Online Appendix to “Dynamic Information Acquisition and Time-Varying Uncertainty”

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This file describes in detail the proof of proposition 4.1 in the main paper. The basic strategy is to show that the value of information is locally increasing when λ (share of informed investors) is equal to zero. To do so, I decompose the slope of the value of information into a "substitutability" and a "complementarity component". The proof then shows that the substitutability component is locally equal to zero (proposition 2.1) and that the complementarity component is upward-sloping (proposition 2.2). These propositions lead to theorem 1, which corresponds to the proposition 4.1 in the paper.

1 Characterization

The equilibrium is characterized by three dynamic relations between investors' beliefs, pricing coefficients, and the share of investors acquiring information. The first is a Kalman filter equation that characterizes the evolution of the conditional expectations of uninformed investors. The second is a financial market clearing condition that pins down pricing coefficients. The third is an information optimality condition to guarantee optimality of investors' information acquisition decision, which pins down the share of informed investors.

1.1 Conditional Expectations of uninformed investors

Uninformed investors form their beliefs based on the entire history of dividends, public signals, and equilibrium stock prices. I here argue that this history can be summarized in two state variables.[1] One is prior fundamental uncertainty.

$$z_{t-1} = Var(F_{t-1}|\Omega^U_{t-1})$$

The other is the informative ratio of the last period:

$$\theta_{t-1} = \frac{p_{Ft-1}}{p_{xt-1}}$$

[1] In principle, the (expected) *level* of stock fundamental and stock supply are also part of the state. With a utility function that exhibits no wealth effect, however, pricing coefficients are determined only by the conditional volatility.

Both are necessary, because they affect the conditional joint distribution of the current fundamental and current price signal. To see this, I arrange the current price signal S_{pt} into a combination of last period fundamental, price signal and current period noise:

Lemma 1.1

$$S_{pt} = \left(\rho^F - \rho^x \frac{\theta_{t-1}}{\theta_t}\right) F_{t-1} + \rho^x \frac{\theta_{t-1}}{\theta_t} S_{pt-1} + \varepsilon_t^F - \frac{1}{\theta_t}\varepsilon_t^x \tag{1.1}$$

Thus, conditional on observing last period's price signal, a greater θ_{t-1} implies that current price signal S_{pt} is less responsive to last period's stock fundamental. This reduces the conditional correlation between the current fundamental and price signal. To see this:

$$\begin{aligned} Cov(F_t, S_{pt}|\Omega_{t-1}^U) &= Cov(\rho^F F_{t-1} + \varepsilon_t^F, S_{pt}|\Omega_{t-1}^U) && (1.2) \\ &= \rho^F \left(\rho^F - \rho^x \frac{\theta_{t-1}}{\theta_t}\right) Var(F_{t-1}|\Omega_{t-1}^U) + \sigma_F^2 && (1.3) \end{aligned}$$

The next proposition characterizes how the conditional belief evolves:

Proposition 1.1 *Given the sequence of $\{\theta_t\}$, the law of motion for conditional volatility z_t is*

$$\frac{1}{z_t} = \frac{1}{Var(F_t|\{P_t\} \cup \Omega_{t-1}^U)} + \frac{1}{\sigma_D^2} + \frac{1}{\sigma_S^2}, \tag{1.4}$$

where $Var(F_t|\{P_t\} \cup \Omega_{t-1}^U)$ is the fundamental volatility upon observing past history and the current price signal (but not the dividend and public signal), and is a function of z_t and pricing coefficients p_{Ft}, p_{xt}:

$$Var(F_t|\{P_t\} \cup \Omega_{t-1}^U) = \left(\rho^F\right)^2 z_{t-1} + \sigma_F^2 - \frac{\left[\rho^F \left(\rho^F - \rho^x \frac{\theta_{t-1}}{\theta_t}\right) z_{t-1} + \sigma_F^2\right]^2}{\left(\rho^F - \rho^x \frac{\theta_{t-1}}{\theta_t}\right)^2 z_{t-1} + \sigma_F^2 + \left(\frac{1}{\theta_t}\right)^2 \sigma_x^2} \tag{1.5}$$

