Public Disclosure, Private Information Acquisition, and Complementarity: A Global-Games Approach

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Abstract

I study the effect of public information disclosure in a market setting where private information acquisition exhibits strategic complementarity. To overcome the issue of equilibrium multiplicity, I introduce heterogeneous information cost and imperfect information on the cost distribution. The resulting unique equilibrium features nonlinear responses to information disclosure. In particular, the classic “crowding-out” result can be reversed and public disclosure “crowd in” more private information acquisition. This effect is most prominent when there is high uncertainty about economic fundamental. The theory predicts that public disclosure of intermediate precision (neither too precise nor too vague) is most effective in stimulating private information acquisition.

Keywords: Information Disclosure; Information Acquisition; Dynamic complementarity; Global Games

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1 Introduction

Public information release has always been a crucial component of modern public policy conduct. Since the 2008 Crisis, financial transparency was brought to the forefront and there are increased regulatory efforts, such as the Dodd–Frank Act of 2010, aiming at improving various aspects of disclosure qualities. On the other hand, private information acquisition activities are ubiquitous in modern economies, in particular financial markets. Thus, to study the impact of public disclosure it is important to understand how it interacts with private information acquisition activities.

Conventional models of information disclosure are mostly based on classic noisy rational expectation models (Grossman and Stiglitz, 1980; Hellwig, 1980), where information acquisitions are strategic substitutes due to market learning. Building on this framework, the literature (e.g. Diamond, 1985; Bushman, 1991; Lundholm, 1991) generally find that public information disclosure “crowds out” private information production. Since then, a separate and growing literature studies rich strategic interactions in private information markets. This literature finds that strategic complementarity in information acquisition arises due to various reasons. What, then, is the implication of public disclosure when the private information market features interactions that render information strategic complements? What additional insights can we gain from such an environment?

This paper fills the gap by analyzing the effect of public information disclosure in a market setting where private information acquisition exhibits strategic complementarity. A major challenge is that the information complementarity typically leads to equilibrium multiplicity. This raises issues related to equilibrium selection and comparative statics. Moreover, it is generally difficult to extend noisy rational expectation models while maintain its tractability. In this paper I propose a tractable method to apply global game techniques to noisy rational expectation models with endogenous information markets. For that purpose, I introduce a small amount of heterogeneity in agents’ information cost and endow them with private signals about the cost distribution. The Gaussian-linear structure remain valid due to a separatability property: because there is no wealth effect in the utility function, information costs only affects the agents’ information choices, but drop out of the equation when investors make their portfolio choices. This implies that the linear-Gaussian structure is preserved.

I apply this technique to a multi-period extension of Grossman and Stiglitz (1980) in which

1For instance, information complementarity can arise because of increasing returns in the information sector (Veldkamp, 2006), private information on endowment (Ganguli and Yang 2009), relative wealth concerns (García and Strobl 2011), non-normal distribution (Breon-Drish 2015), multiple sources of information (Goldstein and Yang 2015), Knightian uncertainty (Mele and Sangiorgi 2015) and dynamic coordination (Chamley 2007; Avdis 2016).

2An isomorphic assumption is that different investors face different funding costs and they have private information on the distribution of borrowing costs.
information complementarity arises. The model can be thought of as the Grossman and Stiglitz (1980) with an additional round of trade conducted by newly arrived investors. Investors are short-lived and the first-generation of investors are allowed to acquire information: i.e. observe the asset fundamental at a cost. Their information choices exhibits strategic complementarity due to a dynamic feedback effect: As more agents get informed, current price signal becomes a more precise indicator of asset fundamental. This raises the information content of future resale asset prices as future investors now observe a more precise (period-1) price signal. The more informative resale stock price feeds back into today’s value of information, inducing more agents to acquire information. I adopt this model as the baseline because: 1) This framework admits a very tractable expression for the value of information and 2) the complementarity in information choice is not induced by complementarity in actions (Hellwig and Veldkamp, 2009) and thus, conditional on information choice there is a unique financial market equilibrium and 3) the two-period framework enables me to draw a close comparison to related works by Kim and Verrecchia (1991), Demski and Feltham (1994), and McNichols and Trueman (1994).

I start the analysis with the case where there is complete information and no cost heterogeneity. I first show that, due to the dynamic complementarity in information acquisition, the value of information can be upward-sloping with respect to the share of informed investors. This leads to equilibrium multiplicity for appropriate information costs. I then conduct comparative statics analysis for each of these equilibria. Surprisingly, public disclosure always crowds out private information acquisition regardless of which equilibrium one selects. This result is nontrivial because the local slope of the value of information is different across different equilibria. In one equilibrium, the value of information is downward-sloping. Public disclosure reduces the value of information, thus crowds out private information acquisition, this corresponds to the conventional theory of public disclosure as in Diamond (1985). In another equilibrium, the value of information is upward-sloping. Here public disclosure increases the value of information. But because the value of information is upward-sloping, an upward-shift reduces the equilibrium share of informed investors. Thus the result is always crowding out.

Given that the crowding-out result is robust across different types of equilibria, does this imply that information complementarity deliver same prediction as information substitutability? No. Note that we obtain the results fixing a certain equilibrium. This effectively assumes away the link between coordination in the information market and economic primitives. To explore this, I assume that there is a small amount of heterogeneity in information cost and that there is private information regarding the cost distribution. In this environment. I derive conditions under which there exists a unique symmetric equilibrium where private agents follow a threshold strategy:

\footnote{This dynamic complementarity in information acquisition is not new and has been explored in papers by Chamley (2007), Avdis (2016) and Cai (2018).}
they choose to acquire information if and only if their private information cost is below a certain threshold. Having established existence and uniqueness, I then explore how public information disclosure affects private information acquisition.

