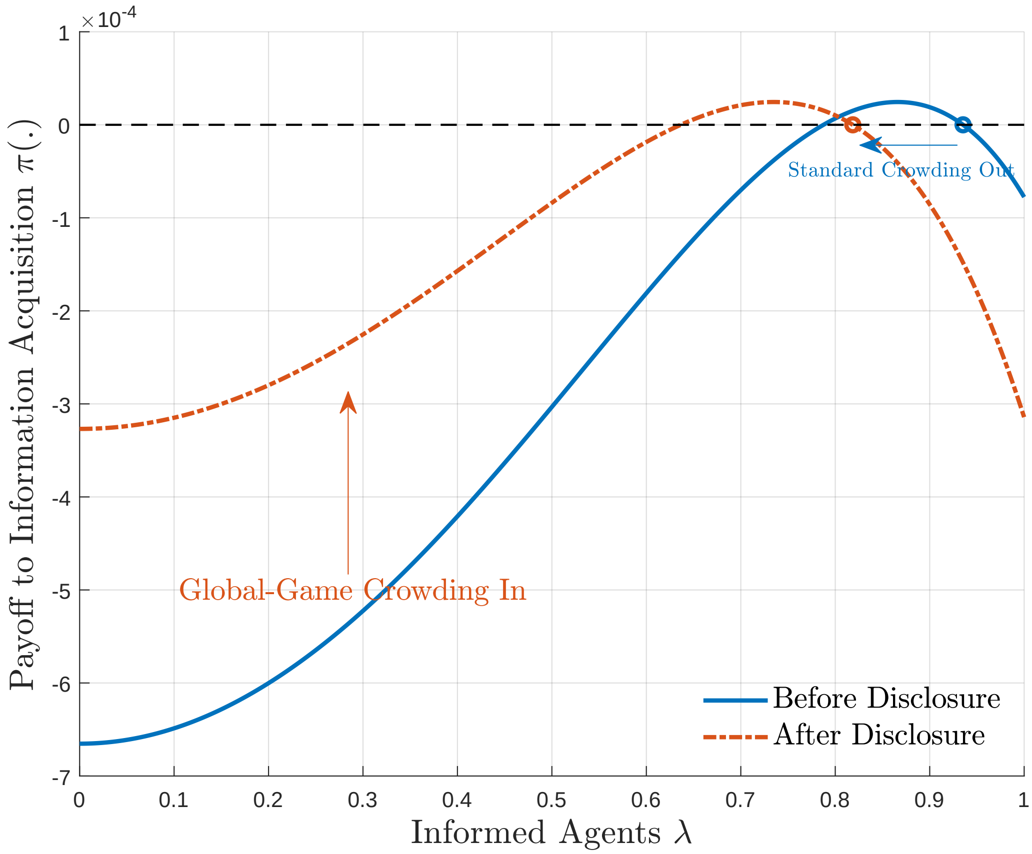
Public disclosure shifts the payoff function from the solid to the dashed line, depressing the value of information at the very top and raising the value at the bottom, i.e., $\frac{\partial \pi}{\partial \tau} < 0$ only for high values of $\lambda$. Under the common-knowledge equilibrium (blue circled point), investors know perfectly that they are coordinating on a very high level of $\lambda$, and therefore they optimally reduce their information acquisition activities upon public disclosure, as $\frac{\partial \pi}{\partial \tau} < 0$ around precisely that level of $\lambda$. This is not the case under the global-game equilibrium where investors need to account for the impact of $\tau$ on all possible values of $\lambda$. Moreover, because the payoff function shifts up substantially at the bottom, the integral $\int_0^1 \frac{\partial \pi}{\partial \tau} d\lambda$ is positive, and thus the *expected* payoff increases, inducing more investors to acquire information.

Note that the crowding-in result does not necessarily follow from *any* source of information complementarity. To obtain crowding-in, it is required that $\frac{\partial \pi}{\partial \tau} > 0$, which *in this particular model* follows from $\frac{\partial \pi}{\partial \lambda} > 0$ but may not be the case for other sources of complementarity. To illustrate, in Online Appendix D, we reproduce the model of Manzano and Vives (2011) in which information complementarity arises due to private information on stock endowment, as in Ganguli and Yang (2009). There, we show that the public-private complementarity condition is violated: injecting more public information always reduces the value of information. Thus, crowding-in cannot occur even under the global-game refinement.[17]

Figure 6 plots the global-game equilibrium along with all common-knowledge equilibria and how they vary with the public information parameter $\tau_F$. Due to the nonmonotonicity in the value of information, three types of common-knowledge equilibria can emerge. The dashed line is the good information equilibrium as studied in Grossman and Stiglitz (1980), where investors coordinate on the highest possible value of $\lambda$. The dotted line is the intermediate equilibrium, and the dash-dotted line at the very bottom depicts the boundary equilibrium where $\lambda$ is always equal to zero. There exists an intermediate region where multiple equilibria exist. Note that regardless of which interior equilibrium one selects, more public information always leads to fewer informed investors. On the other hand, the global-game equilibrium

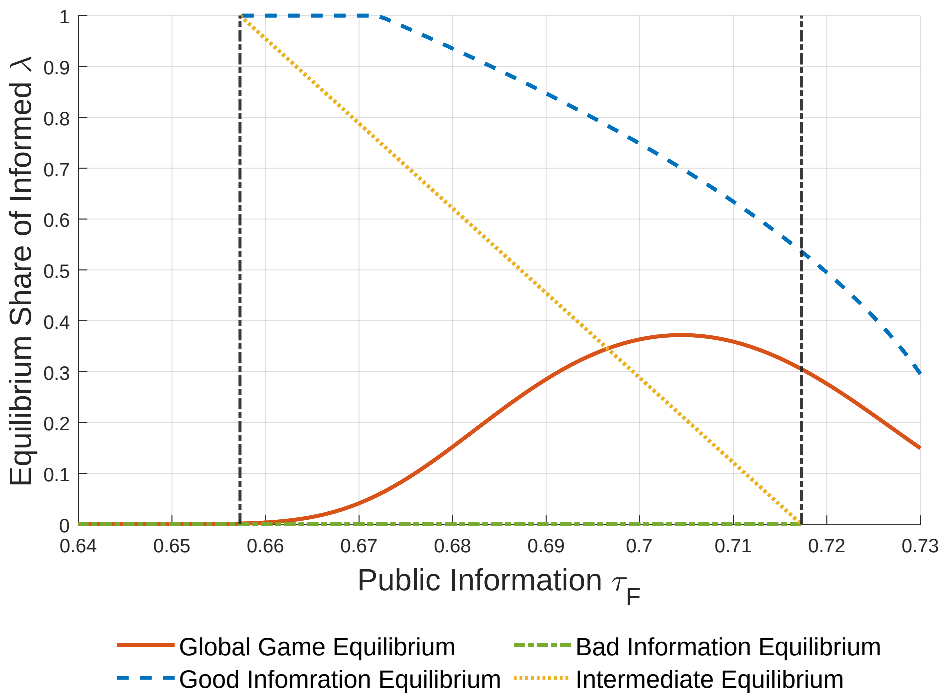
[17] In addition to the model of Manzano and Vives (2011), we also evaluate another model where information complementarity arises due to relative wealth concerns as in García and Strobl (2011). Because their information complementarity also follows from action complementarity, one is unable to obtain the crowding-in result under the global-game refinement because public information disclosure tends to reduce the value of information even with information complementarity. When fundamental uncertainty ($\sigma_x^2$ in their notation) increases, the share of informed investors $\lambda$ also increases (p. 184, figure 3).

Figure 5: Equilibrium Comparison




**Notes**: This figure illustrates the crowding-in result under the global-game refinement. Public disclosure (increases in $\tau_F$) affects the value of information in a highly nonlinear way: it raises the value of information except for the very top values of $\lambda$. With complete cost information, there is no strategic uncertainty, and investors know perfectly that they are coordinating at the good information (the circle point) equilibrium at the very top. In that region, public disclosure depresses the local value of information and hence crowds out private information (horizontal arrow). Under the global-game, strategic uncertainty induces investors to account for the relatively low values of $\lambda$ (vertical arrow), at which disclosure raises the value of information. This crowds in more private information acquisition. $\tau_D = 2, \tau_{x1} = \tau_{x2} = 0.5, R = 1.01, \alpha = 1, \bar{\chi} = 0.195, \sigma = 0.0001, \tau_F = 0.68$ (solid line), $\tau_F = 0.693$ (dashed line).

Figure 6: Impact of Information Disclosure Under Global-game Equilibria




**Notes**: This figure plots how global-game equilibria (solid line) change in response to $\tau_F$, along with other common-knowledge equilibria. At the global-game equilibria, share of informed $\lambda$ increases with $\tau_F$ when $\tau_F$ is sufficiently low, then decreases with it. $\tau_D = 2, \tau_{x1} = \tau_{x2} = 0.5, R = 1.01, \alpha = 1, \bar{\chi} = 0.195, \sigma = 0.0001$.

(solid line) displays crowding-in for relatively low values of $\tau_F$, that is, under relatively high uncertainty.

The global-game equilibrium also displays interesting nonmonotonicity. When there is abundant public information ($\tau_F$ is greater than 0.71), releasing even more information would crowd out private information acquisition. This is because we are approaching the region where information substitutability prevails, and the numerator of Equation 14 ( $\int_0^1 \frac{\partial \pi}{\partial \lambda} d\lambda$) could become negative. This leads to crowding-out because more public information reduces the payoff from information acquisition.[18]

This endogenous nonmonotonicity suggests that for a regulator that wants to learn from market information, there is typically an "optimal" precision of public information disclosure. It is easy to understand the conventional argument that if one releases too much public information, the value of private information deteriorates, which would directly depress the market's effort at collecting information. This model suggests another force at work: when one releases too little public information, there is too much unlearnable risk in stock resale prices. This makes risk-averse investors trade less aggressively. As traders now have less " skin in the game," their incentive to acquire information decreases. This forces them to coordinate toward the bad information equilibria where no one acquires information. This tradeoff gives rise to a unique optimal public information precision that induces most market participants to learn.

## 5 Conclusion

This paper shows that by incorporating strategic uncertainty into a model with endogenous information acquisition, public disclosure can crowd in private information acquisition. We obtain this result by developing a dynamic model to study the impact of public information disclosure on private information acquisition, where private information choices exhibit dynamic strategic complementarity in addition to the conventional static substitutability. To overcome the issue of equilibrium multiplicity, we propose a tractable way of applying global games to the class of noisy rational expectations models with endogenous strategic information acquisition. We find that the classic crowding-out result can be overturned: public disclosure may crowd in private information acquisition when fundamental uncertainty is sufficiently high. This is due to a novel coordination effect: more public information makes it easier for investors to coordinate their information acquisition activities, which enhances price informativeness.

[18]One may wonder whether the monotone equilibrium remains valid with sufficiently low uncertainty and thus when strategic substitutability prevails. Indeed, when $\tau_F$ is too high, the monotone equilibrium breaks down. In Appendix B, it is numerically verified that *for the range of $\tau_F$ considered in this numerical example,* the monotone equilibrium is valid. To do so, we compute the individual payoff gain function for each realization of the information cost, under the market belief that all other agents follow a certain monotone equilibrium. We then numerically verify that, *given the market belief,* the cutoff strategy is indeed an equilibrium.

Our theory provides a new rationale for why regulators should disclose more information from a coordination perspective. The general message of this paper may shed light on the implications of information disclosure in other settings, such as corporate disclosures and stress tests (Goldstein and Leitner, 2018).

The general method developed in this paper can be applied to other models with information complementarity, as different sources of information complementarity can give rise to different predictions under a global-game refinement. It would be particularly natural to apply the methodology in dynamic environments as in this paper, where there are much richer strategic interactions in information gathering than in static environments. Additionally, if one regards fundamental uncertainty as not only affected by disclosure but also inherited from the past, the mechanism explored in this paper, where higher prior uncertainty leads to fewer agents acquiring information, could serve as a propagating force of uncertainty shocks in financial markets. All these issues are intriguing but are beyond the scope of this paper and therefore left for future research.

## References


**Allen, Franklin, Stephen Morris, and Hyun Song Shin**, "Beauty Contests and Iterated Expectations in Asset Markets," *Review of Financial Studies*, 2006, *19* (3), 719–752.

**Amador, Manuel and Pierre-Olivier Weill**, "Learning from prices: public communication and welfare," *Journal of Political Economy*, 2010, *118* (5), 866–907.

**Angeletos, George-Marios and Iván Werning**, "Crises and Prices: Information Aggregation, Multiplicity, and Volatility," *American Economic Review*, December 2006, *96* (5), 1720–1736.

**Athey, Susan**, "Single crossing properties and the existence of pure strategy equilibria in games of incomplete information," *Econometrica*, 2001, *69* (4), 861–889.

**Avdis, Efstathios**, "Information tradeoffs in dynamic financial markets," *Journal of Financial Economics*, 2016, *122* (3), 568 – 584.

**Bai, Jennie, Thomas Philippon, and Alexi Savov**, "Have financial markets become more informative?," *Journal of Financial Economics*, 2016, *122* (3), 625–654.

**Banerjee, Snehal, Jesse Davis, and Naveen Gondhi**, "When transparency improves, must prices reflect fundamentals better?," *The Review of Financial Studies*, 04 2018, *31* (6), 2377–2414.

**Benhabib, Jess and Pengfei Wang**, "Private information and sunspots in sequential asset markets," *Journal of Economic Theory*, 2015, *158*, 558–584.

**_, Xuewen Liu, and Pengfei Wang**, "Endogenous information acquisition and countercyclical uncertainty," *Journal of Economic Theory*, 2016, *165*, 601–642.

**Bond, Philip and Itay Goldstein**, "Government intervention and information aggregation by prices," *The Journal of Finance*, 2015, *70* (6), 2777–2812.

**Breon-Drish, Bradyn**, "On existence and uniqueness of equilibrium in a class of noisy rational expectations models," *The Review of Economic Studies*, 2015, *82* (3), 868–921.

**Cai, Zhifeng**, "Dynamic information acquisition and time-varying uncertainty," *Journal of Economic Theory*, 2019, *184*, 104947.

**Campbell, John Y. and Albert S. Kyle**, "Smart Money, Noise Trading and Stock Price Behaviour," *The Review of Economic Studies*, 1993, *60* (1), 1–34.

**Carlsson, Hans and Eric van Damme**, "Global games and equilibrium selection," *Econometrica*, 1993, *61* (5), 989–1018.

**Chamley, Christophe**, "Complementarities in information acquisition with short-term trades," *Theoretical Economics*, December 2007, *2* (4).

**Colombo, Luca, Gianluca Femminis, and Alessandro Pavan**, "Information acquisition and welfare," *The Review of Economic Studies*, 2014, *81* (4 (289)), 1438–1483.

**Dai, Liang**, "Asset bundling and information acquisition of investors with different expertise," *Journal of Economic Theory*, 2018, *175*, 447–490.

**Dutta, Sunil and Alexander Nezlobin**, "Dynamic effects of information disclosure on investment efficiency," *Journal of Accounting Research*, 2017, *55* (2), 329–369.

**Dye, Ronald A**, "Mandatory versus voluntary disclosures: The cases of financial and real externalities," *Accounting Review*, 1990, pp. 1–24.

**Farboodi, Maryam, Adrien Matray, Laura Veldkamp, and Venky Venkateswaran**, "Where Has All the Data Gone?," April 2020, (26927).

**Froot, Kenneth A., David S. Scharfstein, and Jeremy C. Stein**, "Herd on the street: informational inefficiencies in a market with short-term speculation," *Journal of Finance*, 1992, *47* (4), 1461–1484.

**Gaballo, Gaetano and Guillermo L. Ordoñez**, "The two faces of information," *Working Paper*, 2021.

**Gandhi, Priyank and Hanno Lustig**, "Size anomalies in U.S. bank stock returns," *The Journal of Finance*, 2015, *70* (2), 733–768.

**Ganguli, Jayant Vivek and Liyan Yang**, "Complementarities, multiplicity, and supply information," *Journal of the European Economic Association*, 03 2009, *7* (1), 90–115.

**Gao, Pingyang**, "Disclosure quality, cost of capital, and investor welfare," *The Accounting Review*, 2010, *85* (1), 1–29.

**García, Diego and Günter Strobl**, "Relative wealth concerns and complementarities in information acquisition," *The Review of Financial Studies*, 2011, *24* (1), 169–207.

**Glasserman, Paul, Harry Manmaysky, and Yiwen Shen**, "Dynamic Information Regimes in Financial Markets," *Working Paper*, 2021.

**Goldstein, Itay and Liyan Yang**, "Information diversity and complementarities in trading and information acquisition," *The Journal of Finance*, 2015, *70* (4), 1723–1765.

— **and** — , "Information disclosure in financial markets," *Annual Review of Financial Economics*, 2017, *9* (1), 101–125.

— **and** — , "Good disclosure, bad disclosure," *Journal of Financial Economics*, 2019, *131* (1), 118 – 138.

— **and Yaron Leitner**, "Stress tests and information disclosure," *Journal of Economic Theory*, 2018, *177*, 34 – 69.

— **, Yan Li, and Liyan Yang**, "Speculation and Hedging in Segmented Markets," *The Review of Financial Studies*, March 2014, *27* (3), 881–922.

**Grossman, Sanford J. and Joseph E. Stiglitz**, "On the impossibility of informationally efficient markets," *American Economic Review*, June 1980, *70* (3), 393–408.

**Han, Bing and Liyan Yang**, "Social networks, information acquisition, and asset prices," *Management Science*, 2013, *59* (6), 1444–1457.

_ **, Ya Tang, and Liyan Yang**, "Public information and uninformed trading: implications for market liquidity and price efficiency," *Journal of Economic Theory*, 2016, *163*, 604 – 643.

**Hauswald, Robert and Robert Marquez**, "Competition and strategic information acquisition in credit markets," *The Review of Financial Studies*, 2006, *19* (3), 967–1000.

**Hirshleifer, Jack**, "The private and social value of information and the reward to inventive activity," in "Uncertainty in economics," Elsevier, 1978, pp. 541–556.

**Huang, Shiyang**, "Delegated information acquisition and asset pricing," *Available at SSRN 2666878*, 2015.

_ **, Zhigang Qiu, and Liyan Yang**, "Institutionalization, delegation, and asset prices," *Journal of Economic Theory*, 2020, *186*, 104977.

**Kurlat, Pablo and Laura Veldkamp**, "Should we regulate financial information?," *Journal of Economic Theory*, 2015, *158*, 697–720.

**Kyle, Albert S.**, "Continuous auctions and insider trading," *Econometrica*, 1985, *53* (6), 1315–1335.

**Liu, Xuewen**, "Interbank market freezes and creditor runs," *The Review of financial studies*, 2016, *29* (7), 1860–1910.

**Manzano, Carolina and Xavier Vives**, "Public and private learning from prices, strategic substitutability and complementarity, and equilibrium multiplicity," *Journal of Mathematical Economics*, 2011, *47* (3), 346 – 369. Mathematical Economics II : Special Issue in honour of Andreu Mas-Colell.