Detailed proofs are relegated to the appendix. Observing that $z_t = Var(F_t|\Omega_t^U)$, equation 1.4 follows immediately from the standard Kalman filter formula for normal variables. Equation 1.5 can be understood as follows. First, observing an additional price signal reduces fundamental uncertainty

$$Var(F_t|\{P_t\} \cup \Omega_{t-1}^U) < Var(F_t|\Omega_{t-1}^U) = \left(\rho^F\right)^2 z_{t-1} + \sigma_F^2$$

unless the current stock price is not uninformative ($\theta_t \to 0$). Second, when stock supply x_t is i.i.d.

($\rho^x = 0$), the price signal is just the sum of true fundamental F and some noise term uncorrelated with past history. Thus, equation 1.4 collapses to the standard formula, whereby ex post precision is equal to the sum of ex ante precision and the precision of the price signal.

$$\frac{1}{z_t} = \frac{1}{\left(\rho^F\right)^2 z_{t-1} + \sigma_F^2} + \frac{1}{\left(\frac{1}{\theta_t}\right)^2 \sigma_x^2} + \frac{1}{\sigma_D^2} + \frac{1}{\sigma_S^2} \tag{1.6}$$

Note that θ_{t-1} drops out of the expression. This implies that the state space collapses to a single dimension of z_{t-1}. When stock supply is persistent, x_t is correlated with past history, and thus one needs to invoke the projection theorem of normally distributed variables to obtain equation 1.5. This suggests that in general, we need to keep track of both z_t and θ_t when solving for model dynamics.

Next we consider the evolution of conditional mean $\hat{F}_t$. This is useful when we get to the characterization of optimal portfolio choices.

Lemma 1.2 *Given the sequence of pricing coefficients $\{\theta_t, p_{xt}\}$ and state variables $\{z_t\}$, the law of motion for the conditional mean $\hat{F}_t$ is given by:*

$$\hat{F}_{t+1} = f_{\hat{F}t+1}\hat{F}_t + f_{Ft+1}F_t + \vec{f}_{\varepsilon t+1}\vec{\varepsilon}_{t+1}, \tag{1.7}$$

where $f_{\hat{F}t+1}$ and f_{Ft+1} are scalars and $\vec{f}_{\varepsilon t+1}$ is a 1 by 4 vector of coefficients. These coefficients depend on $\theta_t, z_t, \theta_{t+1}, z_{t+1}$. $\vec{\varepsilon}_{t+1} = [\varepsilon_{t+1}^D, \varepsilon_{t+1}^F, \varepsilon_{t+1}^x, \varepsilon_{t+1}^S]$ is the 4 by 1 shock vector realized at time t+1. In particular, its sensitivity with respect to F_t is given by

$$\begin{aligned} f_{Ft+1} \;=\; & \underbrace{\frac{Var\left(F_{t+1}|\Omega_{t+1}^U\right)}{Var\left(F_{t+1}|\left\{P_{t+1}\right\}\cup\Omega_t^U\right)} \frac{\left[\left(\rho^F - \rho^x\frac{\theta_t}{\theta_{t+1}}\right)\rho^F Var\left(F_t|\Omega_t^U\right) + \sigma_F^2\right]^2}{\left(\rho^F - \rho^x\frac{\theta_t}{\theta_{t+1}}\right)^2 Var\left(F_t|\Omega_t^U\right) + \sigma_F^2 + \frac{1}{\theta_{t+1}^2}\sigma_x^2} \left(\rho^F - \rho^x\frac{\theta_t}{\theta_{t+1}}\right)}_{\text{weight on the price signal}} \\ & + \underbrace{\frac{Var\left(F_{t+1}|\Omega_{t+1}^U\right)}{\sigma_D^2}\rho^F}_{\text{weight on the dividend signal}} + \underbrace{\frac{Var\left(F_{t+1}|\Omega_{t+1}^U\right)}{\sigma_S^2}\rho^F}_{\text{weight on the public signal}}. \end{aligned} \tag{1.8}$$

That the law of motion for $\hat{F}_t$ is linear is a standard property of the Kalman filter. Equation 1.8 measures how the current stock fundamental affects the future expected fundamental. A good realization of the current fundamental implies on average a favorable price signal, dividend signal, and public signal, all three of which imply a higher future expected fundamental. The three terms in the expression f_{Ft+1} capture the weight investors assign to each signal. These weights are determined by the relative precision of each signal and thus depend on price informativeness θ and fundamental uncertainty z.