The main finding of the paper is that this refined equilibrium features crowding-in: more public information increases the private incentives to acquire information. The parameter space is sharply divided into two regions: with sufficiently high fundamental uncertainty, no one acquires information; with relatively low uncertainty, everyone acquires information. There exists an intermediate region where abrupt changes occur: the share of informed investors jumps from zero to one. Thus, the model features nonlinear responses to public disclosure and in particular, “crowding-in” occurs when the level of public information precision passes through the intermediate region.

The “crowding-in” result hinges on two elements. First, the possibility of coordination failure in the information market is linked to fundamental uncertainty. In the absence of this link, the equilibrium would always feature “crowding-out” as in the complete-information case. Second, the value of information is increasing in the amount of available public information. This, together with the first element, implies that more precise public information makes coordination easier to achieve in the information market, hence inducing more investors to acquire information. This “state monotonicity requirement” (Morris and Shin, 2003) is achieved through a dynamic feedback channel: public disclosure provides valuable information not only to current investors, but also to future investors. As a result future investors trade more aggressively in the financial market, making future resale stock price more sensitive to fundamental. This raises the value of information today. Note that the classic static substitutability effect still presents: holding fixed future resale stock price sensitivity, a more precise public signal per se lowers the value of information. Whether public information crowds in or crowds out private information gathering depends crucial on whether the dynamic feedback effect dominates.

I then explore conditions under which the dynamic feedback channel dominates and thus, the crowding-in result holds. I find that higher fundamental uncertainty leads to stronger dynamic feedback. When there is sufficiently high uncertainty regarding economic fundamental, informed trading almost freeze due to investor risk aversion. This is the situation where public disclosure has its most impact on stimulating future informed trading: a little bit extra information could substantially reduce the uncertainty faced by future investors, inducing them to trade more aggressively and increasing the information content of future resale asset price by a large margin. On the other hand, when fundamental uncertainty is sufficient low, static substitutability dominates and

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4When I shut down the dynamic feedback channel by forcing the second-generation investors to “ignore” public signal, the classic conclusion holds despite the fact that there is still information complementarity: public information disclosure always crowds out private information production. See section 3.2.1 for details.
the value of information is monotonically decreasing. The resulting unique equilibrium features crowding out. This implies that the effect of disclosure is not uniform in the parameter space: crowding-in occurs only when there is a sufficiently high degree of fundamental uncertainty.

The theoretical findings of the paper has substantive implications. It speaks to the recent policy debate on the effectiveness of public disclosure by focusing on a particular aspect: its interaction with private information activities. It predicts that information disclosure is most effective when the market is experiencing high uncertainty, because in that case more public information could stimulate even more private information production, countering heightened uncertainty. The theory also predict that the effect of disclosure is not uniform: crowding-out with low uncertainty and crowding-in with high uncertainty. Thus for a regulator aiming at disclosing certain information in a most efficient and cost-effective manner, it is desirable to maintain the precision of public signal at some intermediate region (neither too precise nor too vague), so that private agents are most active in information acquisition activities.

**Related Literature**

The paper is related to three different literature: the literature on information complementarity, on information disclosure, and on global games. First of all, it draws on models with information complementarity. The innovation of this paper is that it develops a tractable global-game technique to study unique predictions in such models. This technique is general and can readily be applied to other models in the noisy rational expectation class. A related paper is by Chamley (2007), who also applies global game techniques to study complementarity in information markets. His model is built on Glosten and Milgrom (1985) and thus the technique is not directly applicable to this class. Moreover, Chamley (2007) does not explore regime switches in response to fundamental changes (e.g. public disclosure), which is the focus of this paper.

Second, it is related to the literature on disclosure. The contribution of this paper is that it illustrates how information complementarity in private information market can overturn the classic crowding-out result (Diamond, 1985). Relatedly, McNichols and Trueman (1994) studies a multi-period trading model with short-term investors. They find that crowding-in can arise when public disclosure happens after private information acquisition. This timing difference is not required here. The crowding-in result of this paper is due to interactions of the two forces: 1) global game forces that link fundamental uncertainty to coordination failure and 2) dynamic feedback channel that makes the value of information increasing in public information. It is also the first model that illustrates the state-dependence nature of information disclosure: crowding-in occurs only when fundamental uncertainty is sufficiently high.

Lastly, the paper relates to the global game literature from which it borrows a number of insights.
and techniques. The key result that departing from common knowledge may restore uniqueness in coordination games stems from the seminal articles of Carlsson and van Damme (1993) and Morris and Shin (1998). I consider an application to information acquisition in financial markets. In this application, agents’ payoff depends nonlinearly on economic fundamentals through general equilibrium forces (instead of assumed in a reduced form way), and I characterize conditions under which global game techniques can be applied.

2 Model

The model can be thought of as the Grossman and Stiglitz (1980) with an additional round of trade conducted by newly arrived investors. There are three dates \( t = 0, 1, 2 \). There is a stock of fixed supply, which pays out a dividend \( D_t \) at date \( t = 1 \) and \( t = 2 \). The dividend stream consists of a persistent component \( F \) and a noise component \( \epsilon^D_t \)

\[
D_t = F + \epsilon^D_t.
\]

The persistent component \( F \) is the asset fundamental and will be priced in equilibrium. The supply of the stock is normalized to 1. There is also a bond of perfectly elastic supply, which delivers return \( R \) across consecutive periods.

In the beginning of period 1, generation-1 agents are born with certain amount of wealth, in the form of bonds and stocks. They don’t directly observe the value of \( F \) but rather are endowed with a noisy public signal of \( F \):

\[
S = F + \epsilon^F.
\]

where the noise \( \epsilon^F \) is unbiased and has variance \( \sigma^2_F \). The coefficient \( \sigma^2_F \) captures the strength of public disclosure. Later when we conduct comparative statics exercises, we vary \( \sigma^2_F \) as a proxy for public information disclosure. The agents are then offered an opportunity to acquire information about the true value of \( F \) at some cost \( \chi \). Investors who choose to purchase this information are labelled “informed” and otherwise “uninformed”. I assume that information, once purchased, cannot be redistributed to others due to copyrights considerations. Thus each agent, if he would like to know the true value of fundamental, has to purchase the information on his own.