**Mason, Robin and Akos Valentinyi**, "The Existence and Uniqueness of Monotone Pure Strategy Equilibrium in Bayesian Games," *Working Paper*, 2010.

**Mele, Antonio and Francesco Sangiorgi**, "Uncertainty, information acquisition, and price swings in asset markets," *The Review of Economic Studies*, 2015, *82* (4), 1533–1567.

**Morris, Stephen and Hyun Song Shin**, "Unique equilibrium in a model of self-fulfilling currency attacks," *The American Economic Review*, 1998, *88* (3), 587–597.

_ **and** _ , "Social value of public information," *American Economic Review*, December 2002, *92* (5), 1521–1534.

_ **and** _ , *Global games: theory and applications*, Vol. 1 of *Econometric Society Monographs*, Cambridge University Press,

**Nieuwerburgh, Stijn Van and Laura Veldkamp**, "Information acquisition and under-diversification," *The Review of Economic Studies*, 2010, *77* (2), 779–805.

**O'Hara, Maureen and Wayne Shaw**, "Deposit insurance and wealth effects: the value of being "too big to fail"," *The Journal of Finance*, 1990, *45* (5), 1587–1600.

**Ordoñez, Guillermo L.**, "Fragility of reputation and clustering of risk-taking," *Theoretical Economics*, 2013, *8* (3), 653–700.

**Pei, Di Harry**, "Communication with endogenous information acquisition," *Journal of Economic Theory*, 2015, *160*, 132–149.

**Peress, Joel**, "Learning from Stock Prices and Economic Growth," *The Review of Financial Studies*, 04 2014, *27* (10), 2998–3059.

_ **and Daniel Schmidt**, "Noise traders incarnate: Describing a realistic noise trading process," *Journal of Financial Markets*, 2021, *54* (C).

**Schaal, Edouard and Mathieu Taschereau-Dumouchel**, "Coordinating business cycles," *Working Paper*, 2018.

**Stambaugh, Robert F.**, "Presidential Address: Investment Noise and Trends," *The Journal of Finance*, 2014, *69* (4), 1415–1453.

**Szkup, Michal and Isabel Trevino**, "Information acquisition in global games of regime change," *Journal of Economic Theory*, 2015, *160*, 387 – 428.

**Veldkamp, Laura L.**, "Media frenzies in markets for financial information," *American Economic Review*, 2006, *96* (3), 577–601.

**Verrecchia, Robert E**, "Information acquisition in a noisy rational expectations economy," *Econometrica*, 1982, pp. 1415–1430.

**Wang, Jiang**, "A model of intertemporal asset prices under asymmetric information," *Review of Economic Studies*, April 1993, *60* (2), 249–282.

_ , "A model of competitive stock trading volume," *Journal of Political Economy*, February 1994, *102* (1), 127–68.

**Xiong, Yan and Liyan Yang**, "Disclosure, competition, and learning from asset prices," *Journal of Economic Theory*, 2021, *197*, 105331.

**Yang, Ming**, "Coordination with flexible information acquisition," *Journal of Economic Theory*, 2015, *158*, 721–738.

_ , "Optimality of debt under flexible information acquisition," *The Review of Economic Studies*, 2020, *87* (1), 487–536.

_ **and Yao Zeng**, "Financing entrepreneurial production: security design with flexible information acquisition," *The Review of Financial Studies*, 2019, *32* (3), 819–863.

**Zou, Junyuan**, "Information traps in over-the-counter markets," *Working Paper*, 2019.

# Appendix

## A Proofs

**Proof of Proposition 1** Total differentiate the equilibrium condition of common knowledge equilibria (equation 1):

$$d\lambda \frac{\partial \pi}{\partial \lambda}\left(\hat{\lambda}, \tau, \bar{\chi}\right) + d\tau \frac{\partial \pi}{\partial \tau}\left(\hat{\lambda}, \tau, \bar{\chi}\right) = 0.$$

Then immediately we have

$$\frac{d\hat{\lambda}}{d\tau} = -\frac{\frac{\partial \pi}{\partial \tau}\left(\hat{\lambda}, \tau, \bar{\chi}\right)}{\frac{\partial \pi}{\partial \lambda}\left(\hat{\lambda}, \tau, \bar{\chi}\right)}.$$

**Proof of Proposition 2** Write $\pi^*_\sigma(\chi, k)$ for the expected payoff gain to acquire information for a player who is of type $\chi$ and knows that all other players will choose to acquire info. if they are of a type less than $k$.

$$\pi^*_\sigma(\chi, k) = \int_{c=-\infty}^{+\infty} \pi\left(\lambda(c, k), \tau, \chi\right) f^\chi(c)\, dc,$$

where

$$\lambda(c, k) = F^c_\varepsilon(k) = F_\varepsilon\left(\frac{k-c}{\sigma}\right),$$

and

$$f^\chi(c) = \frac{1}{\sigma} f\left(\frac{\chi - c}{\sigma}\right).$$

Where $F_\varepsilon(.)$ is the cumulative distribution function for the noise and $f(.)$ denotes the density function. Note there is a subtlety here: $f^\chi(c)$ is the probability of mean equal to $c$ when the signal is $\chi$. To see this, note that

$$\Pr(c < c_0) = \Pr(\chi - \sigma\varepsilon < c_0) = 1 - F_\varepsilon\left(\frac{\chi - c_0}{\sigma}\right).$$

Taking the derivative:

$$f^x(c_0) = \frac{1}{\sigma} F'_\varepsilon\left(\frac{x - c_0}{\sigma}\right) = \frac{1}{\sigma} f\left(\frac{\chi - c_0}{\sigma}\right).$$

To prove the Proposition, we need to show that there exists a unique solution to $\pi^*_\sigma(x, x) = 0$ and this solution is exactly $\chi^*$ that satisfies equation 3. To see this, write $\Psi^*_\sigma(\lambda; x, k)$ for the probability that a player assigns to proportion less than $\lambda$ of the other players observing a signal greater than $k$, if she has a type $x$. Observe that if the true state is $c$, the proportion of players observing a signal greater than $k$ is given by

$$\begin{aligned}\pi_\sigma^*(\chi,k) &= \int_{c=-\infty}^{+\infty} \pi\left(\lambda\left(c,k\right),\tau,\chi\right) f^{\chi}\left(c\right) dc \\ &= \int_{c=-\infty}^{+\infty} \pi\left(F_\varepsilon\left(\frac{k-c}{\sigma}\right),\tau,\chi\right)\frac{1}{\sigma} f\left(\frac{\chi-c}{\sigma}\right) dc.\end{aligned}$$

Thus

$$\pi_\sigma^*(\chi,\chi) = \int_{c=-\infty}^{+\infty} \pi\left(F_\varepsilon\left(\frac{\chi-c}{\sigma}\right),\tau,\chi\right)\frac{1}{\sigma} f\left(\frac{\chi-c}{\sigma}\right) dc.$$

Let $z = \frac{\chi-c}{\sigma}$. Then

$$\pi_\sigma^*(\chi,\chi) = -\sigma\frac{1}{\sigma}\int_{c=+\infty}^{-\infty} \pi\left(F_\varepsilon(z),\tau,\chi\right) f(z)\, dz = -\int_{c=+\infty}^{-\infty} \pi\left(F_\varepsilon(z),\tau,\chi\right) dF(z).$$

Let $\lambda = F_\varepsilon(z)$, hence $z = F_\varepsilon^{-1}(\lambda)$

$$\pi_\sigma^*(\chi,\chi) = -\int_1^0 \pi\left(\lambda,,\tau,\chi\right) d\lambda = \int_0^1 \pi\left(\lambda,\tau,\chi\right) d\lambda.$$

Note that this is also the expected payoff function in the main text:

$$E(\pi(\lambda,\tau,\chi_i)|\chi_i) = \pi_\sigma^*(\chi,\chi) = \int_0^1 \pi\left(\lambda,\tau,\chi\right) d\lambda.$$

Thus, if $\chi^*$ solves $\int_0^1 \pi\left(\lambda,\tau,\chi^*\right) d\lambda$ it must be the case that $\pi_\sigma^*(\chi^*,\chi^*) = 0$. This verifies that $\chi^*$ is the solution to the cutoff equilibrium.

Total differentiate equation 3:

$$d\chi \int_0^1 \frac{\partial \pi}{\partial \chi}\left(\lambda,\tau,\chi^*\right) d\lambda + d\tau \int_0^1 \frac{\partial \pi}{\partial \tau}\left(\lambda,\tau,\chi^*\right) d\lambda = 0.$$

Thus

$$\frac{d\chi^*}{d\tau} = -\frac{\int_0^1 \frac{\partial \pi}{\partial \tau}\left(\lambda,\tau,\chi^*\right) d\lambda}{\int_0^1 \frac{\partial \pi}{\partial \chi}\left(\lambda,\tau,\chi^*\right) d\lambda}.$$

**Proof of Theorem 1** Equation 10 can be seen by comparing the common knowledge equilibrium

$$\frac{d\hat{\lambda}}{d\tau} = -\frac{\frac{\partial \pi}{\partial \tau}\left(\hat{\lambda},\tau,\bar{\chi}\right)}{\frac{\partial \pi}{\partial \lambda}\left(\hat{\lambda},\tau,\bar{\chi}\right)},$$

and the global game equilibrium

$$\frac{d\chi^*}{d\tau} = -\frac{\int_0^1 \frac{\partial \pi}{\partial \tau}\left(\lambda, \tau, \chi^*\right) d\lambda}{\int_0^1 \frac{\partial \pi}{\partial \chi}\left(\lambda, \tau, \chi^*\right) d\lambda}.$$

**Lemma A.1.** *The second-period stock price function $P_2$ is a linear combination of the price signal $S_{P1}$, the public signal $S$, and the noisy stock supply $x_2$, where the coefficients $(a, b, c, d)$ are function of $\theta_1$ and are strictly positive:*

$$P_2 = aS_{P1} + bS - cx_2 + dD_1,$$

**Proof of Lemma A.1:** See online appendix.

**Lemma A.2.** *The value of information $V$ is given by:*

$$V = \frac{Var(Q_1|\Omega_1^U)}{Var(Q_1|\Omega_1^I)} = 1 + \frac{\frac{1}{\Gamma(F|S_{P1},S)}}{\frac{1}{\tau_D} + \frac{1}{\tau_{x2}}\left(\frac{c}{1+d}\right)^2}, \tag{A.1}$$

*where $\Gamma(F|S_{P1}, S)$ is the precision of stock fundamental given the price and public signal, and is given by equation 12. $c$ and $d$ are the price coefficients of the resale stock price $P_2$ with respect to $x_2$ and $D_1$ respectively. The ratio $\frac{c}{1+d}$ is a decreasing function of the fundamental precision $\Gamma(F|S_{P1}, S)$.*

**Proof of Lemma A.2:** See online appendix.

**Proof of Proposition 3:** We will show that under the following conditions

$$\frac{\alpha}{\tau_D} + 1 - R > 0 \tag{A.2}$$

the financial market equilibrium is unique given any $\lambda \in [0,1]$, and the price informativeness $\theta_1$ is always increasing in the share of informed investors $\lambda \in [0,1]$.

We start by deriving the market equilibrium condition. Denote the demand for informed and uninformed investors as $D^I\left(\Omega_1^I\right)$ and $D^U\left(\Omega_1^U\right)$. Thus we can write the first-period market clearing condition as

$$\lambda D^I\left(\Omega_1^I\right) + (1-\lambda)\, D^U\left(\Omega_1^U\right) = x_1.$$

Now, what can uninformed investors learn from the market price? Given that their information set $\Omega_1^U$, they can perfectly predict the uninformed investors' demand $(1-\lambda)\, D^U\left(\Omega_1^U\right)$. Thus the information content of the stock price is contained in the linear combination between informed demand and supply noise $\lambda D^I\left(\Omega_1^I\right) - x_1$. Note note that the informed investors' demand is given by

$$D^I\left(\Omega_1^I\right)=\frac{E\left(Q_1|\Omega_1^I\right)-RP_1}{Var\left(Q_1|\Omega_1^I\right)}.$$

As uninformed investors observe perfectly the price signal and the public signal, this signal becomes:

$$\lambda\frac{F}{\sigma_D^2+C^2\left(\theta_1\right)\sigma_x^2}-x_1.$$

This signal is equivalent to the price signal

$$S_{P1}=F-\frac{1}{\theta_1}x_1.$$

Matching coefficients gives

$$\theta_1=\frac{\lambda}{\alpha\left(\frac{1}{\tau_D}+\left(\frac{c}{1+d}\right)^2\frac{1}{\tau_{x2}}\right)},\tag{A.3}$$

Plug in the expression for function $c$ and $d$, we have:

$$\theta_1=\frac{\lambda}{\alpha\left(\frac{1}{\tau_D}+\left(\frac{\frac{1}{R}\alpha\left(\frac{1}{\tau_F+\theta_1^2\tau_{x1}+\tau_D}+\frac{1}{\tau_D}\right)}{1+\frac{1}{R}\frac{\tau_D}{\tau_F+\theta_1^2\tau_{x1}+\tau_D}}\right)^2\frac{1}{\tau_{x2}}\right)}=\frac{\lambda}{\alpha\left(\frac{1}{\tau_D}+\left(\frac{\alpha\left(\frac{1}{\tau_F+\theta_1^2\tau_{x1}+\tau_D}+\frac{1}{\tau_D}\right)}{R+\frac{\tau_D}{\tau_F+\theta_1^2\tau_{x1}+\tau_D}}\right)^2\frac{1}{\tau_{x2}}\right)},$$

and thus

$$\theta_1\alpha\left(\frac{1}{\tau_D}+\left(\frac{\alpha\left(1+\frac{\tau_F+\theta_1^2\tau_{x1}+\tau_D}{\tau_D}\right)}{R\left(\tau_F+\theta_1^2\tau_{x1}+\tau_D\right)+\tau_D}\right)^2\frac{1}{\tau_{x2}}\right)-\lambda=0.$$

Denote this function to be

$$\Phi\left(\theta_1;\lambda\right)=0.$$

We can also rearrange this equation into a fifth order polynomial of $\theta_1$. Denote this polynomial as

$$H(\theta_1;\lambda)=\frac{1}{\tau_D}\theta_1\alpha\left(R\left(\tau_F+\theta_1^2\tau_{x1}+\tau_D\right)+\tau_D\right)^2+\alpha^2\left(1+\frac{\tau_F+\theta_1^2\tau_{x1}+\tau_D}{\tau_D}\right)^2\frac{1}{\tau_{x2}}\theta_1\alpha\tag{A.4}$$

$$-\lambda\left(R\left(\tau_F+\theta_1^2\tau_{x1}+\tau_D\right)+\tau_D\right)^2\tag{A.5}$$

$$=A_5\theta^5+A_4\theta^4+A_3\theta^3+A_2\theta_1^2+A_1\theta_1^2+A_0\theta_1^2$$

where the some of the coefficients depend on $\lambda$. We look for the real roots of

$$H(\theta_1;\lambda) = 0.$$

Because $H(\theta_1;\lambda)$ is just an elementary rearrangement of $\Phi(\theta_1;\lambda)$, we have[19]

$$\frac{d\theta_1}{d\lambda} = -\frac{\frac{\partial H}{\partial \lambda}}{\frac{\partial H}{\partial \theta_1}} = -\frac{\frac{\partial \Phi}{\partial \lambda}}{\frac{\partial \Phi}{\partial \theta_1}} \tag{A.6}$$

There is generally no formula to determine the number of real roots of a fifth polynomial. This paper takes an approach that explores a special structure of this particular equation. We first derive a condition under which $\frac{d\theta_1}{d\lambda}$ is always positive. We then argue that this condition also implies that there is a unique real root of $\theta_1$.

First, total differentiate the equation $H(.)$:

$$d\theta_1 \frac{\partial H}{\partial \theta_1} + d\lambda \frac{\partial H}{\partial \lambda} = 0,$$

and then we have

$$\frac{d\theta_1}{d\lambda} = -\frac{\frac{\partial H}{\partial \lambda}}{\frac{\partial H}{\partial \theta_1}} = -\frac{\frac{\partial \Phi}{\partial \lambda}}{\frac{\partial \Phi}{\partial \theta_1}}.$$

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[19] To see this, let's consider a simple example. Let $\Phi(\theta_1, \lambda)$ be the following function:

$$\Phi(\theta_1, \lambda) = \frac{1}{\theta_1} - \lambda = 0.$$

It can be arranged trivially into a (first-order) polynomial by multiplying the equation with $\theta_1$:

$$H(\theta_1, \lambda) = \lambda\theta_1 - 1 = 0.$$

Hence:

$$\frac{\frac{\partial H}{\partial \lambda}}{\frac{\partial H}{\partial \theta_1}} = \frac{\theta_1}{\lambda} = \theta_1^2.$$

where the first equality just follows the derivative of $H(\theta_1, \lambda)$ and the second equality follows from $\lambda\theta_1 - 1 = 0$ and hence $\lambda = \frac{1}{\theta_1}$.

Now we can evaluate this for $\Phi(\theta_1, \lambda)$ :

$$\frac{\frac{\partial \Phi}{\partial \lambda}}{\frac{\partial \Phi}{\partial \theta_1}} = \frac{-1}{-\frac{1}{\theta_1^2}} = \theta_1^2.$$

where the first equality follows from the derivative of $\Phi(\theta_1, \lambda)$.