1.2 Excess Stock Return and Optimal Portfolios

Given the equilibrium price function and uninformed investors' beliefs, one can derive the expression for the excess stock return and optimal portfolios. The excess stock return consists of dividends and capital gains, less the interest cost of holding the stock:

$$Q_{t+1} = D_{t+1} + P_{t+1} - RP_t \tag{1.9}$$

$$= \underbrace{F_{t+1} + \varepsilon^D_{t+1}}_{D_{t+1}} + \underbrace{\bar{p}_{t+1} + p_{\hat{F}t+1}\hat{F}_{t+1} + p_{Ft+1}F_{t+1} - p_{xt+1}x_{t+1}}_{P_{t+1}} - RP_t \tag{1.10}$$

where we substitute out stock dividend and equilibrium stock price. Using the law of motion for F_{t+1}, x_{t+1}, and $\hat{F}_{t+1}$, Q_t can be expressed as a linear combination of time t variables and time $t+1$ shocks:

Lemma 1.3 *The excess stock return Q_{t+1} can be expressed as*

$$Q_{t+1} = \bar{q}_{t+1} + q_{\hat{F}t+1}\hat{F}_t + q_{Ft+1}F_t - q_{xt+1}x_t + \vec{q}_{\varepsilon t+1}\vec{\varepsilon}_{t+1} - RP_t$$

where $q_{\hat{F}t+1}, q_{Ft+1}, q_{xt+1} > 0$ are scalars and $\vec{q}_{\varepsilon t+1}$ is a 1 by 4 vector. All coefficients may depend on $\theta_t, z_t, \theta_{t+1}, z_{t+1}$, and p_{xt+1}. In particular:

$$q_{Ft+1} = \rho^F(1 + p_{Ft+1}) + p_{\hat{F}t+1}f_{Ft+1} \tag{1.11}$$

$$q_{xt+1} = \rho^x p_{xt+1}. \tag{1.12}$$

q_{Ft+1} is the sensitivity of the future stock return with respect to the current stock fundamental (equation 1.11). The first term $\rho^F(1 + p_{Ft+1})$ captures the direct favorable impact of current fundamental on future dividends and stock prices due to its persistence. The second term $p_{\hat{F}t+1}f_{Ft+1}$ captures a signaling effect whereby innovations in the stock fundamental impacts uninformed investors' belief through more favorable signals. Equation 1.12 indicates that persistent supply shocks have a negative impact on the future stock return, as it predicts unfavorable future stock supply. Note that knowing the informative ratio is not sufficient to characterize the excess stock return; one also needs information of p_{xt+1}.

Lemma 1.3 suggests that the excess stock return can be decomposed into the following three components in terms of information content:

$$Q_{t+1} = \underbrace{\bar{q}_{t+1} + q_{\hat{F}t+1}\hat{F}_t - RP_t}_{\text{known to all}} + \underbrace{q_{Ft+1}F_t - q_{xt+1}x_t}_{\text{known to informed only}} + \underbrace{\vec{q}_{\varepsilon t+1}\vec{\varepsilon}_{t+1}}_{\text{not known to either}} \tag{1.13}$$

The first component consists of a constant, current stock prices, and uninformed investors' belief $\hat{F}_t$. These are known to all agents in the economy. The second component consists of the realized current stock fundamental and stock supply. This information is known only to the informed investors. The third component consists of future noises that no one at period t could possibly know. Thus the conditional volatility of the excess stock return for informed investors is just the volatility of the noise term:

$$V_t^I = Var(Q_{t+1}|\Omega_t^I) = Var(\vec{q}_{\varepsilon t+1}\vec{\varepsilon}_{t+1}|\Omega_t^I) \tag{1.14}$$