In the beginning of period-1, the financial market opens and generation-1 investors, both informed and uninformed, engaged in trading. There is also a group of noise trades whose demand is denoted by \( x_1 \), which is a normally distributed random variable with mean 0 and variance \( \sigma^2_x \). Without noise trading, stock price would become full-revealing and no equilibrium could exist with positive
information cost. The information set for the uninformed investors is:

$$\Omega_1^U = \{S, P_1\}.$$ 

The information set for the informed investors is:

$$\Omega_1^I = \{S, F, P_1\}.$$ 

Where $P_1$ denotes the equilibrium stock price in the first round of trading. After the trading stage, dividend $D_1$ gets delivered and that concludes period 1.

In the beginning of period 2, the second generation of investors are born. For simplicity I assume that there is no information acquisition choice available to them.\(^5\) Then the financial market opens again and both generation-1 and generation-2 of investors engage in trading. Again there is a group of noise traders with demand $x_2$. Both $x_1$ and $x_2$ are normally distributed with variance $\sigma_x^2$. I assume that noise trading is serially uncorrelated for exposition purposes.\(^6\) Period-2 investors they observe the price history as well as the public signal:

$$\Omega_2 = \{S, P_1, P_2\}.$$ 

Where $P_2$ denotes the equilibrium stock price in period 2. After trading, dividend $D_2$ gets distributed and that concludes period 2 as well as the world. As standard in the literature, all investors are endowed with Constant-Absolute-Risk-Aversion (CARA) utility. As the utility function displays no wealth effect, the mean of all variables does not matter when computing equilibrium price functions. Thus we normalize $\mu_F$ to 0 and asset supply to 0.

A number of simplifying assumptions are made. For instance, investors are short-lived; asset fundamental is time-invariant; noise trading is serially uncorrelated; period-2 investors only observe price history but not past dividends; information acquisition is only allowed in period 0. All of these simplifying assumptions are relaxed in various ways in Avdis (2016) and Cai (2018). These assumptions are made to guarantee that we obtain a relatively transparent expression for the value of information and are not substantial to the existence of information multiplicity.

2.1 Information Complementarity

In this section I characterize the equilibrium and illustrate the source of information complementarity. Note that if the second period resale stock price $P_2$ is zero, the problem faced by generation 1 investors would be exactly the same as in Grossman and Stiglitz (1980). And thus there would  

\(^5\)See Cai (2018) for a version with repeated information acquisition and information complementarity.  
\(^6\)When stock supply is persistent, information complementarity is less likely to arise (Avdis, 2016).
be a unique equilibrium. The source of complementarity and equilibrium multiplicity comes from the feedback effect across two periods. As more investors get informed, first-period stock price becomes more informatives. This provides more information to the generation-2 investors, making them trade more aggressively in the second period. As a result, the resale stock price $P_2$ becomes more sensitive to stock fundamental and less sensitive to noise trading, raising the value of information today.

To characterize the equilibrium, let’s work backwards. In period 2, the second-generation investors are born, and they observe the first period price signal, defined as:

$$S_{P_1} = \theta_1 F - x_1$$

We focus on equilibrium where asset prices are linear functions of fundamental and noise trading. $\theta_1$ is the ratio of fundamental sensitivity and noise sensitivity in the first-period pricing function, and is a key endogenous object. It measures how informative first-period price signal is. Observing $P_1$ and public signal $F$, the posterior variance of fundamental $F$ is

$$Var(F|S,P_1) = \frac{1}{\theta_1^2 + \frac{\sigma_x^2}{\sigma_F^2}}, \quad (2.1)$$

and the posterior mean of $F$ is

$$E(F|S,P_1) = \frac{\theta_1^2}{\theta_1^2 + \frac{\sigma_x^2}{\sigma_F^2}} \left( F - \frac{1}{\theta_1} x_1 \right)$$

Thus, the asset demand by the second generation investors is

$$\frac{E(D_2|S,P_1) - RP_2}{\alpha Var(D_2|S,P_1)} = \frac{E(D_2|S,P_1) - RP_2}{\alpha Var(D_2|S,P_1)} - \frac{1}{\theta_1^2 + \frac{\sigma_x^2}{\sigma_F^2}} \left( F - \frac{1}{\theta_1} x_1 \right) - RP_2$$

$$= \frac{\alpha (Var(F|S,P_1) + \sigma_F^2)}{\alpha Var(D_2|S,P_1)}$$

Where the second equation holds because $D_2 = F + \varepsilon^D_2$. Substitute the asset demand equation into the second-period market clearing condition:

$$\frac{E(D_2|S,P_1) - RP_2}{\alpha Var(D_2|S,P_1)} = x_2$$
Given the expression of $P_2$, we move to the first period. The return to generation-1 investors includes dividend $D_1$ and capital gain $P_2$. Denote the total return $Q_1$:

\[
Q_1 = D_1 + P_2
\]

\[
= F + \varepsilon_{1}^{D} + \frac{1}{\frac{1}{\sigma_{P}^{2}} \sigma_{1}^{2} + 1 + \frac{1}{R\theta_{1}}} \left( F - \frac{1}{\sigma_{1}} x_{1} \right) - \alpha \left( \frac{1}{\frac{1}{\sigma_{P}^{2}} \sigma_{1}^{2} + \sigma_{D}^{2}} \right) x_{2}
\]

\[
= F + \varepsilon_{1}^{D} + M(\theta_1)S_{P1} - \alpha \frac{C(\theta_1)}{R} x_{2}
\]