Hence we conclude that

$$\frac{\frac{\partial H}{\partial \lambda}}{\frac{\partial H}{\partial \theta_1}} = \frac{\frac{\partial \Phi}{\partial \lambda}}{\frac{\partial \Phi}{\partial \theta_1}}.$$

Note that the denominator is always negative:

$$\frac{\partial\Phi}{\partial\lambda} = -1 < 0.$$

We next examine the derivative $\frac{d\theta_1}{d\lambda}$. Total differentiate equation $\Phi$ with respect to $\theta_1$, we have:

$$\begin{aligned}
\frac{\partial\Phi}{\partial\theta_1} &= \alpha\left(\frac{1}{\tau_D} + \left(\frac{\alpha\left(1+\frac{\tau_F+\theta_1^2\tau_{x1}+\tau_D}{\tau_D}\right)}{R\left(\tau_F+\theta_1^2\tau_{x1}+\tau_D\right)+\tau_D}\right)^2\frac{1}{\tau_{x2}}\right) \\
&\quad +\theta_1\alpha\left(2\frac{1}{\tau_{x2}}\frac{\alpha\left(\frac{2\theta_1\tau_{x1}}{\tau_D}\right)\left(R\left(\tau_F+\theta_1^2\tau_{x1}+\tau_D\right)+\tau_D\right)-\alpha\left(1+\frac{\tau_F+\theta_1^2\tau_{x1}+\tau_D}{\tau_D}\right)R2\theta_1\tau_{x1}}{\left(R\left(\tau_F+\theta_1^2\tau_{x1}+\tau_D\right)+\tau_D\right)^2}\right) \\
&= \alpha\left(\frac{1}{\tau_D} + \left(\frac{\alpha\left(1+\frac{\tau_F+\theta_1^2\tau_{x1}+\tau_D}{\tau_D}\right)}{R\left(\tau_F+\theta_1^2\tau_{x1}+\tau_D\right)+\tau_D}\right)^2\frac{1}{\tau_{x2}}\right)+\theta_1\alpha\left(2\frac{1}{\tau_{x2}}\frac{\alpha\left(\frac{2\theta_1\tau_{x1}}{\tau_D}\right)\tau_D-\alpha R2\theta_1\tau_{x1}}{\left(R\left(\tau_F+\theta_1^2\tau_{x1}+\tau_D\right)+\tau_D\right)^2}\right) \\
&= \frac{\alpha}{\tau_D}+\frac{\alpha^3}{\tau_{x2}}\frac{\left(1+\frac{\tau_F+\theta_1^2\tau_{x1}+\tau_D}{\tau_D}\right)^2}{\left(R\left(\tau_F+\theta_1^2\tau_{x1}+\tau_D\right)+\tau_D\right)^2}+2\frac{\theta_1\alpha^2}{\tau_{x2}}\frac{\left(\frac{2\theta_1\tau_{x1}}{\tau_D}\right)\tau_D-2R\theta_1\tau_{x1}}{\left(R\left(\tau_F+\theta_1^2\tau_{x1}+\tau_D\right)+\tau_D\right)^2} \\
&= \frac{\alpha}{\tau_D}+\frac{\alpha^3}{\tau_{x2}}\frac{\left(1+\frac{\tau_F+\theta_1^2\tau_{x1}+\tau_D}{\tau_D}\right)^2}{\left(R\left(\tau_F+\theta_1^2\tau_{x1}+\tau_D\right)+\tau_D\right)^2}+2\frac{\theta_1\alpha^2}{\tau_{x2}}\frac{2\left(1-R\right)\theta_1\tau_{x1}}{\left(R\left(\tau_F+\theta_1^2\tau_{x1}+\tau_D\right)+\tau_D\right)^2} \\
&= \frac{\alpha}{\tau_D}+\frac{\alpha^2}{\tau_{x2}}\frac{1}{\left(R\left(\tau_F+\theta_1^2\tau_{x1}+\tau_D\right)+\tau_D\right)^2}\left[\alpha\left(1+\frac{\tau_F+\theta_1^2\tau_{x1}+\tau_D}{\tau_D}\right)^2-4\theta_1^2\left(R-1\right)\tau_{x1}\right].
\end{aligned}$$

Thus

$$\frac{d\theta_1}{d\lambda} = \frac{1}{\alpha\left(\frac{1}{\tau_D}+\left(\frac{\alpha\left(1+\frac{\tau_F+\theta_1^2\tau_{x1}+\tau_D}{\tau_D}\right)}{R\left(\tau_F+\theta_1^2\tau_{x1}+\tau_D\right)+\tau_D}\right)^2\frac{1}{\tau_{x2}}\right)+\theta_1\alpha\left(2\frac{1}{\tau_{x2}}\frac{\alpha\left(\frac{2\theta_1\tau_{x1}}{\tau_D}\right)\tau_D-\alpha R2\theta_1\tau_{x1}}{\left(R\left(\tau_F+\theta_1^2\tau_{x1}+\tau_D\right)+\tau_D\right)^2}\right)}.$$

So it suffices to check that

$$\alpha\left(\frac{1}{\tau_D}+\left(\frac{\alpha\left(1+\frac{\tau_F+\theta_1^2\tau_{x1}+\tau_D}{\tau_D}\right)}{R\left(\tau_F+\theta_1^2\tau_{x1}+\tau_D\right)+\tau_D}\right)^2\frac{1}{\tau_{x2}}\right)+\theta_1\alpha\left(2\frac{1}{\tau_{x2}}\frac{\alpha2\theta_1\tau_{x1}\left(1-R\right)}{\left(R\left(\tau_F+\theta_1^2\tau_{x1}+\tau_D\right)+\tau_D\right)^2}\right)>0 \tag{A.7}$$

for any $\theta_1(\lambda)$ such that $\lambda \in [0,1]$.

Simplify:

$$\frac{1}{\tau_D} + \left( \frac{\alpha \left(1 + \frac{\tau_F + \theta_1^2 \tau_{x1} + \tau_D}{\tau_D}\right)}{R\left(\tau_F + \theta_1^2 \tau_{x1} + \tau_D\right) + \tau_D} \right)^2 \frac{1}{\tau_{x2}} + \theta_1 \left( 2 \frac{1}{\tau_{x2}} \frac{\alpha 2 \theta_1 \tau_{x1} (1-R)}{\left(R\left(\tau_F + \theta_1^2 \tau_{x1} + \tau_D\right) + \tau_D\right)^2} \right) > 0. \quad \text{(A.8)}$$

Given that $\tau_D > 0$, it suffices to check:

$$\left( \frac{\alpha \left(1 + \frac{\tau_F + \theta_1^2 \tau_{x1} + \tau_D}{\tau_D}\right)}{R\left(\tau_F + \theta_1^2 \tau_{x1} + \tau_D\right) + \tau_D} \right)^2 \frac{1}{\tau_{x2}} + \theta_1 \left( 2 \frac{1}{\tau_{x2}} \frac{\alpha 2 \theta_1 \tau_{x1} (1-R)}{\left(R\left(\tau_F + \theta_1^2 \tau_{x1} + \tau_D\right) + \tau_D\right)^2} \right) > 0.$$

simplify given that $\tau_{x2} > 0$:

$$\left( \alpha \left( 2 + \frac{\tau_F + \theta_1^2 \tau_{x1}}{\tau_D} \right) \right)^2 + 4\alpha \theta_1^2 \tau_{x1} (1-R) > 0.$$

Expand terms:

$$4\alpha + \left( \frac{\tau_F + \theta_1^2 \tau_{x1}}{\tau_D} \right)^2 \alpha + \frac{4\tau_F \alpha}{\tau_D} + \frac{4\theta_1^2 \tau_{x1} \alpha}{\tau_D} + 4\theta_1^2 \tau_{x1} (1-R) > 0.$$

Given that the first three terms are all positive, it suffices to check:

$$\frac{4\theta_1^2 \tau_{x1} \alpha}{\tau_D} + 4\theta_1^2 \tau_{x1} (1-R) > 0.$$

From financial market clearing A.3 it is clear that $\theta_1 \geq 0$ :

$$\frac{\alpha}{\tau_D} + 1 - R > 0.$$

When this condition holds, we have $\frac{\partial H}{\partial \theta_1} > 0$.

Next we show that under this condition, the financial market equilibrium must be unique. Under this condition, we know that

$$\frac{d\theta_1}{d\lambda} > 0.$$

We also know that

$$\frac{d\theta_1}{d\lambda} = -\frac{\frac{\partial H}{\partial \lambda}}{\frac{\partial H}{\partial \theta_1}} > 0.$$

where

$$\frac{\partial H}{\partial \lambda} < 0$$

because

$$\frac{\partial H}{\partial \lambda} = -\left(R\left(\tau_F + \theta_1^2 \tau_{x1} + \tau_D\right) + \tau_D\right)^2 < 0$$

by equation A.4.

Thus it follows that

$$\frac{\partial H}{\partial \theta_1} > 0.$$

However, due to the property of a fifth polynomial, if there exists multiple different real roots, then there must be either three or five real roots, with different derivatives at those real roots crossing the zero line. For example, if there are three real roots, than the derivatives at those three roots must be (from the smallest to the biggest root):

$$\frac{\partial H}{\partial \theta_1} > 0, \frac{\partial H}{\partial \theta_1} < 0, \frac{\partial H}{\partial \theta_1} > 0.$$

In other words, there must exist a root such that $\frac{\partial H}{\partial \theta_1} < 0$. This is a contradiction to our conclusion that $\frac{\partial H}{\partial \theta_1}$ must be greater than zero. Hence, the real root must be unique for $\lambda \in [0, 1]$.

**Proof of Proposition 4:** We will check how the value of information changes with $\theta_1$. For simplicity, we evaluate the following expression, which is a monotonic increasing transformation of the value of information expression:

$$\left(\frac{1}{\tau_D} + \left(\frac{\alpha\left(\frac{1}{\tau_F+\theta_1^2\tau_x} + \frac{1}{\tau_D}\right)}{R + \frac{\tau_D}{\tau_F+\theta_1^2\tau_x+\tau_D}}\right)^2 \frac{1}{\tau_x}\right)\left(\tau_F + \theta_1^2\tau_x\right).$$

Denote $\nu = \tau_F + \theta_1^2\tau_x$. Then the expression becomes:

$$\left(\frac{1}{\tau_D} + \left(\frac{\alpha\left(\frac{1}{\nu} + \frac{1}{\tau_D}\right)}{R + \frac{\tau_D}{\nu+\tau_D}}\right)^2 \frac{1}{\tau_x}\right)\nu = \frac{\nu}{\tau_D} + \left(\frac{\alpha\left(\frac{1}{\nu} + \frac{1}{\tau_D}\right)}{R + \frac{\tau_D}{\nu+\tau_D}}\right)^2 \frac{\nu}{\tau_x}.$$

Taking derivative with respect to $\nu$ yields

$$\begin{aligned}
&= \frac{1}{\tau_D} + \left(\frac{\alpha\left(\frac{1}{\nu}+\frac{1}{\tau_D}\right)}{R+\frac{\tau_D}{\nu+\tau_D}}\right)^2 \frac{1}{\tau_x} + 2\left(\frac{\alpha\left(\frac{1}{\nu}+\frac{1}{\tau_D}\right)}{R+\frac{\tau_D}{\nu+\tau_D}}\right)\left(\frac{\alpha\left(\frac{1}{\nu}+\frac{1}{\tau_D}\right)}{R+\frac{\tau_D}{\nu+\tau_D}}\right)' \frac{\nu}{\tau_x} \\
&= \frac{1}{\tau_D} + \left(\frac{\alpha\left(\frac{1}{\nu}+\frac{1}{\tau_D}\right)}{R+\frac{\tau_D}{\nu+\tau_D}}\right)^2 \frac{1}{\tau_x} + 2\left(\frac{\alpha\left(\frac{1}{\nu}+\frac{1}{\tau_D}\right)}{R+\frac{\tau_D}{\nu+\tau_D}}\right)\frac{\alpha\frac{-1}{\nu^2}\left(R+\frac{\tau_D}{\nu+\tau_D}\right)-\alpha\left(\frac{1}{\nu}+\frac{1}{\tau_D}\right)\frac{-\tau_D}{(\nu+\tau_D)^2}}{\left(R+\frac{\tau_D}{\nu+\tau_D}\right)^2}\frac{\nu}{\tau_x} \\
&= \frac{1}{\tau_D} + \left(\frac{\alpha\left(\frac{1}{\nu}+\frac{1}{\tau_D}\right)}{R+\frac{\tau_D}{\nu+\tau_D}}\right)^2 \frac{1}{\tau_x} + 2\alpha\left(\frac{\alpha\left(\frac{1}{\nu}+\frac{1}{\tau_D}\right)}{R+\frac{\tau_D}{\nu+\tau_D}}\right)\frac{\frac{-1}{\nu^2}\left(R+\frac{\tau_D}{\nu+\tau_D}\right)-\left(\frac{1}{\nu}+\frac{1}{\tau_D}\right)\frac{-\tau_D}{(\nu+\tau_D)^2}}{\left(R+\frac{\tau_D}{\nu+\tau_D}\right)^2}\frac{\nu}{\tau_x} \\
&= \frac{1}{\tau_D} + \left(\frac{\alpha\left(\frac{1}{\nu}+\frac{1}{\tau_D}\right)}{R+\frac{\tau_D}{\nu+\tau_D}}\right)^2 \frac{1}{\tau_x} + 2\alpha\left(\frac{\alpha\left(\frac{1}{\nu}+\frac{1}{\tau_D}\right)}{R+\frac{\tau_D}{\nu+\tau_D}}\right)\frac{\frac{-1}{\nu^2}\left(R+\frac{\tau_D}{\nu+\tau_D}\right)-\left(\frac{1}{\nu}+\frac{1}{\tau_D}\right)\frac{-\tau_D}{(\nu+\tau_D)^2}}{\left(R+\frac{\tau_D}{\nu+\tau_D}\right)^2}\frac{\nu}{\tau_x}.
\end{aligned}$$

We consider the case where $\nu$ is sufficiently low $\nu \to 0$:

$$\frac{1}{\tau_D} + \left(\frac{\alpha\left(\frac{1}{\nu}+\frac{1}{\tau_D}\right)}{R+\frac{\tau_D}{\nu+\tau_D}}\right)^2 \frac{1}{\tau_x} + 2\alpha\left(\frac{\alpha\left(\frac{1}{\nu}+\frac{1}{\tau_D}\right)}{R+\frac{\tau_D}{\nu+\tau_D}}\right)\frac{\frac{-1}{\nu^2}\left(R+\frac{\tau_D}{\nu+\tau_D}\right)-\left(\frac{1}{\nu}+\frac{1}{\tau_D}\right)\frac{-\tau_D}{(\nu+\tau_D)^2}}{\left(R+\frac{\tau_D}{\nu+\tau_D}\right)^2}\frac{\nu}{\tau_x}.$$

Then we can ignore the first term:

$$\begin{aligned}
\propto\; & \left(\frac{1}{R+\frac{\tau_D}{\nu+\tau_D}}\right)^2 \frac{1}{\tau_x}\left(\alpha\left(\frac{1}{\nu}+\frac{1}{\tau_D}\right)\right)^2 \\
& +2\alpha\frac{1}{\left(R+\frac{\tau_D}{\nu+\tau_D}\right)^2}\frac{1}{\tau_x}\left(\frac{\alpha\left(\frac{1}{\nu}+\frac{1}{\tau_D}\right)}{R+\frac{\tau_D}{\nu+\tau_D}}\right)\left(\frac{-1}{\nu^2}\left(R+\frac{\tau_D}{\nu+\tau_D}\right)-\left(\frac{1}{\nu}+\frac{1}{\tau_D}\right)\frac{-\tau_D}{(\nu+\tau_D)^2}\right)\nu
\end{aligned}$$

$$\to \left(\frac{1}{R+1}\right)^2\frac{1}{\tau_x}\left(\alpha\frac{1}{\nu}\right)^2 + 2\alpha\frac{1}{(R+1)^2}\frac{1}{\tau_x}\left(\frac{\alpha\frac{1}{\nu}}{R+1}\right)\left(\frac{-1}{\nu}R\right)$$

$$\to \frac{2R}{R+1} > 0.$$

So this complementarity as long as the fundamental uncertainty is sufficiently high: $\nu \to 0$.

Now we examine the case where $v \to \infty$. In this case:

$$\frac{1}{\tau_D} + \left( \frac{\alpha \left( \frac{1}{\nu} + \frac{1}{\tau_D} \right)}{R + \frac{\tau_D}{\nu + \tau_D}} \right)^2 \frac{1}{\tau_x} + 2\alpha \left( \frac{\alpha \left( \frac{1}{\nu} + \frac{1}{\tau_D} \right)}{R + \frac{\tau_D}{\nu + \tau_D}} \right) \frac{\frac{-1}{\nu^2} \left( R + \frac{\tau_D}{\nu + \tau_D} \right) - \left( \frac{1}{\nu} + \frac{1}{\tau_D} \right) \frac{-\tau_D}{(\nu + \tau_D)^2}}{\left( R + \frac{\tau_D}{\nu + \tau_D} \right)^2} \frac{\nu}{\tau_x}.$$

Note that the third term dominates which is negative, hence substitutability dominates when fundamental uncertainty is sufficiently low.

**Proof of Theorem 2:** From previous analysis we know that

$$\pi = U^I \left( 1 - \sqrt{\frac{Var^U (Q_1)}{Var^I (Q_1)}} \exp(-\alpha R \chi) \right) = U^I \left( 1 - V \exp(-\alpha R \chi) \right).$$

Thus

$$\frac{\partial \pi}{\partial \lambda} = \frac{\partial U^I}{\partial \lambda} \left( 1 - V \exp(-\alpha R \chi) \right) + U^I \left( -\frac{\partial V}{\partial \lambda} \exp(-\alpha R \chi) \right).$$

Given an information equilibrium $\lambda = \hat{\lambda}$, we know that $1 - V \exp(-\alpha R \chi) = 0$. Thus

$$\frac{\partial \pi}{\partial \lambda} = -U^I \frac{\partial V}{\partial \lambda} \exp(-\alpha R \chi).$$

Given that $U^I$ is always negative and $\exp(-\alpha R \chi)$ is always positive, we know that

$$\frac{\partial \pi}{\partial \lambda} \propto \frac{\partial V}{\partial \lambda}.$$

By a similar reasoning:

$$\frac{\partial \pi}{\partial \tau_F} \propto \frac{\partial V}{\partial \tau_F}.$$

Thus we only need to verify that $\frac{\partial V}{\partial \lambda}$ and $\frac{\partial V}{\partial \tau_F}$ always take the same sign. This can be seen by:

$$V = 1 + \frac{\frac{1}{\Gamma(F|S_{P1}, S)}}{\frac{1}{\tau_D} + \left[ \frac{c}{1+d} \right]^2 \frac{1}{\tau_{x2}}}.$$

where

$$\begin{aligned} c &= \frac{1}{R} \alpha \left( \frac{1}{\tau_F + \theta_1^2 \tau_x} + \frac{1}{\tau_D} \right), \\ d &= \frac{1}{R} \frac{\tau_D}{\tau_F + \theta_1^2 \tau_x + \tau_D}. \end{aligned}$$

where both $\lambda$ and $\tau_F$ enter into this expression only through $\tau_F + \theta_1^2 \tau_{x1}$. Thus they must be of the same sign.

To prove proposition 5 and theorem 3, we need the following lemma:

**Lemma A.3.** *When fundamental uncertainty is sufficiently high, the payoff function $\pi$ and the value of information function $V$ has the following monotonically increasing relationship:*

$$\pi(\lambda, \tau_F, \chi) \to 1 - \frac{\exp \alpha R \chi}{V}.$$

**Proof of Lemma A.3** See online appendix.

**Proof of Proposition 5 :** The assumptions A.1 through A.5 in Morris and Shin (2003) are:

1. Action monotonicity: $\pi(\lambda, \tau_F, \chi)$ is nondecreasing in $\lambda$;
2. State monotonicity: $\pi(\lambda, \tau_F, \chi)$ is nonincreasing in $\chi$;
3. Strict Laplacian state monotonicity: there exists unique $\chi^*$ solving
$$\int_0^1 \pi(\lambda, \tau_F, \chi) d\lambda = 0; \tag{A.9}$$
4. Limit dominance: There exist $\underline{\theta}$ and $\overline{\theta}$ such that $\pi(\lambda, \tau_F, \chi) < 0$ for all $\lambda \in [0, 1]$ and $\chi \geq \overline{\theta}$ and $\pi(\lambda, \tau_F, \chi) > 0$ for all $\lambda \in [0, 1]$ and $\chi \leq \underline{\theta}$;
5. Continuity: $\int_0^1 g(\lambda) \pi(\lambda, \tau_F, \chi) d\lambda = 0$ is continuous with respect to type $\chi$ and density $g$.