Since the vector of shock coefficients depends on $p_{\hat{F}t+1}, p_{Ft+1}$ and z_{t+1}, V_t^I also depend on these variables. The conditional volatility faced by the uninformed also reflects the fact that current investors are uncertain about the current stock fundamental and stock supply:

$$V_t^U = Var(Q_{t+1}|\Omega^U) = Var(q_{Ft+1}F_t - q_{xt+1}x_t + \vec{q}_{\varepsilon t+1}\vec{\varepsilon}_{t+1}|\Omega^U) \tag{1.15}$$

$$= Var(q_{Ft+1}F_t - q_{xt+1}x_t|\Omega^U) + V_t^I. \tag{1.16}$$

Note that uninformed investors observe the current price signal. Using the price signal to substitute out supply x_t,

$$\begin{aligned} V_t^U &= Var(q_{Ft+1}F_t - q_{xt+1}\frac{p_{Ft}}{p_{xt}}\left(F_t - S_{pt}\right)|\Omega^U) + V_t^I \\ &= Var(q_{Ft+1}F_t - q_{xt+1}\frac{p_{Ft}}{p_{xt}}F_t|\Omega_t^U) + V_t^I \\ &= \left(q_{Ft+1} - q_{xt+1}\frac{p_{Ft}}{p_{xt}}\right)^2 Var(F_t|\Omega_t^U) + V_t^I \\ &= \left(q_{Ft+1} - q_{xt+1}\theta_t\right)^2 z_t + V_t^I \end{aligned} \tag{1.17}$$

Given the expected stock returns and conditional volatility, we can now derive investors' optimal portfolio. As investors live for two periods with exponential utility, their optimal portfolio choice is particularly simple:

$$s_t^i = \frac{E\left(Q_{t+1}|\Omega_t^i\right)}{\alpha V_t^i}, i = I, U$$

Denoting the time t share of informed investors λ_t, the pricing coefficients p_{Ft} and p_{xt} are then determined by matching coefficients so that the market clearing condition holds for any realizations of the stock fundamental and supply. These conditions implicitly define the law of motion for θ_t and p_{xt}:

Proposition 1.2 *Given a sequence of conditional volatility $\{z_t\}$ and share of informed investors $\{\lambda_t\}$, the following two equations define the law of motion for pricing coefficients θ_t and p_{xt}:*

$$\left[\lambda_t \frac{1}{\alpha V_t^I} + (1-\lambda_t)\frac{1}{\alpha V_t^U}\right]\left(Rp_{xt} - \rho^x p_{xt+1}\right) = 1 \tag{1.18}$$

$$\lambda_t \frac{q_{Ft+1} - \rho^x p_{xt+1}\theta_t}{V_t^I} = \theta_t \tag{1.19}$$

where V_t^I and V_t^U are the conditional volatility of the stock return for informed and uninformed investors defined by 1.14 and 1.15 and are functions of $\theta_t, z_t, \theta_{t+1}, z_{t+1},$ and p_{xt+1}.

Equations 1.18 and 1.19 are forward-looking in nature. Investors make forecasts about future pricing coefficients and conditional distributions of the excess stock return, and their demand through market clearing delivers the current pricing coefficients. It is immediate to see that when there are no informed investors $\lambda_t = 0$, the informative ratio is equal to zero (equation 1.19).

1.3 Value of Information

We define the value of information as the ratio of the expected utility of the informed and uninformed investors.

Definition 1.1 *Define the value of information at time t:*

$$\mathcal{I}_t = W_t^U / W_t^I,$$

where the expected utilities W_t^i *are given by* **??**.

An elegant theoretical result of Grossman and Stiglitz (1980) is that the value of information only depends on second moments and is proportional to the ratio of the conditional volatility of excess stock return. This result carries over to this dynamic model because agents live for two periods, just as in Grossman and Stiglitz (1980).[2] Thus conditional on the distribution of the excess stock return, they solve exactly the same problem as in Grossman and Stiglitz (1980) world, yielding the same expression of expected utility, and hence the same expression for the value of information.