Where

\[
M(\theta_1) = \frac{1}{\frac{1}{\sigma_{P}^{2}} \sigma_{1}^{2} + 1 + \frac{1}{R\theta_{1}}}
\]

\[
C(\theta_1) = \frac{1}{\frac{1}{\sigma_{P}^{2}} \sigma_{1}^{2} + \sigma_{D}^{2}}
\]

Thus, for the informed investors,

\[
E(Q_1|S,F,P_1) = F + M(\theta_1)S_{P1}
\]

\[
Var(Q_1|S,F,P_1) = \sigma_{D}^{2} + \left[ \frac{\alpha}{R} C(\theta_1) \right]^{2} \sigma_{x}^{2}
\]

For uninformed investors, they also need to forecast the value of $F$ given the public and price signal. Thus

\[
Var(Q_1|S,F,P_1) = \frac{1}{\frac{1}{\sigma_{P}^{2}} \sigma_{1}^{2} + \sigma_{D}^{2} + \left[ \frac{\alpha}{R} C(\theta_1) \right]^{2} \sigma_{x}^{2}}
\]

Next we move to the first-period market clearing condition. Denote $\lambda$ the share of informed investors. Then the first-period market clearing implies that

\[
S_{P1} = \theta_1 F - x_1 = \lambda \frac{F}{\alpha Var(Q_1|S,F,P_1)} - x_1
\]

Thus, we have the following characterization for the financial market equilibrium:
Proposition 2.1 \[\text{[Financial Market Equilibrium]}\] Holding fixed the information acquisition stage \((i.e. \text{holding fixed } \lambda)\), the financial market equilibrium \(\theta_1\) can be solved from the following equation:

\[
\theta_1 = \frac{\lambda}{\alpha (\sigma_D^2 + \left[\frac{\alpha}{\bar{R}} C(\theta_1)\right]^{2} \sigma^2)}
\]

(2.6)

where \(C(.)\) is given by equation 2.5.

The advantage of this approach is that \(\theta_1\) is a monotonic transformation of \(\lambda\). Thus, to explore the impact of varying \(\lambda\), we just need to look at the impact of varying \(\theta_1\).

Proposition 2.2 \(\theta_1\) is monotonically increasing in \(\lambda\).

Next, we move to the information acquisition stage. Since agents are short-lived as in Grossman and Stiglitz (1980), the value of information, defined as the ratio of expected utility between informed and uninformed, is proportional to the ratio of stock return volatility. Denote the value of information \(V\). We have the following property:

\[
V = \frac{\text{Var}(Q_1|S,P_1)}{\text{Var}(Q_1|S,F,P_1)}
\]

Substitute in the expressions for the volatility terms, we obtain:

Proposition 2.3 The value of information \(V\) is given by:

\[
V(\lambda) = \frac{\frac{1}{\sigma_F^2 + \sigma^2} + \sigma_D^2 + \left[\frac{\alpha}{\bar{R}} C(\theta_1)\right]^{2} \sigma^2}{\sigma_D^2 + \left[\frac{\alpha}{\bar{R}} C(\theta_1)\right]^{2} \sigma^2}
\]

Where \(\theta_1\) is an implicit function of \(\lambda\) given by equation 2.6

An overall equilibrium is solved whenever the value of information is equated to the cost of information acquisition (unless at boundary):

Proposition 2.4 \(\lambda \in (0,1)\) is an equilibrium in the information market if and only if

\[
V(\lambda) = e^{\alpha R \chi};
\]

\(\lambda = 0(1)\) is an equilibrium if and only if

\[
V(\lambda) \leq (\geq) e^{\alpha R \chi}.
\]
The slope of the value of information can be upward-sloping because a higher value of $\theta_1$ implies more information available to future investors. As a result, future investors trade more aggressively, which reduces the sensitivity of future resale stock price with respect to noise: $C(\theta_1)$ decreases. This reduces the denominator in the value of information expression and thus raises return to acquire information. As illustrated in figure 1, the value of information can be nonlinear with respect to the share of informed investors $\lambda$. Thus, with appropriate information cost (In this case $e^{\alpha R_x} = 1.51$) there exists three equilibrium: two interior equilibrium and a boundary equilibrium where $\lambda = 0$.

### 2.2 The Impact of Information Disclosure under Complete Information

Next, I study how information disclosure affects private information acquisition, holding fixed the type of equilibria. Typically, one might expect that the comparative statics are different across different types of equilibria. This is not the case here, as shown in figure 2. Both interior equilibria feature crowding-out: public disclosure ($\sigma^2_F$ decreases from 1 to 0.95) reduces the equilibrium share of informed investors. This result can be understood as follows. In one equilibrium, the value of information is downward-sloping. Public disclosure reduces the value of information, thus crowds out private information acquisition, this corresponds to the conventional theory of public disclosure as in Diamond (1985). In another equilibrium, the value of information is upward-sloping. Here public disclosure increases the value of information. But because the value of information is upward-sloping, an upward-shift reduces the equilibrium share of informed investors. Thus the result is always crowding out.
In fact, we can prove a stronger version of the result where public disclosure completely crowds out private information acquisition, leaving posterior fundamental uncertainty unchanged. Write down the expression for the value of information, plug in the future sensitivity function $C(.)$:

$$V(\lambda) = \frac{1}{\sigma_F^2 + \sigma_x^2} + \sigma_D^2 + \left[ \frac{\alpha}{\mathcal{R}} \left( \frac{1}{\sigma_F^2 + \sigma_x^2} + \sigma_D^2 \right) \right]^2 \sigma_x^2$$