We first verify that these five requirements are satisfied. By lemma A.3, given that $\tau_F \to 0$, the payoff gain function
$$\pi \to 1 - \frac{\exp(\alpha R \chi)}{V}.$$

We can check the five properties using equation $1 - \frac{\exp(\alpha R \chi)}{V}$. First, note that when fundamental uncertainty is sufficiently high, we know that the value of information is monotonically increasing in $\lambda$. This proves the first requirement. The second and the third requirement is immediate by observing that value of information does not depend on $\chi$. Thus the payoff gain function is monotonically decreasing in $\chi$. For the limit dominance requirement, observe that when $\chi \to 0$, for any values of $\lambda$ we have

$$\pi\left(\chi, \lambda; \sigma_F^2\right) \to 1 - \sqrt{\frac{Var^I(Q_1)}{Var^U(Q_1)}} > 0.$$

When $\chi \to \infty, \pi \to -\infty$ regardless of values of $\lambda$. These two properties imply that the state limit dominance condition is satisfied. The last condition follows from the fact that all equations are differentiable.

We next prove that given $\pi$ satisfies all five properties, a global game equilibrium exists and is unique. The proof largely follows Morris and Shin Proposition 2.1 and works through iterated deletion of strictly dominated strategies. Write $\pi_\sigma^*(\chi, k)$ for the expected payoff gain to acquire information for a player who has a type $\chi$ and knows that all other players will choose to acquire info. if they observe signals less than $k$ :

$$\pi_\sigma^*(\chi, k) = \int_{c=-\infty}^{+\infty} \pi(\lambda(c,k), \chi; \tau_F) f^\chi(c)\, dc,$$

where

$$\lambda(c,k) = F^c(k) = F_\varepsilon\left(\frac{k-c}{\sigma}\right),$$

and

$$f^\chi(c) = \frac{1}{\sigma} f\left(\frac{\chi - c}{\sigma}\right).$$

Note there is a subtlety here: $f^\chi(c)$ is the probability of mean equal to $c$ when the signal is $\chi$. To see this, note that

$$\Pr(c < c_0) = \Pr(\chi - \sigma\varepsilon < c_0) = 1 - F_\varepsilon\left(\frac{\chi - c_0}{\sigma}\right).$$

Taking the derivative:

$$f^x(c_0) = \frac{1}{\sigma} F_\varepsilon'\left(\frac{x - c_0}{\sigma}\right) = \frac{1}{\sigma} f\left(\frac{\chi - c_0}{\sigma}\right).$$

First, observe that $\pi_\sigma^*(\chi, k)$ is continuous in $\chi$ and $k$, decreasing in $\chi$, and increasing in $k. \pi_\sigma^*(\chi, k) < 0$ if $x$ sufficiently large and vice versa. We will argue by induction that a strategy survives $n$ rounds of iterated deletion of strictly interim dominated strategies if and only if

$$s(\chi) = \begin{cases} 0, \text{ if } \chi > \xi_n^1 \\ 1, \text{ if } \chi < \xi_n^2 \end{cases},$$

where $\xi_0^1 = +\infty, \xi_0^2 = -\infty$, and $\xi_n^1$ and $\xi_n^2$ are defined recursively by:

$$\begin{aligned} \xi_{n+1}^1 &= \max\left\{\chi : \pi_\sigma^*\left(\chi, \xi_n^1\right) = 0\right\}, \\ \xi_{n+1}^2 &= \min\left\{\chi : \pi_\sigma^*\left(\chi, \xi_n^2\right) = 0\right\}. \end{aligned}$$

Suppose the claim was true for $n$. Note that $\xi_n^1$ and $\xi_n^2$ are decreasing and increasing sequences, respectively. Because $\xi_0^1 = +\infty$ and $\xi_1^1 < \bar{c} < +\infty$, and $\pi_\sigma^* (\chi, k)$ is decreasing in $\chi$ and increasing in $k$. Thus $\xi_n^1 \to \xi^1$ and $\xi_n^2 \to \xi^2$. Thus, the second step is to show that there exists a unique solution to $\pi_\sigma^* (x, x) = 0$ and this solution is exactly $\chi^*$.

To see this second step, write $\Psi_\sigma^* (\lambda; x, k)$ for the probability that a player assigns to proportion less than $\lambda$ of the other players observing a signal greater than $k$, if she has a type $x$. Observe that if the true state is $c$, the proportion of players observing a signal greater than $k$ is given by

$$\begin{aligned} \pi_\sigma^* (\chi, k) &= \int_{c=-\infty}^{+\infty} \pi (\lambda (c, k), \chi; \tau_F) f^\chi (c) \, dc \\ &= \int_{c=-\infty}^{+\infty} \pi \left( F_\varepsilon \left( \frac{k - c}{\sigma} \right), \chi; \tau_F \right) \frac{1}{\sigma} f \left( \frac{\chi - c}{\sigma} \right) dc. \end{aligned}$$

Thus

$$\pi_\sigma^* (\chi, \chi) = \int_{c=-\infty}^{+\infty} \pi \left( F_\varepsilon \left( \frac{\chi - c}{\sigma} \right), x; \tau_F \right) \frac{1}{\sigma} f \left( \frac{\chi - c}{\sigma} \right) dc.$$

Let $z = \frac{\chi - c}{\sigma}$. Then

$$\pi_\sigma^* (\chi, \chi) = - \int_{c=+\infty}^{-\infty} \pi (F_\varepsilon (z), \chi; \tau_F) \, dF (z).$$

Let $\lambda = F_\varepsilon (z)$, hence $z = F_\varepsilon^{-1} (\lambda)$. Then

$$\pi_\sigma^* (\chi, \chi) = - \int_1^0 \pi (l, \chi; \tau_F) \, dl = \int_0^1 \pi (l, \chi; \tau_F) \, dl.$$

Given that $\int_0^1 \pi (l, \chi; \tau_F) \, dl = 0$ has a unique solution given by $\chi^*$, so is $\pi_\sigma^* (x, x)$.

**Proof of Theorem 3** By lemma A.3, we know that when $\tau_F$ is sufficiently close to zero, $\pi \to 1 - \frac{\exp(\alpha R \chi)}{V}$. We also know that in this case, $V$ is monotonically increasing in $\lambda$. Thus it can be easily verified that $V$ is also monotonically increasing in $\tau_F$. By Proposition 2 and the fact that

$$\lambda^* = F_\varepsilon \left( \frac{\chi^* - \bar{\chi}}{\sigma} \right).$$

We have

$$\frac{d\lambda^*}{d\tau_F} = -\frac{1}{\sigma} f \left( \frac{\chi^* - \bar{\chi}}{\sigma} \right) \frac{\int_0^1 \frac{\partial \pi}{\partial \tau_F} d\lambda}{\int_0^1 \frac{\partial \pi}{\partial \chi} d\lambda}.$$

We know that the payoff function is always decreasing in the information cost, thus $\int_0^1 \frac{\partial \pi}{\partial \chi} d\lambda < 0$. Also we know that $\frac{\partial \pi}{\partial \tau_F} > 0$ so $\int_0^1 \frac{\partial \pi}{\partial \tau_F} d\lambda > 0$. Probability density function is always strictly positive $f\left(\frac{\chi^* - \bar{\chi}}{\sigma}\right) > 0$. Hence $\frac{d\lambda^*}{d\tau_F} > 0$.

# B Validity of Monotone Equilibrium

This section checks the validity of monotone equilibrium solved in the numerical analysis. To do so, we check that the "single-crossing condition" is satisfied, which leads to the existence of a monotone equilibrium (Athey 2001). Specifically, Figure B.1 plots the payoff function $\pi^*(\chi_i, \bar{\chi}; \tau_F)$ as a function of the player's own type $\chi_i$, holding fixed other players' strategy at the equilibrium cutoff $\bar{\chi}$. This function is monotonically decreasing and crosses the zero axis only once. Thus the single crossing condition is satisfied and a monotone equilibrium is valid. Intuitively, this condition is saying that if one expects the others to follow a monotone strategy, he himself would also find monotone strategy optimal. We check that this single-crossing condition is satisfied for the range of $\tau_F$ considered in the main text.

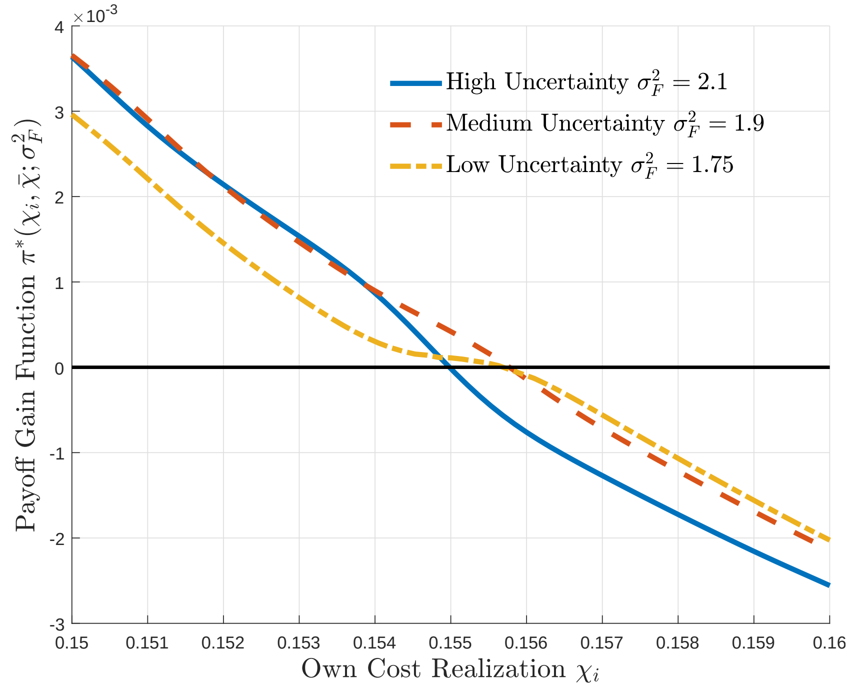
However, note that this property doesn't hold when $\tau_F$ is sufficiently small. This is because with very small uncertainty, the strategic substitutability force would be very strong. This introduces non-monotonicity into the payoff function and could potentially violate the single crossing function. This can be seen in Figure B.1, when fundamental uncertainty is relatively low (dash-dotted yellow line), the payoff function starts curving towards the 0 axis. In this case, the payoff function is still monotonic. When $\tau_F$ increases further, this curvature would be strong enough to break the monotonicity property and lead to multiple crossings between the payoff function and zero axis.

One may wonder why monotone equilibria is still valid when substitutability presents. This is because in this numerical example we introduce strictly positive level of cost heterogeneity $\sigma = 0.0006$. The heterogeneity turns out helps in establishing the existence and uniqueness of monotone equilibrium, even in the absence of strategic complementarity (Mason and Valentinyi, 2010). In fact, one could prove the existence and uniqueness of a monotone equilibrium when: (1) there is no imperfect information, and (2) when there is substantial cost heterogeneity in the cost distribution.

## B.1 A Model with Cost Heterogeneity and Perfect Information

In this section we evaluate the equilibrium refinement of cost heterogeneity, but with perfect information in the generic model of information acquisition as in Section 2. It is again assumed

Figure B.1: Payoff Gain as function of own cost realization




that agents' information costs are drawn from the distribution

$$\chi_i = \bar{\chi} + \sigma\varepsilon_i.$$

But unlike global game where we assume that $\bar{\chi}$ is private information at the information game stage, it is now assumed that $\bar{\chi}$ is public observable to all investors. All other aspects of the model is the same as in Section 4. Note that now there is no imperfect information regarding the cost distribution. Therefore investors can perfect predicts what future $\lambda$ is when making information acquisition decision. Thus the payoff function is just $\pi(\lambda, \tau, \chi)$ without the expectations taken over $\lambda$.

We first define an equilibrium in this environment. Similar to a global game setting, we will be looking for a cutoff equilibrium $\hat{\chi}$ such that agents acquire information if and only if its cost is below $\hat{\chi}$. Given that $\bar{\chi}$ is public information, in such an equilibrium the share of informed investors is $F_\varepsilon\left(\frac{\hat{\chi}-\bar{\chi}}{\sigma}\right)$. Thus, the payoff to an investor of cost $\chi$ is $\pi\left(F_\varepsilon\left(\frac{\hat{\chi}-\bar{\chi}}{\sigma}\right), \tau, \chi\right)$.

The equilibrium is pinned down by the condition that the threshold investor of type $\hat{\chi}$ has payoff equal to zero:

$$\pi\left(F_\varepsilon\left(\frac{\hat{\chi}-\bar{\chi}}{\sigma}\right), \tau, \hat{\chi}\right) = 0. \tag{B.1}$$

Next we prove the main existence and uniqueness theorem. The key observation is that equilibrium would be unique if there is sufficient heterogeneity, namely when $\sigma$ is sufficiently large.

**Theorem B.1.** *Suppose the cost heterogeneity is sufficiently large $\sigma \to \infty$. Then there exists a unique equilibrium in which investors acquire information if and only if their information cost is below some $\hat{\chi}$.*

This theorem can be seen as follows. As $\sigma \to \infty$, the equilibrium share of informed given any finite $\hat{\chi}$ is given by:

$$\lambda = F_{\varepsilon}\left(\frac{\hat{\chi} - \bar{\chi}}{\sigma}\right) \to F(0) = \frac{1}{2}.$$

This implies that the payoff gain function

$$\pi\left(F_{\varepsilon}\left(\frac{\hat{\chi} - \bar{\chi}}{\sigma}\right), \tau, \hat{\chi}\right) \to \pi\left(\frac{1}{2}, \tau, \hat{\chi}\right).$$

Given that the payoff function $\pi$ is monotonically decreasing in the information cost $\chi$, is positive when $\chi \to -\infty$, and is negative when $\chi \to +\infty$, there exists a unique finite value of the cutoff $\bar{\chi}$ such that

$$\pi\left(\frac{1}{2}, \tau, \hat{\chi}\right) = 0.$$

This proves the existence of a unique equilibrium in which investors' action takes the cutoff form when there exists sufficient heterogeneity.

One way to intuitively see this theorem is to notice that when heterogeneity is sufficiently strong, strategic interaction is sufficiently weak, as the share of informed $\lambda$ moves very little. In the absence of the strategic interaction, a cutoff equilibrium would obtain because everyone's action only depend on his own cost realization. See Mason and Valentinyi (2010) for details of this discussion.

We next explore the comparative statics in this setting. Through implicit differentiation of equation B.1:

$$\left(\frac{\partial \pi}{\partial \lambda} f\left(\frac{\hat{\chi} - \bar{\chi}}{\sigma}\right)\frac{1}{\sigma} + \frac{\partial \pi}{\partial \chi}\right) d\hat{\chi} + \frac{\partial \pi}{\partial \tau} d\tau = 0.$$

Thus, changes in the aggregate state $\tau$ will have an impact on the cutoff $\hat{\chi}$ through:

$$\frac{d\hat{\chi}}{d\tau} = -\frac{\frac{\partial \pi}{\partial \tau}}{\frac{\partial \pi}{\partial \lambda} f\left(\frac{\hat{\chi} - \bar{\chi}}{\sigma}\right)\frac{1}{\sigma} + \frac{\partial \pi}{\partial \chi}}.$$

Note that, when $\sigma \to \infty$, the first term in the denominator $\frac{\partial \pi}{\partial \lambda} f\left(\frac{\hat{\chi} - \bar{\chi}}{\sigma}\right)\frac{1}{\sigma} \to 0$ given that the derivative $\frac{\partial \pi}{\partial \lambda}$ and the density functions are all bounded. This implies that as $\sigma \to \infty$:

$$\frac{d\hat{\chi}}{d\tau} \to -\frac{\frac{\partial \pi}{\partial \tau}\left(F_\varepsilon\left(\frac{\hat{\chi}-\bar{\chi}}{\sigma}\right), \tau, \hat{\chi}\right)}{\frac{\partial \pi}{\partial \chi}\left(F_\varepsilon\left(\frac{\hat{\chi}-\bar{\chi}}{\sigma}\right), \tau, \hat{\chi}\right)}.$$

Note that, similar to the global game setting, there is no strategic effect $\frac{\partial \pi}{\partial \lambda}$ in this term. Whether public information $\tau$ crowds in or crowds out $\lambda$ only depend on the fundamental effect $\frac{\partial \pi}{\partial \tau}$, as the denominator $\frac{\partial \pi}{\partial \chi}\left(F_\varepsilon\left(\frac{\hat{\chi}-\bar{\chi}}{\sigma}\right), \tau, \hat{\chi}\right)$ is always negative. Now, given that the fundamental effect is always positive with sufficiently high fundamental uncertainty, the crowding-in result still holds in this environment. We summarize this argument into the following theorem.

**Theorem B.2.** *Suppose the cost heterogeneity is sufficiently large ($\sigma \to \infty$) and fundamental uncertainty is sufficiently high ($\tau_F \to 0$), public information always raise the payoff to information acquisition $\frac{\partial \pi}{\partial \tau} > 0$. Therefore, public information release always crowds in more private information acquisition, i.e., $\frac{d\hat{\chi}}{d\tau} > 0$.*

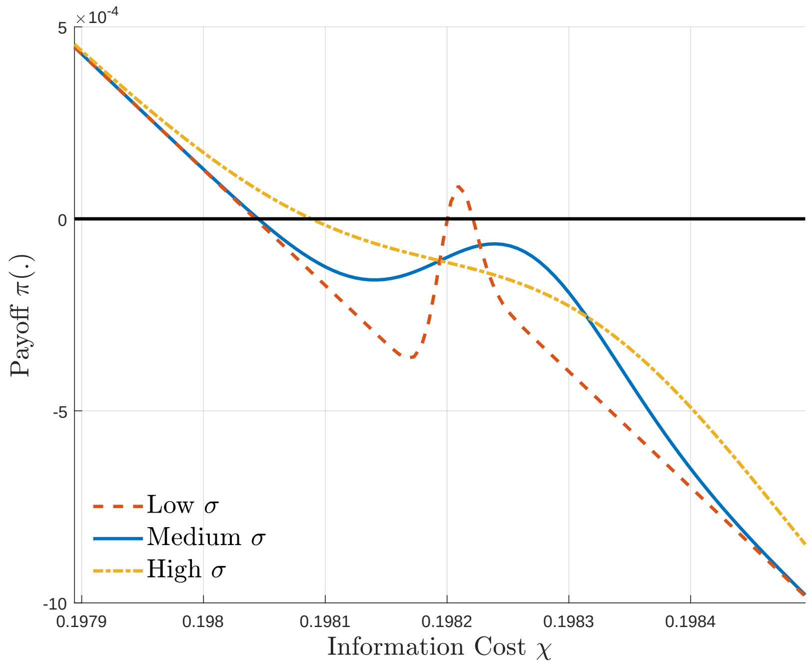
Next we numerically confirm the theoretical results just derived, using parameter values as in the numerical section in the main text. Figure B.2 plots the equilibrium payoff function B.1 as the information cost $\chi$ varies. The intersection between the payoff function and zero horizontal line is an equilibrium. With low heterogeneity (dashed red line), there exists three equilibrium, similar to the homogeneous-information case. With medium and high heterogeneity, the equilibrium function becomes more monotonic and hence admits a unique equilibrium.