Proposition 1.3 *The value of information is proportional to the ratio of conditional volatility of the excess stock return and is given by*

$$\mathcal{I}_t = e^{-\alpha R \chi} \sqrt{\frac{V_t^U}{V_t^I}},$$

where V_t^I *and* V_t^U *are the conditional volatility of the stock return for informed and uninformed investors defined by equations 1.14 and 1.15 and are functions of* $\theta_t, z_t, \theta_{t+1}, z_{t+1}$, *and* p_{xt+1}.

Information optimality means that investors have no incentive to deviate from their information status. In view of Proposition 1.3 and the fact that conditional volatilities are functions of $\theta_t, z_t, \theta_{t+1}, z_{t+1}$, and p_{xt+1}, we can write the information optimality condition as follows:

$$\begin{aligned} \mathcal{I}(\theta_t, z_t, \theta_{t+1}, z_{t+1}, p_{xt+1}) &= 1 \quad \text{if} \quad \lambda_t \in (0,1) \\ \mathcal{I}(\theta_t, z_t, \theta_{t+1}, z_{t+1}, p_{xt+1}) &\leq (\geq) 1 \quad \text{if} \quad \lambda_t = 0(1) \end{aligned} \tag{1.20}$$

[2] When agents' horizon extends to more than two periods, this result no longer holds. In particular, the value of information would presumably depend on first moments of the model.

1.4 Summary

To summarize, the equilibrium is fully characterized by three dynamic relations between conditional volatility, pricing coefficients, and share of informed investors. The first relation is a Kalman filter equation that characterizes the evolution of conditional volatility (Proposition 1.1):

$$\mathcal{K}(z_{t-1}, \theta_{t-1}, z_t, \theta_t) = 0 \tag{1.21}$$

The second relation is a set of market clearing conditions that pin down the evolution of the pricing coefficients and thus informative ratio (Proposition 1.2):

$$\mathcal{M}(\lambda_t, \theta_t, z_t, \theta_{t+1}, z_{t+1}, p_{xt+1}) = 0 \tag{1.22}$$

The third relation is an information optimality condition that is given by equation 1.20.

The equilibrium is both backward-looking and forward-looking. On the one hand, the current market equilibrium depends on the entire history of information, summarized by the state variables z_{t-1} and θ_{t-1}. On the other hand, trading and information acquisition choice depend on future stock returns, which in turn depend on future pricing coefficients θ_{t+1}, p_{xt+1} and future fundamental uncertainty z_{t+1}. The next section starts by characterizing the steady-state property of the system.

2 Steady-state Multiplicity

This section analyzes the steady states of the model. The target is to uncover a key force in the dynamic information market: the dynamic complementarity in information acquisition whereby the value of information today is increasing in the share of informed investors in the future. This has two implications: First, multiple steady states could arise for appropriate levels of information cost. With steady-state multiplicity, The variation of shooting method cannot be applied in solving for model dynamics. This motivates the recursive method introduced in the following section. Second, this dynamic complementarity effect is also the source of endogenous uncertainty shocks discussed later.

I define the steady-state value of information as a function of the share of informed investors.

Intuitively, we ask what the value of information is in a world in which the share of informed investors is fixed and the financial market is in equilibrium.

Definition 2.1 *The steady-state value of information is defined as*

$$\mathcal{I}(\theta_t(\lambda), z_t(\lambda), \theta_{t+1}(\lambda), z_{t+1}(\lambda), p_{xt+1}(\lambda)),$$

where all implicit functions are defined by $\mathcal{K} = 0$ *and* $\mathcal{M} = 0$ *with stationarity imposed.* λ *denotes the steady-state share of informed investors and is exogenously fixed.*

The strategy is to show that the steady-state value of information can increase with the steady-state share of informed investors. As the steady-state share of informed investors varies, it affects both today's and future stock price. The future stock price then feeds back into today's information choice. This introduces an important feedback channel that is absent in Grossman and Stiglitz (1980) where future stock payoff is an exogenous function.