Inspecting this equation reveals that the value of information depends on endogenous variables only through the posterior variance of stock fundamental (equation 2.1). For brevity denote the variance $\nu_F$:

$$\nu_F = \text{Var}(F|S,P_1) = \frac{1}{\sigma_F^2 + \sigma_x^2}$$

And we can write the value of information only as a function of $\nu_F$:

$$V = \frac{\nu_F + \sigma_D^2 + \left[ \frac{\alpha}{\mathcal{R}} (\nu_F + \sigma_D^2) \right]^2 \sigma_x^2}{\sigma_D^2 + \left[ \frac{\alpha}{\mathcal{R}} (\nu_F + \sigma_D^2) \right]^2 \sigma_x^2}.$$  \hspace{1cm} (2.8)

The message from this equation is that any change in prior uncertainty $\sigma_F^2$ and price informativeness $\theta_1$ affects the value of information only through the posterior variance $\nu_F$. Thus, at any interior equilibrium, posterior variance is pinned down by the equilibrium relation that the value of information is equal to the information cost:

$$V = e^{\alpha R\chi}$$

Thus, the effect of disclosure will be fully offset by a reduction in private information gathering, leaving the posterior variance of stock fundamental unchanged. We summarize it into the following proposition:

**Proposition 2.5** Fix any interior equilibrium in the complete-information model. Then information disclosure crowds out private information acquisition:

$$\frac{\partial \lambda}{\partial \sigma_F^2} > 0, \frac{\partial \theta_1}{\partial \sigma_F^2} > 0$$
Its impact on posterior fundamental uncertainty is exactly offset by the crowding-out effect.

\[
\frac{\partial \nu_F}{\partial \sigma^2_F} = 0
\]

This result extends the original Grossman and Stiglitz (1980) result to a multi-period setting. A more general result can be found in Cai (2018) where it is derived in a standard infinite-horizon framework similar to Wang (1994). The crucial assumption for the “complete crowding-out” result is that the noise trading shock is serially uncorrelated. In case the noise trading \( x_t \) is persistent, the crowding-out effect would be less dramatic but nevertheless all equilibria would still feature crowding-out.

In the next section I will show that such prediction is not robust to a minimal perturbation of the structure of the economy. In particular, with introduction of a small heterogeneity in information cost and private information on the cost distribution, the resulting equilibrium would instead feature “crowding-in” effect of public information disclosure. The reason behind this difference is that, the complete-information model ignores the possibility that public disclosure could affect the strength of coordination across agents. This force is absent once we fix an equilibrium. By applying the global game technique, we link strategic coordination across agents to economic primitives, and this delivers new insights into the issue.
3 Cost Heterogeneity and Incomplete Information

Now imagine that agents’ information costs are heterogeneous and are drawn from a type space $\Phi$ which is a uniform distribution with mean $\mu_\chi$ and variance $\sigma_\chi^2$. Denote the cumulative distribution function $F^{\mu_\chi}(\cdot)$. Upon birth, agents observe the variance of the cost distribution but not the mean. Thus they use their own realization of cost, denoted by $\chi_i$, to infer the entire cost distribution. In the absence of any public signal, an agent with a realization of cost $\chi_i$ believes that the information cost is distributed with mean $\chi_i$:

$$\chi|\chi_i \sim U(\chi_i, \sigma_\chi^2) \quad (3.1)$$

Based on this private information, the agent then decide whether or not to acquire information.

The market trading stage in period 1 and period 2 remains unchanged. the only modeling change is that the equilibrium share of informed investors $\lambda$ is publicly observable in the beginning of period 1. That is:

$$\Omega_1^U = \{\lambda, S, P_1\}.$$
$$\Omega_1^{U'} = \{\lambda, S, F, P_1\}.$$
$$\Omega_2 = \{\lambda, S, P_1, P_2\}.$$

Without incomplete information this assumption is not needed as agents can rationally infer the equilibrium share $\lambda$ from the publicly observed cost distribution. This assumption is required here because otherwise agents will form posterior beliefs about $\lambda$ from observing the equilibrium price signal and this implies that the Gaussian-linear framework breaks down. The value of information expression $V(\lambda)$, conditional on information acquisition stage, is still the same because 1) imperfect information is resolved as the equilibrium share $\lambda$ is publicly known and 2) Agents’ utility display no wealth effect and therefore heterogeneous cost does not impact their portfolio choices and asset demands.\(^7\)

We can formulate the information acquisition stage as a symmetric binary-action Bayesian game. There is a continuum of players (investors) $i \in [0, 1]$. For a generic agent $i$, his payoff function depends on his own decision whether or not to acquire information $a_i$, the mass of agents acquiring information $\lambda$, and his information cost $\chi_i$. To analyze best responses, it is enough to know the payoff gain from choosing one action rather than the other. Thus we focus on the net value of information, defined as:

$$U(\lambda, \chi_i) = V(\lambda) - \exp(\alpha R\chi_i) \quad (3.2)$$

\(^7\)An equivalent way of formulating this problem is to assume that agents face different funding opportunities $R_i$ and they have private information about the distribution of funding costs. In essence, we need agents to have private information about the transformed information cost $\exp(\alpha R\chi_i)$. Note that we cannot use heterogeneous risk aversion, as it would change agents’ portfolio choices at the trading stage.
The net value of information measures the payoff gain from acquiring information, as it substracts the information cost from the value of information.

A symmetric Bayesian equilibrium is a strategy $s(.)$ which is a mapping from the type space $\Phi$ to the probability of acquiring information. Given others’ strategy $s(.)$, the share of informed investors $\lambda$ conditional on private state $\chi_i$ is

$$\lambda(\chi_i; s(.)) = \int \chi s(\chi) dF^{\chi_i}(\chi)$$  \hspace{1cm} (3.3)

where $F^{\chi_i}(\chi)$ is the posterior CDF of information cost from equation 3.1. Note that there is no uncertainty regarding the value of $\lambda$, as the law of large number washes out all the residual noises.