Next we explore the impact of information disclosure. Figure B.3 plots the equilibrium payoff function B.1 before and after information disclosure. Both functions are plotted with high cost heterogeneity ($\sigma$ is high), so that a unique equilibrium arises. As shown by the figure, a disclosure shifts the equilibrium $\hat{\chi}$ to the right, implying that more information disclosure raises the threshold under which investors would acquire information, therefore increases private information production. Hence, this figure confirms the result as in Theorem B.2.

# C Robustness and Empirical Relevance

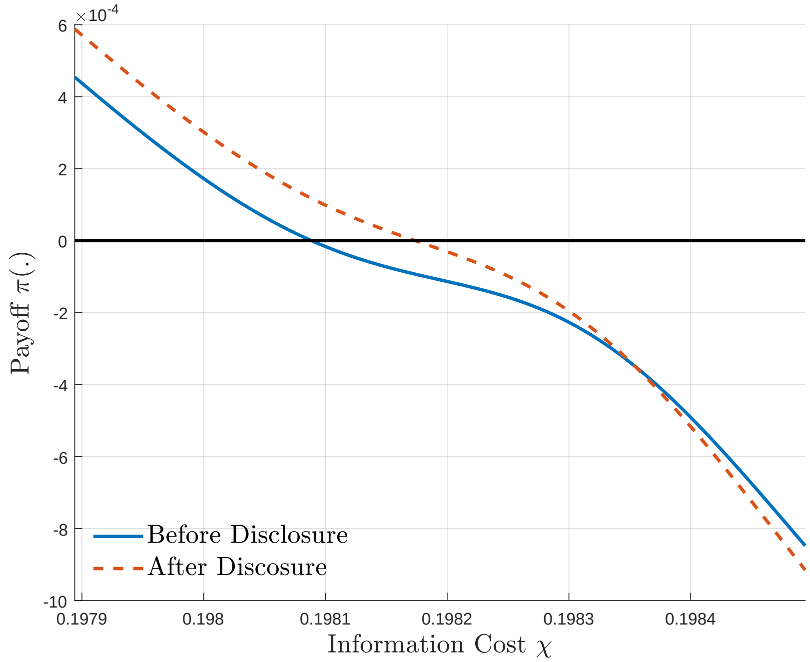
This section discuss the robustness of the mechanism as well as its empirical predictions. The numerical analysis is by no means a fully quantitative evaluation of the implications of information complementarity, but we can still check how prevalent crowding in would be under various parameter combinations. Is the crowding-in result a knife-edge case that only arises with extremely high uncertainty, or it could arise with a larger parametric space? How would this region vary with different parameters?

Figure B.2: Payoff Gain with Cost Heterogeneity




**Notes**: Payoff function with cost heterogeneity and perfect information. $\rho = 0.99$, $\tau_\phi = \tau_F/(1-\rho^2)$, $\tau_D = 2$, $\tau_{x1} = \tau_{x2} = 0.51$, $R = 1.01$, $\alpha = 1$, $\bar{\chi} = 0.1982$, $\tau_F = 0.68$, $\sigma = 0.00002$ (Low $\sigma$), $\sigma = 0.0001$ (Medium $\sigma$), $\sigma = 0.0002$ (High $\sigma$).

Figure B.3: Payoff Gain with Cost Heterogeneity




**Notes**: Information disclosure affects equilibrium share of informed investors in the case of cost heterogeneity but no private information. $\rho = 0.99, \tau_\phi = \tau_F/(1-\rho^2), \tau_D = 2, \tau_{x1} = \tau_{x2} = 0.51, R = 1.01, \alpha = 1, \bar{\chi} = 0.1982, \sigma = 0.0002, \tau_F = 0.68$(Before Disclosure)$, \tau_F = 0.69$(After Disclosure).

This robustness check is displayed in Figure C.1, where we plot the crowding-in region against various model parameters including the supply shock precision $\tau_x$, dividend noise precision $\tau_D$, the fundamental persistence $\rho$, and the risk-averse parameter $\alpha$. The top-left panel plots the how variations in the supply volatility $\tau_x$ affect the crowding-in region (in blue). It is shown that when the supply shock is less volatile ($\tau_x$ is bigger), crowding in is less likely to arise. This can be understood by inspecting the return uncertainty faced by informed investors. We know that

$$Var\left(Q_1|\Omega_1^I\right) = Var\left(D_2 + a(\theta_1)\, S_{P1} + b(\theta_1)\, S - c(\theta_1)\, x_2 + d(\theta_1)\, D_2|\Omega_1^I\right).$$

The informed investors perfectly observe the stock fundamental, along with other public information. Thus their residual uncertainty comes solely from dividend noise and future supply shock, augmented with some equilibrium coefficients:

$$Var\left(Q_1|\Omega_1^I\right) = [1 + d(\theta_1)]^2 \frac{1}{\tau_D} + [c(\theta_1)]^2 \frac{1}{\tau_x}.$$

From the previous analysis we know that the key mechanism of complementarity works through the endogenous coefficient on the supply shock: $c(\theta_1)$. Thus, the supply precision $\tau_x$ serves as a weight on how much this complementarity channel affects the return uncertainty faced by informed investors and hence their incentive to trade and acquire information. A larger value of $\tau_x$ reduces the relative importance of trading risks and therefore, crowding in is less likely to arise.

By similar reasoning, an increase in dividend volatility also reduces the relative importance of future trading risks in affecting the uncertainty faced by informed investors, thereby reducing the incidence of crowding in. This is confirmed in the top-right panel, where it is shown that larger values of $\tau_D$ increase the crowding-in region.

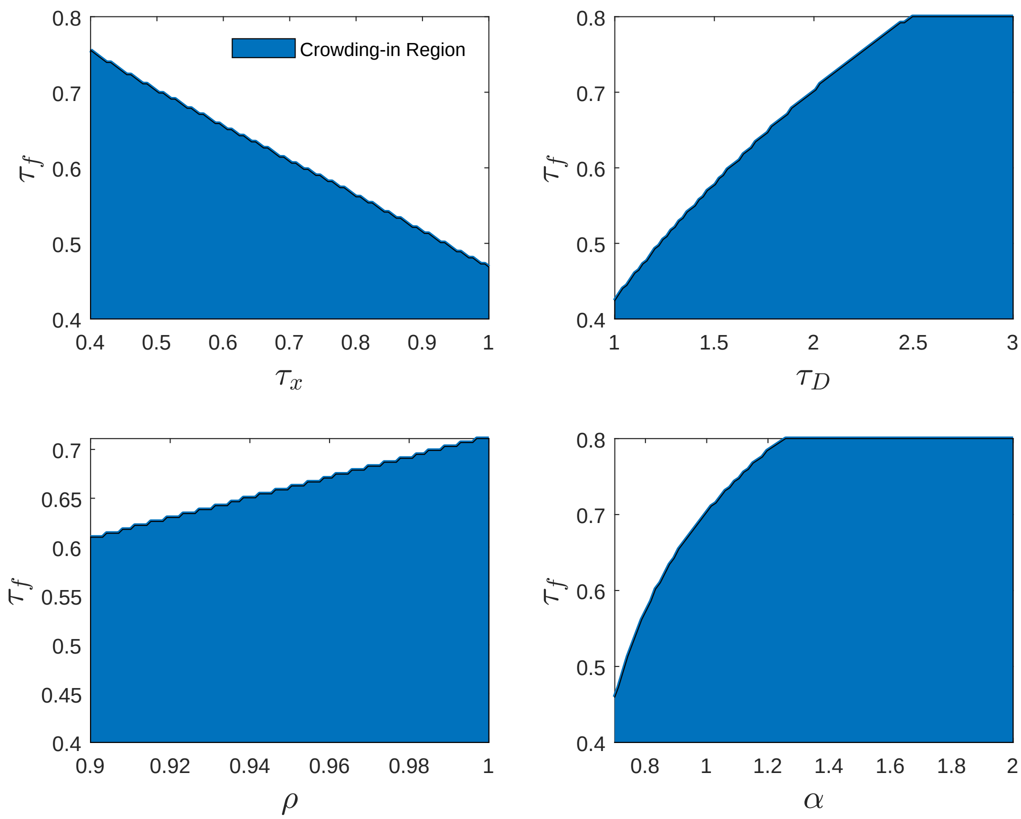
The bottom-left panel reveals that a less persistent stock fundamental (lower $\rho$) reduces the incidence of complementarity. This is expected because the dynamic feedback channel requires that the future resale stock price reflect the current stock fundamental. In the bottom-right panel we explore how changes in investors risk aversion affect the crowding-in region. We find that lower risk aversion reduces the crowding-in region. This is due to the feedback of uncertainty to investors' incentive to acquire information. With lower risk aversion, investors' incentive to trade does not vary as much with change in stock payoff uncertainty. As a result, the value of information is less affected by future trading risks, making the dynamic complementarity weaker.

Lastly, we conclude that the crowding-in result is robust in all the numerical experiments and does not seems to be a knife-edge case. Thus, the crowding-in channel warrants further investigation of a fully quantitative model of financial markets and information acquisition.

We now explore empirical predictions of the model, and contrast them to the standard model of Grossman and Stiglitz (1980). Over the past decades, improvements in information technology have perhaps led to substantial improvements in the investors' ability to collect information, or a reduction in the cost of information acquisition. We thus first explore the impact of a reduction in the (mean) information cost $\bar{\chi}$. In the unique equilibrium of Grossman and Stiglitz (1980) and also in the good information equilibrium of this model, lower information cost $\bar{\chi}$ naturally leads to more informed investors, hence greater price informativeness. The global-game equilibrium also generates similar realistic predictions. As can be seen from equation 3, the equilibrium cutoff associated with the global-game equilibrium is independent of $\bar{\chi}$. Hence, as $\bar{\chi}$ decreases, share of informed investors increases, leading to greater stock price informativeness.

Our model also generates different predictions from Grossman and Stiglitz (1980). A salient trend of financial market in the past decades is the decline in retail trading (Stambaugh, 2014). This can be modeled as a reduction in noise trading volatility. In Grossman and Stiglitz (1980), the decrease in noise trading is exactly offset by less informed trading, resulting in no change in

Figure C.1: Robustness of Global-game Crowding In




**Notes**: Robustness check on how changing model parameters affect the crowding-in result. Benchmark parameters (unless varied in the experiment) $\tau_D = 2, \tau_{x1} = \tau_{x2} = 0.5, R = 1.01, \alpha = 1, \bar{\chi} = 0.195, \sigma = 0.0001$.

equilibrium price informativeness. In this model, however, due to the dynamic complementarity component in the value of information (Equation A.1), a decrease in noise trading reduces the noise in the future resale stock price. This channel encourages more informed trading and hence higher value of information and greater price informativeness. This is consistent with the time-series evidence in Bai et al. (2016). By similar reasoning, in the cross-section our model predicts that more liquid stocks (in the sense that its price is less subject to noise trader's impact, see Kyle (1985)) should attract more analysts' coverage, hence leading to greater price informativeness. This is consistent with cross-sectional tests in Bai et al. (2016) and predictions from Farboodi et al. (2020) that stock prices of larger, more liquid firms tend to be more informative.

# D Extension on Persistent Supply Shocks

In this section we consider an environment where the supply shock is serially correlated. There are two points we want to make. First, a serially correlated stock supply weakens strategic complementarity. In the sense that the value of information is less likely to be increasing in $\lambda$ the share of informed investors. This replicates the existing finding in the literature as in Avdis (2016). Second, we argue that the value of information can still be increasing in the amount of public information provided, even when stock supply is serially correlated. Hence the crowding-in result still holds under persistent stock supply, whenever a global game equilibrium exists.

Specifically we assume that:

1. $x_1$ is normally distributed with precision $\tau_{x1}$;
2. $x_2 = \rho_x x_1 + \varepsilon_{x2}$, where $1 > \rho_x \geq 0$ is the persistence parameter and $\varepsilon_{x2}$ is normally distributed with precision $\tau_{x2}$.

Note that when $\rho_x = 0$, it collapses to our benchmark model.

At day 2, a continuum of investors are born, and they observe the first period price signal

$$S_{p1} = F - \frac{1}{\theta_1} x_1.$$

The dividend payment at the end of the period $D_1$ and the public signal $S$. All of those signals can help to predict the future dividend $D_2 = F + \varepsilon_2^D$.

Thus the posterior variance of $F$ is given by:

$$Var\left(F|S, D_1, S_{p1}\right) = \frac{1}{\tau_F + \theta_1^2 \tau_{x1} + \tau_D}.$$

And the posterior mean is

$$E\left(F|S_1, D_1, S_{p1}\right) = \frac{\tau_F}{\tau_F + \theta_1^2\tau_{x1} + \tau_D} S_1 + \frac{\theta_1^2\tau_{x1}}{\tau_F + \theta_1^2\tau_{x1} + \tau_D} S_{p1} + \frac{\tau_D}{\tau_F + \theta_1^2\tau_{x1} + \tau_D} D_1.$$

Thus, the second period stock demand is

$$\begin{aligned}
\frac{E\left(D_2|S_1, D_1, S_{p1}\right) - Rp_2}{\alpha Var\left(D_2 S_1, D_1, S_{p1}\right)} &= \frac{E\left(D_2|S_1, D_1, S_{p1}\right) - Rp_2}{\alpha Var\left(D_2|S_1, D_1, S_{p1}\right)} = \frac{E\left(F + \varepsilon_2^D|S_1, D_1, S_{p1}\right) - Rp_2}{\alpha Var\left(F + \varepsilon_2^D|S_1, D_1, S_{p1}\right)} \\
&= \frac{\left[\frac{\tau_F}{\tau_F+\theta_1^2\tau_{x1}+\tau_D} S_1 + \frac{\theta_1^2\tau_{x1}}{\tau_F+\theta_1^2\tau_{x1}+\tau_D} S_{p1} + \frac{\tau_D}{\tau_F+\theta_1^2\tau_{x1}+\tau_D} D_1\right] - Rp_2}{\alpha\left[\frac{1}{\tau_F+\theta_1^2\tau_{x1}+\tau_D} + \frac{1}{\tau_D}\right]}.
\end{aligned}$$

Using the market clearing condition, we have an expression for second period resale stock price:

$$p_2 = aS_1 + bS_{p1} - cx_2 + dD_1,$$

where

$$\begin{aligned}
a &= \frac{1}{R}\frac{\tau_F}{\tau_F + \theta_1^2\tau_x + \tau_D}, \\
b &= \frac{1}{R}\frac{\theta_1^2\tau_x}{\tau_F + \theta_1^2\tau_x + \tau_D}, \\
c &= \frac{1}{R}\alpha\left(\frac{1}{\tau_F + \theta_1^2\tau_x + \tau_D} + \frac{1}{\tau_D}\right), \\
d &= \frac{1}{R}\frac{\tau_D}{\tau_F + \theta_1^2\tau_x + \tau_D}.
\end{aligned}$$

Thus, the excess stock return is

$$\begin{aligned}
Q &= D_1 + p_2 = D_1 + aS_1 + bS_{p1} - cx_2 + dD_1 \\
&= (1+d)\, D_1 + aS_1 + bS_{p1} - cx_2 \\
&= (1+d)\left(F + \varepsilon_1^D\right) + aS_1 + bS_{p1} - c\left(\rho_x x_1 + \varepsilon_2^x\right).
\end{aligned}$$

Turning into the first period. The first-period informed investors observes the price signal, the public signal, and the true fundamental. Hence the conditional expectation is

$$\begin{aligned} Var\left(Q|F,S_1,S_{p1}\right) &= (1+d)^2\frac{1}{\tau_D}+c^2\frac{1}{\tau_{x2}}+Var((1+d)\,F-c\rho_x x_1|F,S_1,S_{p1}) \\ &= (1+d)^2\frac{1}{\tau_D}+c^2\frac{1}{\tau_{x2}}. \end{aligned}$$

Consider the problem of uninformed investors:

$$Var\left(Q|S_1,S_{p1}\right)=(1+d)^2\frac{1}{\tau_D}+c^2\frac{1}{\tau_{x2}}+Var((1+d)\,F-c\rho_x x_1|S_1,S_{p1}).$$

Note

$$Var((1+d)\,F-c\rho_x x_1|S_1)=(1+d)^2\frac{1}{\tau_F}+c^2\rho_x^2\frac{1}{\tau_{x1}}.$$

Meanwhile, we have

$$\begin{aligned} & Cov\left((1+d)\,F-c\rho_x x_1,S_{p1}|S_1\right) \\ =\; & Cov\left((1+d)\,F-c\rho_x x_1,F-\frac{1}{\theta}x_1|S_1\right) \\ =\; & (1+d)\frac{1}{\tau_F}+\frac{c\rho_x}{\theta}\frac{1}{\tau_{x1}}, \end{aligned}$$

and

$$Var\left(S_{p1}|S_1\right)=\frac{1}{\tau_F}+\frac{1}{\theta^2\tau_{x1}}.$$

Thus, due to the Projection Theorem of jointly normal variables:

$$\begin{aligned} & Var((1+d)\,F-c\rho_x x_1|S_1,S_{p1}) \\ =\; & (1+d)^2\frac{1}{\tau_F}+c^2\rho_x^2\frac{1}{\tau_{x1}}-\frac{\left((1+d)\,\frac{1}{\tau_F}+\frac{c\rho_x}{\theta}\frac{1}{\tau_{x1}}\right)^2}{\frac{1}{\tau_F}+\frac{1}{\theta^2\tau_{x1}}} \\ =\; & \frac{\left((1+d)^2\frac{1}{\tau_F}+c^2\rho_x^2\frac{1}{\tau_{x1}}\right)\left(\frac{1}{\tau_F}+\frac{1}{\theta^2\tau_{x1}}\right)-\left((1+d)\,\frac{1}{\tau_F}+\frac{c\rho_x}{\theta}\frac{1}{\tau_{x1}}\right)^2}{\frac{1}{\tau_F}+\frac{1}{\theta^2\tau_{x1}}} \\ =\; & \frac{\begin{array}{c}(1+d)^2\left(\frac{1}{\tau_F}\right)^2+(1+d)^2\frac{1}{\tau_F}\frac{1}{\theta^2\tau_{x1}}+c^2\rho_x^2\frac{1}{\tau_{x1}}\frac{1}{\tau_F}+c^2\rho_x^2\frac{1}{\tau_{x1}}\frac{1}{\theta^2\tau_{x1}} \\ -\left(\left((1+d)\,\frac{1}{\tau_F}\right)^2+\left(\frac{c\rho_x}{\theta}\frac{1}{\tau_{x1}}\right)^2+2\,(1+d)\,\frac{1}{\tau_F}\frac{c\rho_x}{\theta}\frac{1}{\tau_{x1}}\right)\end{array}}{\frac{1}{\tau_F}+\frac{1}{\theta^2\tau_{x1}}} \\ =\; & \frac{\left((1+d)\,\frac{1}{\theta}-c\rho_x\right)^2}{\frac{1}{\tau_F}+\frac{1}{\theta^2\tau_{x1}}}\left(\frac{1}{\tau_{x1}}\frac{1}{\tau_F}\right). \end{aligned}$$

Hence

$$Var\left(Q|S_1, S_{p1}\right) = (1+d)^2 \frac{1}{\tau_D} + c^2 \frac{1}{\tau_{x2}} + (1+d)^2 \frac{1}{\tau_F} + c^2 \rho_x^2 \frac{1}{\tau_{x1}} - \frac{\left((1+d)\frac{1}{\tau_F} + \frac{c\rho_x}{\theta}\frac{1}{\tau_{x1}}\right)^2}{\frac{1}{\tau_F} + \frac{1}{\theta^2 \tau_{x1}}}.$$

Given the expressions of $Var\left(Q|S_1, S_{p1}\right)$ and $Var\left(Q|F, S_1, S_{p1}\right)$, we can derive the value of information

$$\begin{aligned} V &= \frac{Var\left(Q|S_1, S_{p1}\right)}{Var\left(Q|F, S_1, S_{p1}\right)} \\ &= 1 + \frac{\frac{(1+d-c\rho_x\theta)^2}{\tau_F + \theta^2 \tau_{x1}}}{(1+d)^2 \frac{1}{\tau_D} + c^2 \frac{1}{\tau_{x2}}}. \end{aligned}$$

Hence we arrive at the following value of information expression:

$$V = 1 + \underbrace{\left(1 - \frac{c}{1+d}\rho_x \theta\right)^2}_{\text{Extra Term}} \frac{\frac{1}{\tau_F + \theta^2 \tau_{x1}}}{\frac{1}{\tau_D} + \left(\frac{c}{1+d}\right)^2 \frac{1}{\tau_{x2}}}.$$

Compared to the value of information expression in the benchmark model, there is an extra term

$$\left(1 - \frac{c}{1+d}\rho_x \theta\right)^2 \tag{D.1}$$

when $\rho_x = 0$, this term becomes 1 and we obtain the original value of information expression as in proposition A.2.