To gain intuition, I focus on the *information gain* component of the value of information defined as the amount of uncertainty reduced through information acquisition:

$$\Delta V_t \quad = \quad V_t^U - V_t^I \tag{2.1}$$

Using equation 1.17 and substituting in expressions q_{Ft+1} and q_{xt+1} given by lemma 1.3, we arrive at the following expression:

$$\Delta V_t \quad = \quad \left(\rho^F(1 + p_{Ft+1}) + p_{\hat{F}t+1} f_{Ft+1} - \rho_x p_{xt+1} \frac{p_{Ft}}{p_{xt}}\right)^2 Var(F_t|\Omega_t^U) \tag{2.2}$$

The information gain hinges upon two aspects: First, how much fundamental uncertainty faced by investors — the term $Var(F_t|\Omega_t^U)$ — and second, how sensitive the future stock return is with respect to current fundamental. When there is higher fundamental uncertainty or when return is more sensitive to the fundamental, there is greater gain from acquiring information. The endogenous sensitivity term is what distinguishes this paper from Grossman and Stiglitz (1980) in which sensitivity only comes from exogenous dividends. Here the sensitivity is endogenous, because the return also depends on the future resale stock price which in term determined by future investors' information acquisition and trading activities. This introduces a dynamic coordination motive within the dynamic information market.

As the share of steady-state informed investors increases, two opposing effects emerge. On the one hand, there are more informed investors *today*. This implies that the current price signal becomes more precise and thus there is less fundamental uncertainty. This is the classic static substitutability effect in Grossman and Stiglitz (1980). On the other hand, there are more informed investors *in the future*. Thus, future stock prices become more sensitive to fundamental: p_{Ft+1} increases. This tends to increase the endogenous sensitivity component in the information gain. This dynamic complementarity force raises the value of information. Whether the value of information is upward-sloping generally depends on the relative strength of these two forces.

A crucial observation of this paper is that the static substitutability effect is locally absent around $\lambda = 0$. That is, $Var(F_t|\Omega_t^U)$ does not vary with λ as it tends to zero. This observation follows from two properties of the Kalman filer equation $\mathcal{K}$ (see proposition 1.1): First, λ does not enter into the equation directly, but only indirectly through the pricing coefficients. Second, pricing coefficients p_F and p_x enter the equation only through the informative ratio squared θ^2. Thus the derivatives always contain this ratio θ, which converges to 0 when λ tends to 0. These two properties, taken together, imply that the conditional volatility of the stock fundamental is not affected by changes in the share of informed investors, directly or indirectly. We can thus treat it as a constant when λ is very close to 0.

We summarize this observation in the following proposition

Proposition 2.1 *(Local absence of static substitutability) Denote λ the steady-state share of informed investors. The total derivative of conditional volatility with respect to the share of informed investors, when λ tends to 0, converges to zero:*

$$\frac{dVar(F_t|\Omega_t^U)}{d\lambda} \to 0.$$

Thus, when there are few informed investors, the dynamic complementarity effect always dominates. Rewrite equation 2.2 omitting the time script, and cancel out terms with respect to p_x:

$$\Delta V = \left(\rho^F(1+p_F) + p_{\hat{F}} f_F - \rho_x p_F\right)^2 Var(F|\Omega^U)$$

A standard property of dynamic noisy rational expectation models is that p_F and $p_{\hat{F}}$ sum to a constant (Wang, 1994). Denoting this constant a, we have:

$$\Delta V = \left(\rho^F(1+p_F) + (a-p_F)f_F - \rho_x p_F\right)^2 Var(F|\Omega^U)$$