**Definition 3.1** A symmetric Bayesian equilibrium is a strategy $s : \Phi \rightarrow [0, 1]$ such that this strategy is optimal if others follow the same strategy:

$$s(\chi_i) \begin{cases} 
= 1, & \text{if } U(\lambda(\chi_i; s(.)), \chi_i) > 0. \\
\in [0, 1], & \text{if } U(\lambda(\chi_i; s(.)), \chi_i) = 0. \\
= 0, & \text{if } U(\lambda(\chi_i; s(.)), \chi_i) < 0.
\end{cases}$$  \hspace{1cm} (3.4)

Where function $U(.)$ is the net value of information given by equation 3.2 and function $\lambda(.)$ is the agents’ posterior belief of the share of informed investors given his individual state and the strategies of other players (equation 3.3).

### 3.1 Monotone Equilibrium: Existence and Uniqueness

In this section I accord with the global game literature and focus on symmetric monotone equilibria in which agents follow a threshold strategy: they choose to acquire information if and only if their information cost $\chi$ is below certain threshold $\bar{\chi}$. I will first sketch the idea and then derive conditions under which such an equilibrium exists and is unique. For an agent with realization of cost $\chi_i$, if everyone else in this economy is following this cutoff strategy, his net value of information is

$$U(\lambda(\chi_i), \chi_i) = V(F^{\chi_i}(\bar{\chi})) - \exp(\alpha R \chi_i)$$

The $F^{\chi_i}(\bar{\chi})$ is the expected share of informed investors given that all others are following the $\bar{\chi}$ threshold strategy and the cost distribution has mean $\chi_i$.

Given any threshold $\bar{\chi}$, one can compute the marginal $\chi_i$ above which the net value of information is negative and below which the net value is positive. This marginal $\chi_i$ satisfies:

$$V(F^{\chi_i}(\bar{\chi})) - \exp(\alpha R \chi_i) = 0$$
This defines an implicit mapping \( \chi_i(\bar{\chi}) \), which is a mapping from market belief to market outcome. Once we have this mapping, we look for a fixed point:

\[
\chi_i(\bar{\chi}) = \bar{\chi}
\]

This threshold \( \bar{\chi} \) represents the equilibrium threshold strategy that agents will follow. Thus equilibrium share of informed investors is

\[
\lambda = F^{\mu_x}(\bar{\chi})
\]

Depending on model parameters, the information game may not have a monotone equilibrium. Next, I state a condition that guarantees the existence and uniqueness of such equilibrium.

**Condition 1**

\[
\frac{1}{\sigma_F^2 + \frac{\sigma_2^2}{\sigma_x^2}} > \sqrt{\left( \frac{R\sigma_D}{\alpha\sigma_x} \right)^2 + \sigma_D^4}
\]

Where \( \hat{\theta}_1 \) is the price informativeness when all investors are informed: \( \lambda = 1 \). This \( \hat{\theta}_1 \) is characterized by the following equation:

\[
\hat{\theta}_1 = \frac{1}{\alpha \left( \sigma_D^2 + \left( \frac{\alpha}{\pi} C \left( \hat{\theta}_1 \right) \right)^2 \sigma_x^2 \right)}
\]

where function \( C(.) \) is given by 2.5.

Condition 1 guarantees *global strategic complementarity*, as shown in the following proposition:

**Proposition 3.1** Under condition 1, the value of information \( V \) is monotonically increasing in the share of informed investors.

The proof of proposition 3.1 consists of three parts: first, an increase in the share of informed investors \( \lambda \) leads to an increase in the price informativeness \( \theta_1 \). This is guaranteed by proposition 2.2. Second, an increase in \( \theta_1 \) leads to a decrease in posterior variance \( \nu_F \). This is trivial given equation 2.7. The last step is to show that, under condition 1, the value of information is decreasing in the posterior variance \( \nu_F \). This is done by taking derivatives with respect to the value of information expression in equation 2.8:

\[
\frac{\partial V}{\partial \nu_F} = \frac{\sigma_D^2 + \left( \frac{\alpha}{\pi} \sigma_x \right)^2 (\sigma_D^4 - \nu_F^2)}{\left( \sigma_D^2 + \left( \frac{\alpha}{\pi} \sigma_x \right)^2 (\nu_F + \sigma_D^2)^2 \right)^2}
\]
Thus, the sign of this derivative is determined by the term

\[ \sigma_D^2 + \left( \frac{\alpha}{R} \sigma_x \right)^2 \left( \sigma_D^4 - \nu_F^2 \right) \]

For the value of information to be upward sloping, we need this term to be negative. This implies:

\[ \nu_F > \sqrt{\left( \frac{R \sigma_D}{\alpha \sigma_x} \right)^2 + \sigma_D^4} \]

Plug in the expression of \( \nu_F \) (equation 2.7), and observe that the lower bound of the left hand side is obtained when \( \theta_1 \) is the biggest, i.e. when \( \lambda = 1 \), we arrive at condition 1.

The interpretation of condition 1 is that the fundamental uncertainty \( \sigma_F^2 \) needs to be sufficiently high. To see this, consider two extremes. When uncertainty vanishes \( \sigma_F^2 \to 0 \), the left hand side of condition 1 tends to zero whereas the right hand side is bounded as long as the noise in the dividend does not vanish. Thus, the condition is violated.\(^8\) When fundamental uncertainty is sufficiently high, the left hand side tends to infinity whereas the right hand side is unaffected. Thus this condition is satisfied.

Next, we show that condition 1 implies that the value of information is increasing in the public information precision. In fact, we are able to prove a stronger statement that there exists an upper dominance region and a lower dominance region: when fundamental uncertainty is sufficiently low, everyone chooses to acquire information and vice versa:

**Proposition 3.2** Under condition 1, the following two statements are true:

1. (**State Monotonicity**) The value of information is increasing in public information precision (or equivalently decreasing in \( \sigma_F^2 \)).