The first point we want to make is that more persistent stock supply (a larger value of $\rho_x$) weakens the dynamic complementarity. Note that the complementarity works because coefficient $\frac{c}{1+d}$ decreases when $\theta$ increases. In this case there is an extra force $1 - \frac{c}{1+d}\rho_x\theta$, which may change with $\theta$. Expanding this expression yields

$$1 - \frac{\frac{1}{R}\alpha\left(\frac{1}{\tau_F + \theta_1^2 \tau_x + \tau_D} + \frac{1}{\tau_D}\right)}{1 + \frac{1}{R}\frac{\tau_D}{\tau_F + \theta_1^2 \tau_x + \tau_D}} \rho_x \theta = 1 - \frac{\alpha\left(1 + \frac{\tau_F + \theta_1^2 \tau_x + \tau_D}{\tau_D}\right)\theta}{R\left(\tau_F + \theta_1^2 \tau_x + \tau_D\right) + \tau_D} \rho_x.$$

Take derivative with respect to $\theta$:

$$
\begin{aligned}
& -\frac{\begin{array}{c}\left[\alpha\left(1+\frac{\tau_F+\theta_1^2\tau_x+\tau_D}{\tau_D}\right)+\alpha\left(2\frac{\theta_1\tau_x}{\tau_D}\right)\theta_1\right]\left(R\left(\tau_F+\theta_1^2\tau_x+\tau_D\right)+\tau_D\right)\\ -\alpha\left(1+\frac{\tau_F+\theta_1^2\tau_x+\tau_D}{\tau_D}\right)\theta\left(2R\theta\tau_x\right)\end{array}}{\left(R\left(\tau_F+\theta_1^2\tau_x+\tau_D\right)+\tau_D\right)^2}\rho_x \\
= \; & -\alpha\frac{\left(1+\frac{\tau_F+\theta_1^2\tau_x+\tau_D}{\tau_D}\right)\left(R\tau_F+R\tau_D+\tau_D\right)+\theta_1^2\tau_x R\left(\frac{\left(\tau_F+\theta_1^2\tau_x+\tau_D\right)}{\tau_D}+1\right)}{\left(R\left(\tau_F+\theta_1^2\tau_x+\tau_D\right)+\tau_D\right)^2}\rho_x \\
< \; & 0.
\end{aligned}
$$

Hence when stock supply is persistent, this extra term tends to reduce the derivative of $\frac{dV}{d\theta}$, hence it weakens the complementarity. Thus, when $\rho_x$ is sufficiently large, the value of information would not be increasing in $\theta$: substitutability always dominates. This replicates the finding as in Avdis (2016).

On the other hand, note that crowding-in can still arise when stock supply is persistent. To see this, note that when we take derivative with respect to $\tau_F$, this extra term D.1 increases because $\frac{c}{1+d}$ decreases with $\tau_F$, and $\theta$ is unchanged as it captures private information provision. To see $\frac{c}{1+d}$ decreases with $\tau_F$:

$$
\begin{aligned}
& \left(\frac{\frac{1}{R}\alpha\left(\frac{1}{\tau_F+\theta_1^2\tau_x+\tau_D}+\frac{1}{\tau_D}\right)}{1+\frac{1}{R}\frac{\tau_D}{\tau_F+\theta_1^2\tau_x+\tau_D}}\right)' \\
= \; & \frac{\alpha\left(R\frac{\left(\tau_F+\theta_1^2\tau_x+\tau_D\right)}{\tau_D}+1\right)-\alpha\left(R+R\frac{\tau_F+\theta_1^2\tau_x+\tau_D}{\tau_D}\right)}{\left(R\left(\tau_F+\theta_1^2\tau_x+\tau_D\right)+\tau_D\right)^2} \\
= \; & \frac{\alpha\left(1-R\right)}{\left(R\left(\tau_F+\theta_1^2\tau_x+\tau_D\right)+\tau_D\right)^2}<0.
\end{aligned}
$$

given that the gross interest rate $R > 1$. Thus, when $\rho_x$ is persistent it is in fact more likely that the value of information increases with $\tau_F$. And by the property of the global game equilibrium we know that when the payoff function increases with $\tau_F$, the crowding-in effect arises.

Thus, it is crucial to empirically identify the degree of stock supply persistence. Peress and Schmidt (2021) find that this persistence parameter can be large under high frequency settings, but tends to be small at monthly or lower frequency. In our numerical simulation, we set the per-period interest rate to be 1% intending to capture relatively lower (*i.e.*, quarterly) frequency movements in asset prices and investor behaviors. Thus, we view our benchmark assumption (*i.i.d.* stock supply shock) to be a reasonable one.

# Online Appendix

## A Additional Proofs

**Lemma A.1.** *The second-period stock price function $P_2$ is a linear combination of the price signal $S_{P1}$, the public signal $S$, and the noisy stock supply $x_2$, where the coefficients $(a, b, c, d)$ are function of $\theta_1$ and are strictly positive:*

$$P_2 = aS_{P1} + bS - cx_2 + dD_1.$$

**Proof of Lemma A.1:** At day 2, a continuum of investors are born, and they observe the first period price signal

$$p_1 = \theta_1 F - x_1.$$

This prior is the same as period-1 investors. The dividend is

$$D_2 = F + \varepsilon_2^D.$$

where the $\varepsilon_2^D$ follows $N\left(0, \sigma_D^2\right)$ , Thus the posterior precision of $F$ is

$$\tau_{F2} = Var\left(F|S, D_1, S_{p1}\right) = \tau_F + \theta_1^2\tau_x + \tau_D.$$

As the investors also observe dividend payments $D_1$. as well as the first period public signal $S = F + \varepsilon_F$. Thus the posterior variance of $F$ after observing all available information, is:

$$Var\left(F|S, D_1, S_{p1}\right) = \frac{1}{\tau_F + \theta_1^2\tau_x + \tau_D}.$$

And the posterior mean is

$$E\left(F|S_1, D_1, S_{p1}\right) = \frac{\tau_F}{\tau_F + \theta_1^2\tau_x + \tau_D}S_1 + \frac{\theta_1^2\tau_x}{\tau_F + \theta_1^2\tau_x + \tau_D}S_{p1} + \frac{\tau_D}{\tau_F + \theta_1^2\tau_x + \tau_D}D_1.$$

Thus, the demand is

$$\begin{aligned}\frac{E\left(D_2|S_1, D_1, S_{p1}\right) - Rp_2}{\alpha Var\left(D_2 S_1, D_1, S_{p1}\right)} &= \frac{E\left(D_2|S_1, D_1, S_{p1}\right) - Rp_2}{\alpha Var\left(D_2|S_1, D_1, S_{p1}\right)} = \frac{E\left(F + \varepsilon_2^D|S_1, D_1, S_{p1}\right) - Rp_2}{\alpha Var\left(F + \varepsilon_2^D|S_1, D_1, S_{p1}\right)} \\ &= \frac{\left[\frac{\tau_F}{\tau_F+\theta_1^2\tau_x+\tau_D}S_1 + \frac{\theta_1^2\tau_x}{\tau_F+\theta_1^2\tau_x+\tau_D}S_{p1} + \frac{\tau_D}{\tau_F+\theta_1^2\tau_x+\tau_D}D_1\right] - Rp_2}{\alpha\left[\frac{1}{\tau_F+\theta_1^2\tau_x+\tau_D} + \frac{1}{\tau_D}\right]}.\end{aligned}$$

The second period price $p_2$ is given by

$$\frac{E\left(D_2|S_1, D_1\right) - Rp_2}{\alpha Var\left(D_2|S_1, D_1\right)} = x_2.$$

Thus

$$\frac{\left[\frac{\tau_F}{\tau_F+\theta_1^2\tau_x+\tau_D}S_1 + \frac{\theta_1^2\tau_x}{\tau_F+\theta_1^2\tau_x+\tau_D}S_{p1} + \frac{\tau_D}{\tau_F+\theta_1^2\tau_x+\tau_D}D_1\right] - Rp_2}{\alpha\left[\frac{1}{\tau_F+\theta_1^2\tau_x+\tau_D} + \frac{1}{\tau_D}\right]} = x_2.$$

Collect terms:

$$p_2 = aS_1 + bS_{p1} - cx_2 + dD_1,$$

where

$$\begin{aligned}
a &= \frac{1}{R}\frac{\tau_F}{\tau_F + \theta_1^2\tau_x + \tau_D}, \\
b &= \frac{1}{R}\frac{\theta_1^2\tau_x}{\tau_F + \theta_1^2\tau_x + \tau_D}, \\
c &= \frac{1}{R}\alpha\left(\frac{1}{\tau_F + \theta_1^2\tau_x + \tau_D} + \frac{1}{\tau_D}\right), \\
d &= \frac{1}{R}\frac{\tau_D}{\tau_F + \theta_1^2\tau_x + \tau_D}.
\end{aligned}$$

**Lemma A.2.** *The value of information $V$ is given by:*

$$V = \frac{Var(Q_1|\Omega_1^U)}{Var(Q_1|\Omega_1^I)} = 1 + \frac{\frac{1}{\Gamma(F|S_{P1},S)}}{\frac{1}{\tau_D} + \frac{1}{\tau_{x2}}\left(\frac{c}{1+d}\right)^2}, \tag{A.1}$$

*where $\Gamma(F|S_{P1}, S)$ is the precision of stock fundamental given the price and public signal, and is given by equation 12. $c$ and $d$ are the price coefficients of the resale stock price $P_2$ with respect to $x_2$ and $D_1$ respectively. The ratio $\frac{c}{1+d}$ is a decreasing function of the fundamental precision $\Gamma(F|S_{P1}, S)$.*

**Proof:** This proposition is an extension of Theorem 2 in Grossman and Stiglitz (1980) into a multi-period framework. First define expected utility of agents net information cost $\hat{W}^i$. Plug agents' budget constraint: $c_t = (D_t + P_{t+1} - RP_t)s$ into the utility function, we obtain the expected utility of each type of agent conditional on the realized market price $P_t$:

$$\hat{J}^i = \max_s EU((Q_1 - RP_1)s|\Omega_t^i).$$

Given CARA utility and normally distributed random variables:

$$\begin{aligned}
\hat{J}^i &= \max_s EU((Q_1 - RP_1)s|\Omega_t^i) \\
&= \max_s EU(-\exp\left(-(D_1 + P_2 - RP_1)s\right)|\Omega_t^i) \\
&= \max_s -\exp[-\alpha(E[D_1 + P_2 - RP_1|\Omega_t^i]s - \frac{1}{2}\alpha s^2 Var(D_2 + P_2 - RP_1))].
\end{aligned}$$

Hence, maximizing over the objective function is equivalent to maximizing

$$\max_s E[D_1 + P_2 - RP_1|\Omega_1^i]s - \frac{1}{2}\alpha s^2 Var(D_1 + P_2 - RP_1|\Omega_1^i).$$

Solving for optimal $s^*$ yields

$$s^{i*} = \frac{E[D_1 + P_2 - RP_1|\Omega_1^i]}{\alpha Var(\ D_1 + P_2 - RP_1|\Omega_1^i)}.$$

Plugging back into the original objective function:

$$\hat{J}^i = -\exp\left(-\frac{1}{2}\frac{(E[D_1 + P_2|\Omega_1^i] - \ RP_1)^2}{Var(Q_1|\Omega_1^i)}\right).$$

Let

$$h = Var(Q_1|\Omega_1^U) - Var(Q_1|\Omega_1^I) > 0.$$

The reason why $h$ is greater than 0 is that the information set of the uninformed investors is more coarse then that of the informed investors. Taking the ex-ante conditional expectation of the informed $\hat{W}^I(P)$ with respect to the uninformed's information set $\Omega_t^U$:

$$\begin{aligned}
E_{\Omega_t^U}[\hat{J}^I] &= E\left[-\exp\left(-\frac{1}{2}\frac{(E[\ D_1 + P_2|\Omega_1^I] - RP_1)^2}{Var(Q_1|\Omega_1^I)}\right)|\Omega_t^U\right] \\
&= E\left[-\exp\left(-\frac{1}{2}\frac{h}{Var(Q_1|\Omega_1^I}z^2\right)|\Omega_t^U\right],
\end{aligned}$$

where $z = \frac{(E[D_1+P_2|\Omega_t^U]-RP_1)}{\sqrt{h}}$.

Thus, by the moment-generating function of a noncentral chi-squared distribution (formula $A21$ of Grossman and Stiglitz (1980)):

$$
\begin{aligned}
E_{\Omega_t^U}[\hat{J}^I] &= \frac{1}{\sqrt{1+\frac{h}{Var(Q_{t+1}|\Omega_t^I)}}} \exp\left(\frac{-E[z|\Omega_t^U]^2 \frac{1}{2}\frac{h}{Var(Q_{t+1}|\Omega_t^I)}}{1+\frac{h}{Var(Q_{t+1}|\Omega_t^I)}}\right) \\
&= \sqrt{\frac{Var(Q_{t+1}|\Omega_t^I)}{Var(Q_{t+1}|\Omega_t^U)}} \exp\left(\frac{-E[z|\Omega^U]^2 \frac{1}{2}\frac{h}{Var(Q_{t+1}|\Omega_t^I)}}{1+\frac{h}{Var(Q_{t+1}|\Omega_t^I)}}\right) \\
&= \sqrt{\frac{Var(Q_1|\Omega_1^I)}{Var(Q_1|\Omega_1^U)}} \hat{J}^U.
\end{aligned}
$$

Integrating on both sides with respect to $\Omega_t^U$, one gets:

$$
\frac{E_{\Omega_t^U}\left[\hat{J}^I\right]}{E_{\Omega_1^U}\left[\hat{J}^U\right]} = \sqrt{\frac{Var(Q_1|\Omega_1^I)}{Var(Q_1|\Omega_1^U)}}.
$$

Lastly we can integrate the left-hand-side with respect to $\Omega_t^I$, we obtain:

$$
\left(\frac{E_{\Omega_t^I}\left[\hat{J}^I\right]}{E_{\Omega_1^U}\left[\hat{J}^U\right]}\right)^2 = \frac{Var(Q_1|\Omega_1^I)}{Var(Q_1|\Omega_1^U)}.
$$

Next we derive expression of $Var(Q_1|\Omega_1^I)$ and $Var(Q_1|\Omega_1^U)$. Given that

$$
\begin{aligned}
Q_1 &= D_1 + P_2 \\
&= (1+d)\, D_1 + aS_{p1} + bS - cx_2 \\
&= (1+d)\, F + aS_{p1} + bS - cx_2 + (1+d)\, \varepsilon_{D1}.
\end{aligned}
$$

Thus

$$
\begin{aligned}
Var(Q_1|\Omega_1^I) &= (1+d)^2 \frac{1}{\tau_D} + c^2 \frac{1}{\tau_{x2}}, \\
Var(Q_1|\Omega_1^U) &= (1+d)^2 \frac{1}{\tau_F + \theta_1^2 \tau_{x1}} + (1+d)^2 \frac{1}{\tau_D} + c^2 \frac{1}{\tau_{x2}}.
\end{aligned}
$$

Hence

$$
\frac{Var(Q_1|\Omega_1^I)}{Var(Q_1|\Omega_1^U)} = \frac{(1+d)^2 \frac{1}{\tau_F+\theta_1^2\tau_{x1}} + (1+d)^2 \frac{1}{\tau_D} + c^2 \frac{1}{\tau_{x2}}}{(1+d)^2 \frac{1}{\tau_D} + c^2 \frac{1}{\tau_{x2}}} = 1 + \frac{\frac{1}{\tau_F+\theta_1^2\tau_{x1}}}{\frac{1}{\tau_D} + \left[\frac{c}{1+d}\right]^2 \frac{1}{\tau_{x2}}}.
$$

**Lemma A.3.** *When fundamental uncertainty is sufficiently high, the payoff function $\pi$ and the value* *of information function V has the following monotonically increasing relationship:*

$$\pi(\lambda, \tau_F, \chi) \to 1 - \frac{\exp \alpha R \chi}{V}.$$

**Proof of Lemma A.3** Our goal is to derive an expression for the information payoff gain function $\pi(\lambda, \tau_F, \chi)$.

These are the expressions when investors are informed and uninformed, respectively. We start with the general expression for the expected utility

$$U^i = -E \exp\left[-\frac{1}{2}\frac{\left(E\left[Q_1|\Omega^i\right] - RP_1\right)^2}{\alpha Var\left(Q_1|\Omega^i\right)}\right],$$

where $Q_1 = D_1 + aS + bS_{p1} - cx_2 + dD_1 = F + \varepsilon_1^D + aS + bS_{p1} - cx_2 + d\left(F + \varepsilon_1^D\right)$.