The coefficient f_F depends on various volatility terms (see equation 1.8), which are also just functions of θ^2. Thus, by the similar logic of Proposition 2.1, we know that the sensitivity parameter f_F does not vary with λ when λ is near zero. Thus, λ can affect information gain ΔV only through pricing coefficient p_F. Taking this derivative yields the following expression:

$$\frac{d\Delta V}{d\lambda} = 2\left(\rho^F(1+p_F) + (a-p_F)f_F - \rho_x p_F\right)\left(\rho^F - f_F - \rho_x\right)\frac{dp_F}{d\lambda} \tag{2.3}$$

$$= 2\left(\rho^F + af_F\right)\left(\rho^F - f_F - \rho_x\right)\frac{dp_F}{d\lambda} \tag{2.4}$$

where in the second line I substitute in the fact that p_F tends to zero when λ tends to zero. To the extent that increasing λ naturally increases p_F, the sign of this derivative hinges critically on the term

$$\rho^F - f_F - \rho_x$$

When there are no informed investors, market learning does not arise. Thus, uninformed investors' beliefs only depend on the precision of dividend and public signals. Thus the sensitivity parameter f_F collapses to:

$$f_F = \rho^F\left(\frac{Var(F|\Omega^U)}{\sigma_D^2} + \frac{Var(F|\Omega^U)}{\sigma_S^2}\right)$$

These steps deliver the following proposition characterizing the slope of information gain:

Proposition 2.2

$$\frac{d\Delta V}{d\lambda} > 0 \textit{ for } \lambda \textit{ sufficiently small}$$

if and only if

$$\left(1 - \frac{Var(F|\Omega^U)}{\sigma_D^2} - \frac{Var(F|\Omega^U)}{\sigma_S^2}\right)\rho^F - \rho^x > 0$$

where $Var(F|\Omega^U)$ is the steady-state conditional volatility of the stock fundamental given by Proposition 1.1 with $\lambda = 0$.

The proposition reveals that the dynamic complementarity force generally depends on three aspects. First, a more persistent stock fundamental strengthens dynamic complementarity, as the future stock return would be more sensitive to the current fundamental; second, a more persistent stock supply weakens dynamic complementarity. This is because with persistent supply, information about the stock fundamental is not that useful for predicting the future stock return *in addition to the current price signal*. The logic is that conditional on current price signal, observing a good fundamental also implies a large stock supply. If this large supply persists into the future, the future stock return would not be very high despite the favorable fundamental information. Thus, more persistent stock supply reduces the *sensitivity* of the future stock return with respect to the fundamental, making information acquisition less attractive. Third, it also depends on the precision of exogenous signals, as this affects how aggressively uninformed investors trade. When the exogenous signals are very precise, varying the share of informed investors would not have a substantial effect on the loading coefficients of the stock fundamental as both types of investors trade equally aggressively on fundamental information. This reduces the strength of dynamic complementarity.

In general, the value of information $\mathcal{I}$ not only depends on the information gain ΔV but also on the *level* of conditional volatility faced by the informed investors V^I:

$$\mathcal{I} = e^{-\alpha R \chi}\sqrt{\frac{V^U}{V^I}} = e^{-\alpha R \chi}\sqrt{1 + \frac{\Delta V}{V^I}}.$$

This introduces an additional term in the complete characterization of the value of information, as stated by the following theorem:

Theorem 1 *Fix a financial market equilibrium $j = h, l$:*

$$\frac{d\mathcal{I}}{d\lambda} > 0 \text{ for } \lambda \text{ sufficiently small.}$$

if and only if

$$\left(1 - \frac{Var(F|\Omega^U)}{\sigma_D^2} - \frac{Var(F|\Omega^U)}{\sigma_S^2}\right)\rho^F - \rho^x + \phi^j > 0$$

where $Var(F|\Omega^U)$ is the steady-state conditional volatility of the stock fundamental given by Proposition 1.1 taking $\lambda = 0$. Parameter ϕ^j depends on the type of financial market equilibrium $j = h, l$.

The additional term ϕ^j generally depends on the type of financial market equilibrium. This is because the volatility introduced by noise trading enters into the uncertainty faced by informed investors V^I. This leads to interesting interactions between the financial market equilibrium and information market equilibrium. As the focus of this paper is on dynamics, interested readers are referred to the previous version of this paper for results in this regard.

References

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