2. (**Dominance Region**) For appropriate information cost \( \chi \), there exists a lower bound \( \sigma_F^2 \) and an upper bound \( \bar{\sigma}_F^2 \) such that for all \( \sigma_F^2 < \sigma_F^2 \), all investors choose to acquire information: \( \lambda = 1 \) and for all \( \sigma_F^2 > \bar{\sigma}_F^2 \), no investors choose to acquire information: \( \lambda = 0 \). Both the upper dominance and the lower dominance region are nonempty.

The proof of this proposition hinges on the crucial observation that the value of information \( V \) tends to 1 when fundamental uncertainty \( \sigma_F^2 \) tends to infinity. Thus, we can construct the bounds and the dominance regions as follows (Figure 3): first fixed a lower bound \( \sigma_F^2 \) that satisfies condition 1 (blue curve depicts the value of information conditional on \( \sigma_F^2 \)). This guarantees that the lower

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\(^8\)If fact, when fundamental uncertainty is sufficiently low, there is global strategic substitutability in the information market and the value of information is monotonically decreasing.
dominance region is nonempty. Given $\sigma^2_F$, pick a strictly positive but sufficiently small information cost $\chi$ such that the value of information given $\sigma^2_F = \sigma^2_F$ is always above the information cost (the red flat curve in figure 3). This is achievable because the value of information at $\lambda = 0$: $V(0; \sigma^2_F = \sigma^2_F)$ is strictly greater than 1. Next, we pick $\bar{\sigma}^2_F$ such that the entire value of information function conditional on $V(0; \sigma^2_F = \sigma^2_F)$ is strictly less than the information cost (the yellow curve in figure 3). This is achievable because of the observation highlighted in the very beginning: value of information can be arbitrarily close to 1 for sufficiently large fundamental uncertainty.

Given the global strategic complementary, state monotonicity, and the existence of upper and lower dominance region, it is easy to show that the model satisfies the A.1 through A.5 conditions highlighted in Morris and Shin (2003) and thus we have the following result:

**Theorem 1** Under condition 1, the model has a unique equilibrium in which agents acquire information if and only if their information cost is below a certain threshold $\bar{\chi}$.

Note that condition 1 is sufficient but not necessary. When this condition is not met, it is possible that the value of information has a decreasing portion in which classic substitutability force dominates. The existence result still holds as long as the decreasing (substitutability) portion is bounded (Goldstein and Pauzner, 2005). To illustrate this, I plot the net value of information conditional on agents’ private realization of $\chi_i$, holding fixed $\bar{\chi}$. For monotone strategy to be valid,
the net value of information

\[ V(F^{\chi_1}(\bar{\chi})) - \exp(\alpha R_{\chi_i}) \]

needs to satisfy a “single crossing property”: it crosses the zero horizontal line only once. The blue curve plots the net value of information when condition 1 is met. Given that there is global strategic complementarity, it is easy to show that the net value of information is monotonically decreasing. The yellow curve depicts the case where fundamental uncertainty is very low and thus condition 1 is not met. The net value of information in this case has an increasing portion because of strategic substitutability. In this case, there is no pure strategy monotonic equilibrium. The intermediate case depicts the scenario where fundamental uncertainty is marginally lower than required by condition 1. There is a wiggle in the red curve as the value of information now features both complementarity and substitutability. However, the single crossing property is still satisfied and thus monotone strategy is still valid. In other words, as long as the substitutability region is far away from the cutoff point, we still have a unique equilibrium featuring monotone strategy.

### 3.2 Results

Having established equilibrium existence and uniqueness, we explore implications of public information disclosure in this equilibrium. The equilibrium response of private information market is depicted in figure 5. When fundamental uncertainty \( \sigma_F^2 \) is very low or very high, there is no
strategic uncertainty as the dominance strategy is to acquire (not acquire) information. Multiple equilibrium occurs in the intermediate region, including a mixed equilibrium (yellow curve). The unique equilibrium under global game refinement (red curve) transits smoothly from the lower dominance region to the upper dominance region. Thus, strategic coordination in the information market is linked to fundamental uncertainty. Higher fundamental uncertainty predicts less investors acquiring information, as it pushes equilibrium closer to the case of coordination failure ($\lambda = 0$). Conversely, public information disclosure could stimulate private information gathering, as investors are more likely to coordinate.

**Theorem 2** As $\sigma_\chi^2 \to 0$, there exists a cutoff $\sigma_F^2$ such that equilibrium share of investors is $1(0)$ if and only if $\sigma_F^2 < (>\sigma_F^2$. Thus, the equilibrium features “crowding-in”: lowering $\sigma_F^2$ induces more investors to acquire information.

As the noise in the cost distribution vanishes ($\sigma_\chi^2 \to 0$), the parameter space is divided into two regimes: one with high uncertainty and no investors acquire information and the other with low uncertainty and everyone acquires information (figure 6). Correspondingly, there is a region where posterior fundamental variance $\nu_F$ is highly sensitive to prior variance $\sigma_F^2$. This implies that the crowding-in effect is highly nonlinear: in most of the parameter space, private information acquisition is irresponsive to public disclosure; there exists a small region where private information acquisition is highly sensitive to fundamental uncertainty.
3.2.1 Dynamic Feedback

Lastly we argue that the dynamic feedback channel whereby public signal affects the value of information through the trading behavior of future investors is crucial. Without this feature, public disclosure would crowd out private information acquisition, despite the presence of information complementarity. To illustrate this, consider an alternative scenario where the second-generation investors do not observe the initial public signal $S$ but an alternative signal $S'$ of the same precision. Thus, their information set is:

$$\Omega_2 = \{\lambda, S', P_1, P_2\}.$$ 

This implies that public disclosure in $S$ signal does not affect the trading behavior of the future investors. Nonetheless, since future investors still observe previous price signal $P_1$, the dynamic complementarity in information acquisition still presents and this leads to multiple equilibria.