Let's start with the case with informed investors. For informed investors:

$$\begin{aligned} E\left(Q_1|\Omega^I\right) &= (1+d)F + aS + bS_{p_1}, \\ Var\left(Q_1|\Omega^I\right) &= (1+d)^2\frac{1}{\tau_D} + c^2\frac{1}{\tau_{x2}}. \end{aligned}$$

where for uninformed investors

$$Var\left(Q_1|\Omega^U\right) = (1+d)^2\frac{1}{\tau_F + \theta_1^2\tau_{x1}} + (1+d)^2\frac{1}{\tau_D} + c^2\frac{1}{\tau_{x2}}.$$

Thus the expected utility for informed investor is given by:

$$U^I = -E \exp\left[-\frac{1}{2}\frac{\left((1+d)F + aS + bS_{p_1} - RP_1\right)^2}{\alpha\left((1+d)^2\frac{1}{\tau_D} + c^2\frac{1}{\tau_{x2}}\right)}\right] * \exp(\alpha R x).$$

Note that $(1+d)F + aS + bS_{p_1} - RP_1$ is normally distributed. Denote this excess stock return by $a$ which distributed with mean $\mu_a$ and $\sigma_a$. In this model the means are normalized to zero, thus the unconditional mean of the excess stock return $\mu_a = 0$. Then $a^2$ is just a non-central chi-squared distribution with degree $k = 1$. Thus, the unconditional $W^I$ is given by

$$U^I = -E \exp\left[-\frac{1}{2}\frac{\sigma_a^2 a^2}{\alpha\left((1+d)^2\frac{1}{\tau_D} + c^2\frac{1}{\tau_{x2}}\right)}\right] = -M_a\left(\frac{-\sigma_a^2}{2\alpha\left((1+d)^2\frac{1}{\tau_D} + c^2\frac{1}{\tau_{x2}}\right)}\right).$$

Given the formula for the moment generating function:

$$M_a(t) = \exp\left(\lambda t / (1-2t)\right)(1-2t)^{-\frac{1}{2}},$$

with some algebraic manipulation we have

$$\begin{aligned} U^I &= -\exp\left(\frac{\mu_a^2\left(\frac{-\sigma_a^2}{2\alpha\left((1+d)^2\frac{1}{\tau_D}+c^2\frac{1}{\tau_{x2}}\right)}\right)}{1-2\frac{-\sigma_a^2}{2\alpha\left((1+d)^2\frac{1}{\tau_D}+c^2\frac{1}{\tau_{x2}}\right)}}\right)\left(1-2\frac{-\sigma_a^2}{2\alpha\left((1+d)^2\frac{1}{\tau_D}+c^2\frac{1}{\tau_{x2}}\right)}\right)^{-\frac{1}{2}} * \exp(\alpha R x) \\ &= -\frac{\exp\left(-\frac{1}{2}\frac{\mu_a^2\sigma_a^2}{\alpha\left((1+d)^2\frac{1}{\tau_D}+c^2\frac{1}{\tau_{x2}}\right)+\sigma_a^2}\right) * \exp(\alpha R x)}{\sqrt{1+\frac{\sigma_a^2}{\alpha\left((1+d)^2\frac{1}{\tau_D}+c^2\frac{1}{\tau_{x2}}\right)}}}. \end{aligned}$$

We know that the uninformed investors' expected utility is given by

$$U^U = \sqrt{\frac{Var^U(Q_1)}{Var^I(Q_1)}}\exp(-\alpha R\chi)\,U^I.$$

Then, with some algebraic manipulation, we can derive the expression for $\pi(\cdot)$ as below:

$$\begin{aligned} \pi(\lambda,\tau_F,\chi) &= U^I - U^U \\ &= U^I\left(1-\sqrt{\frac{Var^U(Q_1)}{Var^I(Q_1)}}\exp(-\alpha R\chi)\right) \\ &= -\frac{\exp(\alpha R x)-\sqrt{\frac{Var^U(Q_1)}{Var^I(Q_1)}}}{\sqrt{1+\frac{Var\left(F+\varepsilon_1^D+aS+bS_{p1}-cx_2+d\left(F+\varepsilon_1^D\right)-RP_1\right)}{\alpha\left((1+d)^2\frac{1}{\tau_D}+c^2\frac{1}{\tau_{x2}}\right)}}}. \end{aligned}$$

Next we derive the expression for $Var\left(F+\varepsilon_1^D+aS+bS_{p1}-cx_2+d\left(F+\varepsilon_1^D\right)-RP_1\right)$, which requires a more complete characterization of the first-period stock price function $P_1$.

To do so, we can organize the stock return as follows: $Q_1 = (1+d)F + aS + bS_{p1} + (1+d)\varepsilon_2^D - cx_2$.Thus the informed expectation of $Q_1 = (1+d)F + aS + bS_{p1}$ while the uninformed expectation is $(1+d)R\left(aS+bS_{p1}\right)+aS+bS_{p1}$. Thus the market clearing condition can be given by

$$\lambda\frac{(1+d)F+aS+bS_{p1}-RP_1}{\alpha V^I}+(1-\lambda)\frac{(1+d)R\left(aS+bS_{p1}\right)+aS+bS_{p1}-RP_1}{\alpha V^U} = x_1.$$

Conjecture that the pricing function $P_1$ take the following form:

$$P_1 = \frac{1}{R}\left[MF - \frac{M}{\theta_1}x_1 + NS + aS + bS_{p1}\right].$$

Moreover, we want to get the expressions for $M$ and $N$ by exploiting the market clearing condition:

$$\lambda\frac{(1+d)F + aS + bS_{p1} - RP_1}{\alpha V^I} + (1-\lambda)\frac{(1+d)\,R\left(aS + b\left(F - \frac{1}{\theta_1}x_1\right)\right) + aS + bS_{p1} - RP_1}{\alpha V^U} = x_1,$$

where for abbreviation we write $V^I = Var\left(Q_1|\Omega^I\right)$ and $V^U = Var\left(Q_1|\Omega^U\right)$. Thus one can obtain the coefficients $M$ and $N$ through coefficient matching:

$$\begin{aligned} M &= (1+d)\frac{\lambda\frac{1}{\alpha V^I} + (1-\lambda)\frac{Rb}{\alpha V^U}}{\lambda\frac{1}{\alpha V^I} + (1-\lambda)\frac{1}{\alpha V^U}} = (1+d)\frac{\lambda\frac{1}{\alpha V^I} + (1-\lambda)\frac{\frac{\theta_1^2\tau_x}{\tau_F + \theta_1^2\tau_x + \tau_D}}{\alpha V^U}}{\lambda\frac{1}{\alpha V^I} + (1-\lambda)\frac{1}{\alpha V^U}}, \\ N &= (1+d)\frac{(1-\lambda)\frac{Ra}{\alpha V^U}}{\lambda\frac{1}{\alpha V^I} + (1-\lambda)\frac{1}{\alpha V^U}} = (1+d)\frac{(1-\lambda)\frac{\frac{\tau_F}{\tau_F + \theta_1^2\tau_x + \tau_D}}{\alpha V^U}}{\lambda\frac{1}{\alpha V^I} + (1-\lambda)\frac{1}{\alpha V^U}}. \end{aligned}$$

Define $F + \varepsilon_1^D + aS + bS_{p1} - cx_2 + d\left(F + \varepsilon_1^D\right) - RP_1 = X$. Then with some algebraic manipulation we have

$$\begin{aligned} Var\,(X) &= Var\left(F + aS + bS_{p1} + dF - \left(MF - \frac{M}{\theta_1}x_1 + NS + aS + bS_{p1}\right)\right) \\ &= M^2\frac{1}{\theta_1^2\tau_{x1}} + N^2\frac{1}{\tau_F}. \end{aligned}$$

Now we take $\tau_F \to 0$, Then

$$M = (1+d)\frac{\lambda\frac{1}{\alpha V^I} + (1-\lambda)\frac{\frac{\theta_1^2\tau_x}{\tau_F + \theta_1^2\tau_x + \tau_D}}{\alpha V^U}}{\lambda\frac{1}{\alpha V^I} + (1-\lambda)\frac{1}{\alpha V^U}} \to (1+d)\frac{\lambda\frac{1}{\alpha V^I} + (1-\lambda)\frac{\frac{\theta_1^2\tau_x}{\theta_1^2\tau_x + \tau_D}}{\alpha V^U}}{\lambda\frac{1}{\alpha V^I} + (1-\lambda)\frac{1}{\alpha V^U}}.$$

We need to check the convergence of $N^2\frac{1}{\tau_F}$:

$$(N)^2\frac{1}{\tau_F} = \left((1+d)\frac{(1-\lambda)\frac{\frac{\tau_F}{\tau_F + \theta_1^2\tau_x + \tau_D}}{\alpha V^U}}{\lambda\frac{1}{\alpha V^I} + (1-\lambda)\frac{1}{\alpha V^U}}\right)^2\frac{1}{\tau_F} \to 0.$$

Thus the entire expression converges to

$$Var\left(X\right) \to M^2 \frac{1}{\theta_1^2 \tau_{x1}} \to \frac{1}{\theta_1^2 \tau_{x1}} \text{ as } \tau_D \to 0.$$

which implies that

$$\pi\left(\chi, \lambda; \tau_F\right) = -\frac{\exp(\alpha R x) - \sqrt{\frac{Var^U(Q_1)}{Var^I(Q_1)}}}{\sqrt{1 + \frac{Var\left(F + aS + bS_{p1} + dF - RP_1\right)}{\alpha\left((1+d)^2 \frac{1}{\tau_D} + c^2 \frac{1}{\tau_{x2}}\right)}}} \to -\frac{\exp(\alpha R x) - \sqrt{\frac{Var^U(Q_1)}{Var^I(Q_1)}}}{\sqrt{1 + \frac{\frac{1}{\theta_1^2 \tau_{x1}}}{\alpha\left((1+d)^2 \frac{1}{\tau_D} + c^2 \frac{1}{\tau_{x2}}\right)}}}.$$

Note that the value of information expression

$$V = 1 + \frac{\frac{1}{\tau_F + \theta_1^2 \tau_{x1}}}{(1+d)^2 \frac{1}{\tau_D} + c^2 \frac{1}{\tau_x}} \to 1 + \frac{\frac{1}{\theta_1^2 \tau_{x1}}}{\alpha\left((1+d)^2 \frac{1}{\tau_D} + c^2 \frac{1}{\tau_{x2}}\right)}.$$

as $\tau_F \to 0$. Thus the utility differential converges to

$$\pi\left(\chi, \lambda; \tau_F\right) \to -\frac{1}{\sqrt{\frac{Var^U(Q_1)}{Var^I(Q_1)}}} \left(\exp(\alpha R \chi) - \sqrt{\frac{Var^U\left(Q_1\right)}{Var^I\left(Q_1\right)}}\right) = 1 - \frac{\exp(\alpha R \chi)}{V}.$$

Thus, it follows all the properties of our value of information.

## B Alternative Information Cost Function

We consider a setting where private investors can choose the precision of its signal, $I$, of the economic fundamental $F$, with precision denoted by $\tau_I$. We also know that the price signal is given by

$$S_{P1} = F - \frac{1}{\theta_1} x_1,$$

where $\theta_1$ is an endogenous variable. Given the price signal, the stock payoff $Q_1$ is given by

$$Q_1 = D_2 + P_2 = F + \varepsilon_2^D + AS_{P1} + BS - Cx_2.$$

Given the private signal and the price signal, as well as the public signal, we have the conditional moments for the private investors as:

$$\begin{aligned}E^{i}\left(F\right) &= \frac{\tau_I}{\tau_I+\theta_1^2\tau_{x2}+\tau_F}I+\frac{\theta_1^2\tau_{x2}}{\tau_I+\theta_1^2\tau_{x2}+\tau_F}S_{P1}+\frac{\tau_F}{\tau_I+\theta_1^2\tau_{x2}+\tau_F}S,\\ Var^{i}\left(F\right) &= \frac{1}{\tau_I+\theta_1^2\tau_{x2}+\tau_F}.\end{aligned}$$

Thus the stock return payoff is given by

$$\begin{aligned}E^{i}\left(Q_1\right) &= \frac{\tau_I}{\tau_I+\theta_1^2\tau_{x2}+\tau_F}I+\frac{\theta_1^2\tau_{x2}}{\tau_I+\theta_1^2\tau_{x2}+\tau_F}S_{P1}+\frac{\tau_F}{\tau_I+\theta_1^2\tau_{x2}+\tau_F}S+AS_{P1}+BS,\\ Var^{i}\left(Q_1\right) &= \frac{1}{\tau_I+\theta_1^2\tau_{x2}+\tau_F}+\frac{1}{\tau_D^2}+C^2\frac{1}{\tau_{x2}}.\end{aligned}$$

Thus the stock demand for private investors are:

$$\begin{aligned}d^{i} &= \frac{E^{i}\left(Q_1\right)-RP_1}{\alpha Var^{i}\left(Q_1\right)}\\ &= \frac{\frac{\tau_I}{\tau_I+\theta_1^2\tau_{x2}+\tau_F}I+\frac{\theta_1^2\tau_{x2}}{\tau_I+\theta_1^2\tau_{x2}+\tau_F}S_{P1}+\frac{\tau_F}{\tau_I+\theta_1^2\tau_{x2}+\tau_F}S+AS_{P1}+BS-RP_1}{\alpha\left(\frac{1}{\tau_I+\theta_1^2\tau_{x2}+\tau_F}+\frac{1}{\tau_D^2}+C^2\frac{1}{\tau_{x2}}\right)}.\end{aligned}$$

The market clearing condition is given by

$$\int d^{i}di = x_1.$$

Exploiting symmetry, we obtain

$$\theta_1 = \frac{1}{\alpha\left(\frac{1}{\tau_I+\theta_1^2\tau_{x2}+\tau_F}+\frac{1}{\tau_D^2}+C^2\frac{1}{\tau_{x2}}\right)}.$$

This is an equilibrium condition giving rise to the value of $\theta_1$ given private signal precision $\tau_I$. Now we obtain the expression for the expected utility of private agents, given $\tau_I$ and $\theta_1$. Agents' consumption is given by

$$c = R\left(w_0 - P_1 d^{i} - \kappa\left(\tau_I\right)\right) + \left(P_2 + D_2\right)d^{i},$$

where the information cost $\kappa\left(\tau_I\right)$ is the cost of acquiring a signal of precision $\tau_I$. The expected utility of this agent is given by $\max_{d^i,c} E\left(-\exp\left(-\alpha c\right)\right)$, which is

$$U^i = -\exp\left(-\alpha R\left(w_0 - \kappa\left(\tau_I\right)\right)\right) * E\left[\exp\left(-\frac{1}{2}\frac{\left(E^i\left(Q_1\right) - RP_1\right)}{\alpha Var^i\left(Q_1\right)}\right)\right].$$

Thus we only need to derive the expression for

$$\begin{aligned} & E\left[\exp\left(-\frac{1}{2}\frac{\left(E^i\left(Q_1\right) - RP_1\right)}{\alpha Var^i\left(Q_1\right)}\right)\right] \\ = \ & E\left[\exp\left(-\frac{1}{2}\frac{\left(\frac{\tau_I}{\tau_I+\theta_1^2\tau_{x2}+\tau_F}I + \frac{\theta_1^2\tau_{x2}}{\tau_I+\theta_1^2\tau_{x2}+\tau_F}S_{P1} + \frac{\tau_F}{\tau_I+\theta_1^2\tau_{x2}+\tau_F}S + AS_{P1} + BS - RP_1\right)^2}{\alpha\left[\frac{1}{\tau_I+\theta_1^2\tau_{x2}+\tau_F} + \frac{1}{\tau_D^2} + C^2\frac{1}{\tau_{x2}}\right]}\right)\right]. \end{aligned}$$

Note that the numerator $\frac{\tau_I}{\tau_I+\theta_1^2\tau_{x2}+\tau_F}I + \frac{\theta_1^2\tau_{x2}}{\tau_I+\theta_1^2\tau_{x2}+\tau_F}S_{P1} + \frac{\tau_F}{\tau_I+\theta_1^2\tau_{x2}+\tau_F}S + AS_{P1} + BS - RP_1$ is distributed with mean 0 and some variance denoted by $\sigma_Q^2$. Let $x$ be a random variable such that $\sigma_Q x = \frac{\tau_I}{\tau_I+\theta_1^2\tau_{x2}+\tau_F}I + \frac{\theta_1^2\tau_{x2}}{\tau_I+\theta_1^2\tau_{x2}+\tau_F}S_{P1} + \frac{\tau_F}{\tau_I+\theta_1^2\tau_{x2}+\tau_F}S + AS_{P1} + BS - RP_1$. Then the mean of $x$ is 0 and the variance of $x$ is given by 1. Thus we can express the expected utility as:

$$-E\left[\exp\left(-\frac{1}{2}\frac{\sigma_Q^2}{\alpha\left[\frac{1}{\tau_I+\theta_1^2\tau_{x2}+\tau_F} + \frac{1}{\tau_D^2} + C^2\frac{1}{\tau_{x2}}\right]}x^2\right)\right] = -E\left[\exp\left(\phi x^2\right)\right],$$

where

$$\phi = -\frac{1}{2}\frac{\sigma_Q^2}{\alpha\left[\frac{1}{\tau_I+\theta_1^2\tau_{x2}+\tau_F} + \frac{1}{\tau_D^2} + C^2\frac{1}{\tau_{x2}}\right]}.$$

Using formula for a moment generating function

$$M_a\left(t\right) = \exp\left(\lambda t/\left(1-2t\right)\right)\left(1-2t\right)^{-\frac{1}{2}},$$

we have

$$E\left[\exp\left(\phi x^2\right)\right] = \left(1-2\phi\right)^{-\frac{1}{2}} = \left(1 + \frac{\sigma_Q^2}{\alpha\left[\frac{1}{\tau_I+\theta_1^2\tau_{x2}+\tau_F} + \frac{1}{\tau_D^2} + C^2\frac{1}{\tau_{x2}}\right]}\right)^{-\frac{1}{2}}.$$

Then the expected utility can be formulated as

$$U^i = -\exp\left(-\alpha R\left(w_0 - \kappa\left(\tau_I\right)\right)\right) * \left(1 + \frac{\sigma_Q^2}{\alpha\left[\frac{1}{\tau_I + \theta_1^2\tau_{x2} + \tau_F} + \frac{1}{\tau_D^2} + C^2\frac{1}{\tau_{x2}}\right]}\right)^{-\frac{1}{2}}.$$