The critical difference is that, now the value of information is decreasing in the precision of public signal $S$, as it no longer affects the trading behavior of future investors. This can be seen in the value of information expression, where the precision of the public signal observed by future
investors is exogenous fixed to \( \text{Var}(S') = \Sigma \):

\[
V(\lambda) = \frac{1}{\sigma_F^2 + \sigma_x^2} + \sigma_D^2 + \left[ \frac{\alpha}{R} \left( \frac{1}{\Sigma + \sigma_F^2} + \sigma_D^2 \right) \right]^2 \sigma_x^2
\]

\[
\sigma_D^2 + \left[ \frac{\alpha}{R} \left( \frac{1}{\Sigma + \sigma_F^2} + \sigma_D^2 \right) \right]^2 \sigma_x^2
\]

Thus, varying \( \sigma_F^2 \) only reduces the first term in the numerator and therefore the value of information is monotonically decreasing in the public signal precision. This means that the state monotonicity property (proposition 3.2) is flipped: for sufficiently low fundamental variance, few people would like to acquire information and vice versa. Thus the refined equilibria features crowding-out, consistent with conventional wisdom. This case is numerically solved and plotted in figure 7.

The message from this exercise is that, to get the crowding-in effect it is very important to have some force that makes the value of information upward-sloping with respect to fundamental uncertainty. Otherwise we would still obtain crowding-out even if the model feature very strong strategic complementarity.
The dynamic feedback channel highlighted here has a similar flavor in early papers by Kim and Verrecchia (1991), Demski and Feltham (1994), and McNichols and Trueman (1994). In these papers, public disclosure occurs after the information acquisition decision. Thus, more precise public information could crowd in more private information acquisition as future prices reflect more fundamental. In this model, public disclosure happens at the same time as the time of private information acquisition and is observed to future investors. Thus, the crowding-in does not always happen because it depends on the endogenous tension between static substitutability and dynamic complementarity. As a result, the crowding-in effect is state-dependent: it only occurs when fundamental uncertainty is high and thus the dynamic force dominates. Last but not least, it also depends on the global game force that links fundamental uncertainty to coordination in the information market. In the absence of such a link, proposition 2.5 applies and all equilibria would feature crowding-out. It is the interaction of two forces: 1) global game forces that link fundamental uncertainty to coordination failure and 2) dynamic feedback channel that generate the crowding-in result.

3.2.2 State Dependence

The crowding-in result depends critically on the level of prior fundamental uncertainty. When fundamental uncertainty is relatively low (so that condition 1 is not met), both the global strategic complementarity and dominance region property break down. Thus we need to work with the situation where the value of information is potentially non-monotonic. In particular, when the strategic substitutability is sufficiently strong, monotone strategy is no longer valid (figure 4). We need instead look for mixed strategy equilibrium where agents’ strategy is a mapping from their
own types to a probability distribution over the action space. This is a challenging computational task. We thus focus on the case where fundamental uncertainty is sufficiently low, so that there is global strategic substitutability and, for even lower level of fundamental uncertainty, no one is willing to acquire information.

**Proposition 3.3** For sufficiently low level of fundamental uncertainty $\sigma_F^2$, the value of information is monotonically decreasing. For even lower levels of fundamental uncertainty, the value of information is uniformly below the cost of acquire information.

The proposition is illustrated in figure 8. When fundamental uncertainy is sufficiently low, both the blue curve and the red curve are monotonically decreasing. And for even lower fundamental uncertainty $\sigma_F^2 = 0.5$ (the red curve), it lies uniformly below the information cost. We will focus on characterizing this region. This has two advantages: first, for sufficiently low fundamental uncertainty (the red curve) there is no strategic interactions across agents as the dominance strategy

![Crowding-in v.s. Crowding-out](image.png)

Figure 9: Crowding-in v.s. Crowding-out
is to not acquire any information. Second, global strategic substitutability implies that we can find a pretty good initial guess for the agents’ strategy, which greatly facilitates computation. The result is presented in figure 9 where both the (conventional) crowding-out region and the crowding-in region are plot. Note that we do not compute the middle portion where the value of information is non-monotonic and the numerical solution to mixed strategy equilibrium is inaccurate. But the main message is conveyed in the figure. The impact of disclosure on private information acquisition is state-dependent. When uncertainty is relatively high (red curve), equilibrium features crowding-in; when uncertainty is relatively low (blue curve), equilibrium feature crowding-out. Due to the presence of both a crowding-out and crowding-in region, there exists an intermediate region of public information precision that maximizes private information acquisition activities. Thus, for a regulator aiming at disclosing certain information in a most efficient and cost-effective manner, it could be optimal to deliver a signal of intermediate precision (neither too precise nor too vague), so that private information market is most active.

4 Conclusion

This paper studies the impact of public information disclosure on private information acquisition, when the private information choices exhibit strategic complementarity. To overcome the issue of equilibrium multiplicity, I propose a tractable way of applying global games to the class of noisy rational expectation models with endogenous information choices. I find that the classic crowding-out result is overturned: public disclosure may crowd in private information acquisition when fundamental uncertainty is sufficiently high.

The general method developed in this paper can be applied to other models with information complementarity. It would be interesting to explore how the impact of public disclosure can be different with different sources of complementarities. Also, if one thinks of fundamental uncertainty as not only affected by disclosure but also inherited from the past, the mechanism explored in this paper — higher prior uncertainty leads to less agents acquiring information — can potentially be a propagating force of uncertainty shocks in financial markets. These are beyond the scope of this paper and we leave it for future research.
References