Next we derive the expression for $\sigma_Q^2$. Given that the random variable is

$$\frac{\tau_I}{\tau_I + \theta_1^2\tau_{x2} + \tau_F}I + \frac{\theta_1^2\tau_{x2}}{\tau_I + \theta_1^2\tau_{x2} + \tau_F}S_{P1} + \frac{\tau_F}{\tau_I + \theta_1^2\tau_{x2} + \tau_F}S + AS_{P1} + BS - RP_1,$$

we need to first find an expression for $P_1$. Go back to the market clearing condition:

$$\frac{\frac{\tau_I}{\tau_I + \theta_1^2\tau_{x2} + \tau_F}I + \frac{\theta_1^2\tau_{x2}}{\tau_I + \theta_1^2\tau_{x2} + \tau_F}S_{P1} + \frac{\tau_F}{\tau_I + \theta_1^2\tau_{x2} + \tau_F}S + AS_{P1} + BS - RP_1}{\alpha\left(\frac{1}{\tau_I + \theta_1^2\tau_{x2} + \tau_F} + \frac{1}{\tau_D^2} + C^2\frac{1}{\tau_{x2}}\right)} = x_1.$$

Note that

$$\begin{aligned}
&\frac{\tau_I}{\tau_I + \theta_1^2\tau_{x2} + \tau_F}I + \frac{\theta_1^2\tau_{x2}}{\tau_I + \theta_1^2\tau_{x2} + \tau_F}S_{P1} + \frac{\tau_F}{\tau_I + \theta_1^2\tau_{x2} + \tau_F}S + AS_{P1} + BS - RP_1 \\
= \quad & \alpha\left(\frac{1}{\tau_I + \theta_1^2\tau_{x2} + \tau_F} + \frac{1}{\tau_D^2} + C^2\frac{1}{\tau_{x2}}\right)x_1.
\end{aligned}$$

Thus we have

$$\sigma_Q^2 = \left[\alpha\left(\frac{1}{\tau_I + \theta_1^2\tau_{x2} + \tau_F} + \frac{1}{\tau_D^2} + C^2\frac{1}{\tau_{x2}}\right)\right]^2\frac{1}{\tau_{x1}}.$$

Plug it back to the expression of expected utility, we have:

$$\begin{aligned}
U^i \quad &= \quad -\exp\left(-\alpha R\left(w_0 - \kappa\left(\tau_I\right)\right)\right) * \left(1 + \frac{\left[\alpha\left(\frac{1}{\tau_I + \theta_1^2\tau_{x2} + \tau_F} + \frac{1}{\tau_D^2} + C^2\frac{1}{\tau_{x2}}\right)\right]^2\frac{1}{\tau_{x1}}}{\alpha\left[\frac{1}{\tau_I + \theta_1^2\tau_{x2} + \tau_F} + \frac{1}{\tau_D^2} + C^2\frac{1}{\tau_{x2}}\right]}\right)^{-\frac{1}{2}} \\
&= \quad \frac{-\exp\left(-\alpha R w_0\right)\exp\left(\alpha R\kappa\left(\tau_I\right)\right)}{\left(1 + \alpha\left(\frac{1}{\tau_I + \theta_1^2\tau_{x2} + \tau_F} + \frac{1}{\tau_D^2} + C^2\frac{1}{\tau_{x2}}\right)\frac{1}{\tau_{x1}}\right)^{\frac{1}{2}}}.
\end{aligned}$$

Taking first order conditions, the marginal cost of increasing the precision by a little bit is given by $\exp\left(\alpha R\kappa\left(\tau_I\right)\right)\kappa'\left(\tau_I\right)$. The marginal benefit is given by

$$\frac{\alpha\left(\frac{1}{\left(\tau_I+\theta_1^2\tau_{x2}+\tau_F\right)^2}\right)\frac{1}{\tau_{x1}}}{\frac{1}{2}\left(1+\alpha\left(\frac{1}{\tau_I+\theta_1^2\tau_{x2}+\tau_F}+\frac{1}{\tau_D^2}+C\left(\theta_1\right)^2\frac{1}{\tau_{x2}}\right)\frac{1}{\tau_{x1}}\right)^{\frac{3}{2}}}.$$

Hence there are two competing forces at work, similar to our benchmark model of the corresponding equation A.1 of Proposition A.2:

$$\frac{\alpha\overbrace{\left(\frac{1}{\left(\tau_I+\theta_1^2\tau_{x2}+\tau_F\right)^2}\right)}^{\text{Static Substitutability}}\frac{1}{\tau_{x1}}}{\frac{1}{2}\left(1+\alpha\underbrace{\left(\frac{1}{\tau_I+\theta_1^2\tau_{x2}+\tau_F}+\frac{1}{\tau_D^2}+C\left(\theta_1\right)^2\frac{1}{\tau_{x2}}\right)}_{\text{Dynamic Complementarity}}\frac{1}{\tau_{x1}}\right)^{\frac{3}{2}}}.$$

First, the static substitutability effect shows up in the numerator. As individual's signal gets more precise, the price signal also gets more precise, $\theta_1$ increases. This shifts downward the marginal benefit, leading to lower value of information. Second, the dynamic complementarity effect shows up in the denominator. As $\theta_1$ increases, the denominator tends to decrease, mainly because the coefficient on future trading risk $C=\frac{\alpha}{R}\left(\frac{1}{\tau_F+\theta_1\tau_{x1}}+\frac{1}{\tau_D}\right)$ decreases. This shifts up the marginal benefit term, leading to higher value of information. Thus, the major tradeoff between substitutability and complementarity is preserved under this alternative information cost structure.

# C Government Intervention and Optimal Disclosure

In this section we explicitly model the government's objective function and its disclosure policy. The goal is to lay a micro-foundation for the government's taste of maximizing price informativeness. The approach to model government intervention is similar to Bond and Goldstein (2015). Specifically, we assume that the government can take an action $T$ that affects the second-period fundamental, and hence the second-period dividend payment:

$$\begin{aligned}F_2 &= F+T,\\ D_3 &= F_2+\varepsilon_2^D=F+T+\varepsilon_2^D.\end{aligned}$$

The objective of the government is to regulate the fundamental of the firm to some target level $F^T$, subject to its own information set $\Omega^G$:

$$\max_T -E\left[\left(T-(F^T-F)\right)^2|\Omega^G\right].$$

Thus, in the absence of imperfect information there is perfect regulation: the government sets the optimal action $T^* = F^T - F$ and hence fundamental is equal to the target level $F_2 = F^T$. Following Bond and Goldstein (2015), we interpret this as a reduced-form way of capturing government interventions regarding the macroeconomy. and the banking sector. For example, the government may consider bailing out financial institutions during stress time and there are research showing that such intervention may benefit shareholders by raising the share prices of the institutions (O'Hara and Shaw, 1990; Gandhi and Lustig, 2015). This could impact the investors' behavior and in turn, affect the government's decision by affecting the amount of market information available.

The government makes an intervention based on market information it observes, as well as any private sources of information about the firm to be intervened. It is assumed that the government receives a private noisy signal about fundamental $S_G = F + \varepsilon^G$. It then determines its disclosure policy: $S = S_G + \varepsilon^F$ where the variance of the noise $\tau_F$ is its endogenous choice variable. Then, given the public announcement, the private sector conduct trading and delivers an equilibrium price signal $S_{P1}$. This price signal provides valuable information to the government as it reflects private information from the investors. Thus the government information set at the time of intervention is given by $\Omega^G = \{S_G, S_{P1}\}$. Thus the government faces the following information design problem: Given the $S_G$ and $S_{P1}$, the government maximizes its objective, with indirect utility given by $U(\sigma_F^2)$:

$$U(\tau_F) = \max_T -E\left[\left(T-(F^T-F)\right)^2|S_G, S_{P1}(\tau_F)\right].$$

where it is made explicit that the endogenous price signal depends on the strength of the pubic announcement. The government then picks the disclosure policy to maximize $\max_{\tau_F} U(\tau_F)$.

We solve this problem backwards. Given the information set $S_G$ and $S_{P1}$, the government sets optimal intervention as $T* = F^T - E(F|S_G, S_{P1})$. This implies that the indirect value function:

$$U(\tau_F) = -Var(F|S_G, S_{P1}(\tau_F)) = -\frac{1}{\frac{1}{\sigma_G^2} + \frac{\theta_1^2(\tau_F)}{\sigma_x^2}}.$$

where $\theta_1(\tau_F)$ is the endogenous price informativeness in period 1. Thus, the goal of the government is to design a policy to stimulate the most amount of market information, *i.e.*, to maximize

$\theta_1^2(\tau_F)$. This provides a micro-foundation as to why the government would like to maximize market price informativeness.

## D Derivation of Results in Manzano and Vives (2011)

We use this part to describe the model of Manzano and Vives (2011) and show that in this environment even if information complementarity arises, the fundamental effect always takes a negative sign: more public information always reduces the value of information. Thus the crowding-in result cannot occur even under global-game refinement.

The setup of Manzano and Vives (2011) is as follows. There is a stock that pays off $v$, which follows a normal distribution $N\left(\bar{v}, \tau_v^{-1}\right)$. The parameter $\tau_v$ measures the amount of publicly available information about the stock fundamental. There are $\mu$ share of informed investors and $1-\mu$ share of uninformed investors. Informed investors observe a signal $s_i = v + \varepsilon_i$. The signal can be correlated $cov\left(\varepsilon_i, \varepsilon_j\right) = \rho\tau_\varepsilon^{-1}$. Uninformed investors do not observe such a signal. Each investors have an endowment of $u_i = u + \eta_i$, where the error term is *i.i.d.* with variance $\tau_\eta^{-1}$. $u_i$ is a private signal about the aggregate endowment for investor $i$.

Given the structure, we can derive the precision of information by informed and uninformed investors. For informed investors, he observes a public price signal $p$, a private signal $s_i$, and a signal coming from endowment $u_i$. Denote $\beta$ to be the equilibrium price informativeness, we have

$$
\begin{aligned}
z &= v + \tilde{\varepsilon} - \frac{1}{\beta} u, \\
z_i &= v + \tilde{\varepsilon} - \frac{1}{\beta} \eta_i, \\
s_i &= v + \varepsilon_i.
\end{aligned}
$$

Thus using Proposition 3 yields

$$
\begin{aligned}
\tau^I &= \left(var\left(v|s_i, z_i, z\right)\right)^{-1} = \frac{\tau_\varepsilon\left(\tau_v + \tau_\varepsilon\right) + \beta^2\left(\tau_u + \tau_\eta\right)(1-\rho)\left(\tau_\varepsilon + \rho\tau_v\right)}{\tau_\varepsilon + (1-\rho)\left(\tau_u + \tau_\eta\right)\rho\beta^2}, \\
\tau^U &= \left(var\left(v|z_j, z\right)\right)^{-1} = \frac{\tau_\varepsilon\tau_v + \beta^2\left(\tau_u + \tau_\eta\right)\left(\tau_\varepsilon + \rho\tau_v\right)}{\tau_\varepsilon + \left(\tau_u + \tau_\eta\right)\rho\beta^2}.
\end{aligned}
$$

where $\beta$ solves a fixed point problem independent of $\tau_v$.

The main source of information complementarity depends on action complementarity: when the informed investors put heavy weight on their own private signal, equilibrium price becomes informative, making the endogenous price signal $z_i$ more precise, leading investors to weigh more their private signal. This property leads to strategic complementarity in information acquisition.

Next we examine the value of information $\pi$, defined as the ratio of condition uncertainties $\tau^I$ and $\tau^U$. Then with some algebraic manipulation, we can obtain that

$$\pi = \frac{\tau^I}{\tau^U} = \frac{\tau_\varepsilon + \left(\tau_u + \tau_\eta\right)\rho\beta^2}{\tau_\varepsilon + (1-\rho)\left(\tau_u + \tau_\eta\right)\rho\beta^2}\left(1 + \frac{\tau_\varepsilon^2 - \rho\beta^2\left(\tau_u + \tau_\eta\right)\left(\tau_\varepsilon + \rho\tau_v\right)}{\tau_\varepsilon\tau_v + \beta^2\left(\tau_u + \tau_\eta\right)\left(\tau_\varepsilon + \rho\tau_v\right)}\right).$$

In turn, with further algebraic manipulation, we can check how the value of information changes with the precision of public information:

$$\begin{aligned}
\frac{\partial\pi}{\partial\tau_v} &= \frac{-\rho\beta^2\left(\tau_u + \tau_\eta\right)\rho\left[\tau_\varepsilon\tau_v + \beta^2\left(\tau_u + \tau_\eta\right)\left(\tau_\varepsilon + \rho\tau_v\right)\right]}{\left[\tau_\varepsilon\tau_v + \beta^2\left(\tau_u + \tau_\eta\right)\left(\tau_\varepsilon + \rho\tau_v\right)\right]^2} \\
&\quad - \frac{\left[\tau_\varepsilon^2 - \rho\beta^2\left(\tau_u + \tau_\eta\right)\left(\tau_\varepsilon + \rho\tau_v\right)\right]\left(\tau_\varepsilon + \beta^2\left(\tau_u + \tau_\eta\right)\rho\right)}{\left[\tau_\varepsilon\tau_v + \beta^2\left(\tau_u + \tau_\eta\right)\left(\tau_\varepsilon + \rho\tau_v\right)\right]^2} \\
&= \frac{-\tau_\varepsilon^2\tau_\varepsilon}{\left[\tau_\varepsilon\tau_v + \beta^2\left(\tau_u + \tau_\eta\right)\left(\tau_\varepsilon + \rho\tau_v\right)\right]^2} < 0.
\end{aligned}$$

Consequently, we have $\frac{\partial\pi}{\partial\tau_v} < 0$. This means that no matter whether there exists information complementarity, the value of information is always decreasing in the precision of the public signal. Thus, by Proposition 2, the crowding-in effect cannot occur even under global-game refinement.

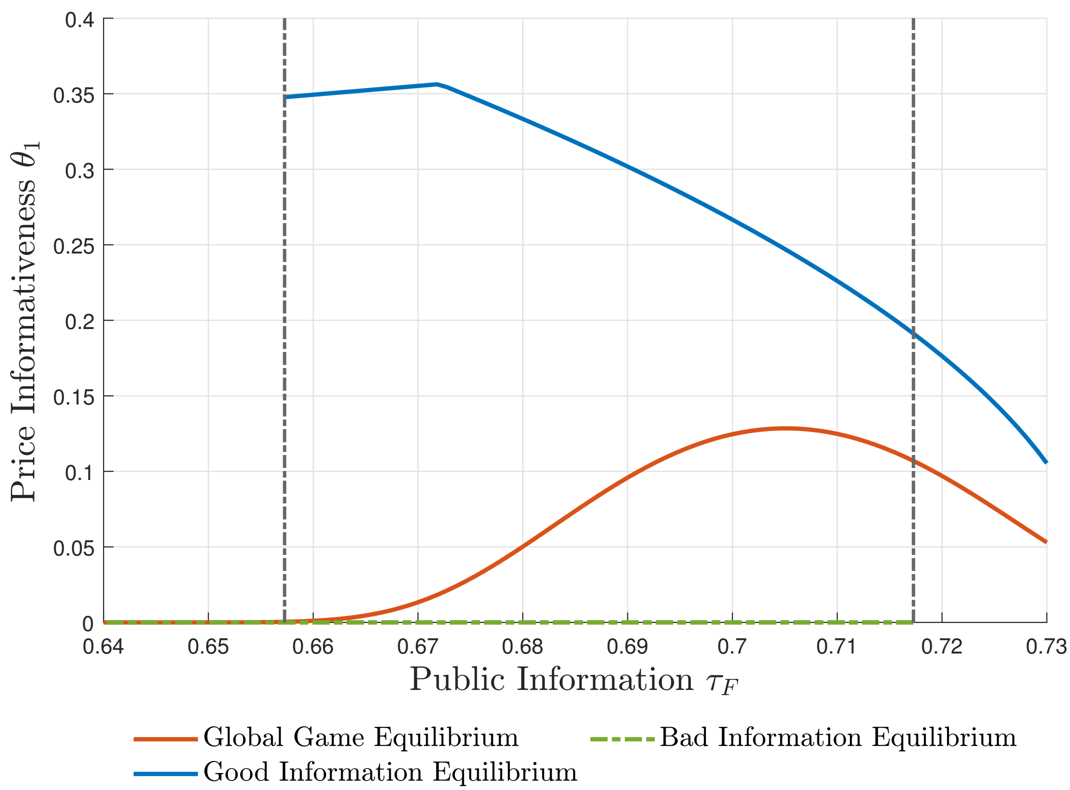
# E Price Informativeness

In many environments, regulators make decisions partly based on market information (e.g., Bond and Goldstein, 2015), and asset price informativeness is a key object of interest because it determines how much regulators can learn from the market. As a result, regulators may have an incentive to maximize stock price informativeness.[20] Figure E.1 plots the equilibrium price informativeness $\theta_1$ as a function of fundamental uncertainty in common-knowledge equilibria and global-game equilibrium.

An interesting feature of the good information equilibrium (solid line) is that price informativeness starts to decline when fundamental uncertainty is sufficiently high. This is due to the trading volume effect working at the *intensive* margin. We know that price informativeness is affected by both the extensive margin (the share of informed investors) and the intensive margin (how aggressively they trade), which is determined by how much residual risk they face.

[20] Appendix C provides a model to microfound this preference, similar to Bond and Goldstein (2015).

Figure E.1: Price Informativeness under Complete Information and Global Game




**Notes**: This figure plots the equilibrium price informativeness $\theta_1$ as a function of public disclosure strength $\tau_F$. In the good information equilibrium under complete cost information (solid blue line), the peak in price informativeness is reached when $\tau_F = 0.672$. Under the global-game equilibrium, due to the coordination force the peak is reached at a much lower level of public uncertainty $\tau_F = 0.705$. $\rho = 0.99, \tau_\phi = \tau_F/(1-\rho^2), \tau_D = 2, \tau_{x1} = \tau_{x2} = 0.5, R = 1.01, \alpha = 1, \bar{\chi} = 0.195, \sigma = 0.0001$.

When fundamental uncertainty is sufficiently high, the share of informed investors $\lambda$ reaches nearly 100%, and hence the extensive margin effect disappears. At the intensive margin, a rise in fundamental uncertainty implies higher future unlearnable risk ($C(\cdot)$ increases), and this makes *existing* informed investors trade more cautiously, reducing the amount of private information incorporated into equilibrium prices. As a result, price informativeness declines.

Note that for all the numerical experiments conducted in this paper, this intensive margin effect tends to be weak and only appears when the extensive adjustment almost reaches its boundary, i.e., when the share of informed investors $\lambda$ reaches nearly 100%. Thus, a regulator seeking to maximize price informativeness should set a relatively low strength of public disclosure in an effort to increase the share of informed investors to the greatest extent possible.

In the global-game equilibrium, on the other hand, the peak of price informativeness arrives with much less fundamental uncertainty. This is due to the coordination force whereby more public information makes the private information market coordinate towards the good equilibrium and away from the bad equilibrium. This force dominates the conventional crowding-out effect much earlier than when the share of informed investors $\lambda$ reaches its boundary, leading to a different maximum of price informativeness. Thus, it is important to account for the coordination force when designing disclosure policies. If a regulator ignores this coordination force, she would mistakenly release too little public information to the financial market, particularly during times of uncertainty.